A hint of AdS/CFT

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ABSTRACT: The aim of these lecture notes is to give a hint of the theoretical concepts underlying the AdS/CFT correspondence to the 2-3 year bachelor students. The knowledge of classical electrodynamics, theoretical mechanics with action principle and the classical gravity is assumed.

We introduce classical Abelian and non-Abelian gauge field theories, basic features of the string theory, including the spectra of open and closed strings. Then we turn to D-branes and construct the holographic duality between gravity on $\mathrm{AdS}_5 \times \mathrm{S}_5$ and large N SYM theory. We discuss the possible generalizations of the duality to less symmetric cases, including finite temperature and chemical potential and introduce the applications to Condensed Matter theory.

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1 Introduction

AdS/CFT duality (gauge/string duality, holography) is lying on a forefront of the modern theoretical physics. This deepest concept relies on an unexpected point of view which was provided by string theory and it allows to relate gauge filed theories and the theory of gravity in nontrivial way. This duality provides unique tools to address the dynamics of the quantum gauge theories in the regime of strong coupling, where the other approaches fail. On the other hand, it can unveil the properties of the quantum gravity, in case when the dual field theory is accessible to study. Among its applications are the treatment of the physics of hadrons and that of the strongly correlated quantum systems, like high temperature superconductors. Nowadays gauge/string duality is treated as a well developed set of rules which allows multitude of applications in diverse subjects of physics. However, knowledge of the origin and the underlying principles of the duality is crucial in order to use this powerful tool properly. The aim of these notes is to give a hint of these principles and provide a starting point for the education of this spectacular concept in the modern theoretical physics.

2 Gauge fields

2.1 Classical electrodynamics as Abelian gauge field theory

Let's start from the concept of gauge field theory. The familiar theory of electrodynamics is described by the field strength tensor [1]

$$F_{i0} = -F_{0i} = E_i \tag{2.1}$$

$$F_{ij} = \epsilon_{ijk} H_k. \tag{2.2}$$

Its equations of motion – the Maxwell equations, follow from the action principle

$$S = -\frac{1}{4} \int d^4x F_{\mu\nu} F^{\mu\nu}, \tag{2.3}$$

where the dynamical variables are the components of the vector potential

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{2.4}$$

The crucial property of the action of electrodynamics is its *gauge symmetry*: the action is symmetric with respect to the *local* (coordinate dependent) transformation of the vector potential of the form

$$A_\mu(x) \to A_\mu(x) + \partial_\mu \lambda(x). \tag{2.5}$$

The gauge invariance plays a fundamental role in the physics of electromagnetic interaction, since it is tightly related to the conservation of charge and restricts significantly the possible form of the equations of motion. Indeed, how to we couple the shift-symmetric field A_μ to matter, keeping the gauge symmetry intact? For this we need to construct a *charged* matter.

For a simple example consider the complex scalar field Φ with the action [2]

$$S \equiv \int d^4 \mathcal{L} \tag{2.6}$$

$$\mathcal{L} = -\partial^\mu \Phi^\dagger \partial_\mu \Phi - m^2 \Phi\Phi^\dagger + O[(\Phi\Phi^\dagger)^2] \tag{2.7}$$

This action is invariant under global (coordinate independent) phase rotations of the complex scalar

$$\Phi \to e^{I\alpha}\Phi \qquad \delta\Phi = i\delta\alpha\Phi \tag{2.8}$$

$$\Phi^\dagger \to e^{-I\alpha}\Phi^\dagger \qquad \delta\Phi^\dagger = -i\delta\alpha\Phi^\dagger \tag{2.9}$$

This is a $U(1)$-transformation group: the transformation which keeps an absolute value of a *single* complex number intact. The action (2.6) can only contain the terms which are invariant under this $U(1)$ symmetry. Therefore **the symmetry restricts the possible form of the action.**

What is a physical consequence of the global $U(1)$ symmetry in the action? The equations of motion coming from variation of the action (2.6) are

$$\frac{\delta S}{\delta \Phi^\dagger} = \frac{\partial \mathcal{L}}{\partial \Phi^\dagger} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \Phi^\dagger} \right) = 0 \tag{2.10}$$

On the other hand the variation of the Lagrangian reads [2](Ch.22)

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\Phi^\dagger}\delta\Phi^\dagger + \frac{\partial\mathcal{L}}{\partial\partial_\mu\Phi^\dagger}\delta\partial_\mu\Phi^\dagger + c.c. \tag{2.11}$$

Using the equations of motion we rewrite this as

$$\delta\mathcal{L} = \partial_\mu\left(\frac{\delta\mathcal{L}}{\partial(\partial_\mu\Phi^\dagger)}\delta\Phi^\dagger\right) + \frac{\delta S}{\delta\Phi^\dagger}\delta\Phi^\dagger + c.c \tag{2.12}$$

The term in parenthesis is called *the Noether current.* Its fundamental feature is that since on the solutions to the equations of motion the second term vanishes, in case when the Lagrangian has a *global* symmetry $\delta\mathcal{L}/\delta\alpha = 0$, Noether current is conserved:

$$\partial_\mu j^\mu = 0 \tag{2.13}$$

$$j^\mu = \frac{\delta\mathcal{L}}{\partial(\partial_\mu\Phi^\dagger)}\frac{\delta\Phi^\dagger}{\delta\alpha} + \frac{\delta\mathcal{L}}{\partial(\partial_\mu\Phi)}\frac{\delta\Phi^\dagger}{\delta\alpha} = i\left(\partial^\mu\Phi\Phi^\dagger - \partial^\mu\Phi^\dagger\Phi\right) \tag{2.14}$$

Rewriting this in terms of space and time components we arrive at the continuity equation for the electromagnetic current, where j^0 is a charge density.

$$\frac{\partial}{\partial t}j^0(x) + \nabla\cdot\boldsymbol{j}(x) = 0 \tag{2.15}$$

We see that **the global U(1) symmetry describes the matter with electric charge and ensures its conservation.**

Once we have a charged matter, we can couple it to the electromagnetic field. We write

$$S = \int d^4x\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + gA_\mu j^\mu - \partial^\mu\Phi^\dagger\partial_\mu\Phi - m^2\Phi\Phi^\dagger\right], \tag{2.16}$$

with g being the coupling constant. Now when we apply a gauge transformation (2.5), the Lagrangian changes by

$$\delta\mathcal{L}_\lambda = gj^\mu\partial_\mu\lambda, \tag{2.17}$$

however, if we simultaneously perform a *coordinate dependent* $U(1)$-transformation (2.8) with parameter $\alpha(x)$, then the kinetic term of the scalar field will result in the same extra term

$$\delta\mathcal{L}_\alpha = -\left(\frac{\delta\mathcal{L}}{\partial(\partial_\mu\Phi^\dagger)}\frac{\delta\Phi^\dagger}{\delta\alpha} + \frac{\delta\mathcal{L}}{\partial(\partial_\mu\Phi)}\frac{\delta\Phi^\dagger}{\delta\alpha}\right)\partial_\mu\alpha(x) \equiv -j^\mu\partial_\mu\alpha(x). \tag{2.18}$$

The action of the electrodynamics coupled to a charged matter remains gauge invariant if the transformations of the gauge field are linked to the simultaneous transformations of the matter field

$$A_\mu \to A_\mu + g\partial_\mu\lambda(x), \qquad \Phi \to e^{i\lambda(x)}\Phi \tag{2.19}$$

In this way the electrodynamics can be built as **U(1) gauge field theory**.

This whole construction gets even more transparent when one introduces the gauge covariant derivative:

$$D_\mu\Phi = \partial_\mu\Phi + igA_\mu\Phi, \tag{2.20}$$

which conveniently encapsulates the current and kinetic terms of the action

$$D_\mu\Phi D^\mu\Phi^\dagger = gA_\mu j^\mu - \partial^\mu\Phi^\dagger\partial_\mu\Phi \tag{2.21}$$

2.2 Non-Abelian gauge field theories

In the previous section we considered a $U(1)$ gauge field theory. As mentioned before, $U(1)$ is a group of transformation leaving the absolute value of a single complex number intact. The question arises: can we generalize the notion of the gauge field theory to a more complicated groups[1]?

Suppose now the scalar field is not a single complex number, but rather a set of 2 complex numbers with independent phases.

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \tag{2.22}$$

Now the global $U(1)$ transformation (2.8) is substituted by $SU(2)$ – the transformation which keeps the norm of the 2-component vector invariant.

$$\Phi \to e^{\alpha^1\sigma_1+\alpha^2\sigma_2+\alpha^3\sigma_3} \cdot \Phi \tag{2.23}$$

$$\delta\Phi = \left(\delta\alpha^1\sigma_1 + \delta\alpha^2\sigma_2 + \delta\alpha^3\sigma_3\right) \cdot \Phi. \tag{2.24}$$

Here σ_i are the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.25}$$

Which form a basis in the $su(2)$ Lie group and $e^{\alpha^i\sigma_i}$ is a generic element of the $SU(2)$ group: a group of 2×2 hermitian matrices with unit determinant.

One says that the 2-component complex vector Φ_i is in a *fundamental* representation of the group $SU(2)$, while the transformation matrix $e^{\alpha^i\sigma_i}$, which has 3 real parameters, is in the *adjoint* representation of the group. In general the $SU(N)$ group has N-dimensional complex valued fundamental representations and $N^2 - 1$-dimensional real valued adjoint representations. The adjoint representation can also be realized by the hermitian $N \times N$ matrices with unit determinant.

By a direct analogy with the previous considerations (2.19) we deduce the form of the gauge field in this case. Now the gauge field A must acquire values in the adjoint representation of the gauge group, i.e. now the gauge field is a $SU(2)$ matrix, parametrized by the 3 components.

$$(A_\mu)_{\alpha\beta} = A^i_\mu(\sigma_i)_{\alpha\beta}, \qquad i = 1\ldots3, \qquad \alpha,\beta = 1\ldots2. \tag{2.26}$$

When writing a Lagrangian for the gauge field theory with larger gauge groups one has to be careful since the matrix-valued fields don't commute anymore. These theories are called non-Abelian, in contrast to the Abelian $U(1)$ case, which commutes. Nonetheless the Lagrangian turns out to be quite similar to the one we saw previously (2.16).

$$S = \int d^4x \left[-\frac{1}{4}Tr[F_{\mu\nu}F^{\mu\nu}] - D^\mu\Phi D_\mu\Phi^\dagger - m^2\Phi\Phi^\dagger\right] \tag{2.27}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g_{YM}[A_\mu, A_\nu] \tag{2.28}$$

$$D_\mu\Phi = \partial_\mu\Phi + g_{YM}A_\mu\Phi \tag{2.29}$$

This action is invariant with respect to non-Abelian gauge transformations (*check*)

$$\Phi \to \omega\Phi, \qquad \omega \equiv e^{\alpha^i \sigma_i} \tag{2.30}$$

$$A_\mu \to \omega A_\mu \omega^{-1} + \omega \partial_\mu \omega^{-1} \tag{2.31}$$

This field theory with non-Abelian group is often called Yang-Mills gauge field theory. The major difference with the Abelian one is seen in the expression for the gauge field strength, which now contains a nonlinear term. This renders the equations of motion of this theory nonlinear even in case without matter and when the coupling constant is large, it turns into major obstacle, since the perturbation theory is not applicable and the exact solution to the nonlinear equations is absent.

The Yang-Mills SU(2) theory has originally been introduced in order to describe protons and neutrons in the nucleus. Even though that particular approach didn't work, now the Yang-Mills theory is underlying the modern theory of the strong interactions: the Quantum Chromodynamics [3], where the gauge group is $SU(3)$, and 8 different types of gluons couple together 3 colors of quarks. The QCD is known to be particularly hard to solve in the low energy limit, where the coupling constant is large, the quarks are confined and the perturbative treatment fails.

Quite remarkably, it is in the applications to QCD, where the first versions of the String theory have been introduced back in 1960s.

3 Strings

3.1 String theory: open and closed strings

Now we turn to the String theory, which is a conceptual basis of the AdS/CFT correspondence. In a nut shell, the string theory as a theory of 1-dimensional objects propagating in time. While the usual particle is represented by a world line in the space-time, the string spans the two-dimensional surface, extended in both spatial and time directions – the world sheet. There are two types of strings: the open ones with free ends and the closed ones with two ends joined together. The quantum strings have a particular finite tension $\mathcal{T}$, which sets the mass scale of the theory. Due to this tension the strings tend to shrink to zero size (apart from the remaining quantum fluctuations) and the lowest energy states of them behave as point like particles. The consistent theory of super strings[1] can be formulated in 9+1 dimensional target spacetime

Similarly to the usual mechanical string, the quantum strings support the vibrational modes. This is a new crucial feature as compared to the ordinary particle, which doesn't provide enough degrees of freedom for those vibrations. Remarkably the **lowest energy excitations of the closed strings appear as the gravitons** from the point of view of the target space, where the string propagates. On the other hand, the **lowest energy excitation of the open string is a gauge vector particle** [4], see Table 1.

[1]This involves a further generalization of the symmetry of the theory: the supersymmetry, which we will not explain here.

	Closed strings	Open strings
Lowest excitation	graviton	gauge vector boson
Effective theory	Gravity	Gauge field theory

Table 1: Low energy excitations in the string theory.

An important aspect is related to the open string. Unlike a closed one, **the open string has to end somewhere**. The end points of the open strings can be attached to the additional extended multidimensional objects of the string theory: *the D-branes*. One can think of a D-brane as a multidimensional analogue of the plane in 3 dimensions. A plane has 2 internal directions and one direction in the space is perpendicular to it. Similarly, a D3-brane is an object with $3+1$ dimensional world volume and 6 directions transverse to it. If one end of an open string is attached to the D3 brane, it can propagate along the $3+1$ internal directions, but it can not leave the brane and move along the 6 transverse directions. **An open string with both ends attached to the brane has a lowest energy excitation which looks like a gauge vector particle living in the world volume of this brane.**

3.2 Two points of view on a stack of D3-branes

It gets even more interesting when we consider a stack of coincident N D3 branes [5, 6]. In order to keep track on which end of the string is attached to which particular brane, one has to introduce an additional index for each end – the Chan-Paton factor. The open string on a stack of N branes carries two such indices, ranging from 1 to N. These indices are inherited by the gauge vectro particle as well and we recognize the familiar structure of the non-Abelian $SU(N)$ gauge potential (c.f. (2.26)), which carries two indices in the fundamental representation, which are equivalent to one adjoint. Hence we arrive to a crucial finding: **the open strings attached to a stack of N D3-branes realize the SU(N) Yang-Mills gauge field theory as their low energy effective theory.**

However, if N is large enough, one can look at the stack of D3-branes from a completely different point of view. The D3-branes, much like the strings themselves, have a finite tension. Therefore they carry the energy density and if one puts many of them in the same place, they will affect the curvature of the outer 9+1 dimensional space, in complete analogy to the usual black hole, sourced by a point-like mass sitting in its center. The only difference is the dimensionality of the outer space and the extra dimensions of the branes themselves. The resulting metric of the curved 10-dimensional space, affected by the stack of Dp branes is [6, 7].

$$ds^2 = H_p^{-1/2} dx \cdot dx + H_p^{1/2} dy \cdot dy, \qquad H_p(r) = 1 + \left(\frac{r_p}{r}\right)^{7-p}, \tag{3.1}$$

where in our case we will focus of $p = 3$, $dx \cdot dx$ is the (3+1)-dimensional metric along the world volume of the branes and $dy \cdot dy = dr^2 + r^2 d\Omega_5^2$ is the metric of the 6 perpendicular directions (r being the radial distance to the stack in this transverse space).

Let us focus on the near horizon asymptotic form of the D3-brane metric (3.1). We set $r_3 = R$ and consider $r \to 0$. In this case the metric reads

$$ds^2 \sim (r/R)^2 dx \cdot dx + (R/r)^2 dr^2 + R^2 d\Omega_5^2. \tag{3.2}$$

Changing the variables $z = R^2/r$ it acquires the form

$$ds^2 \sim R^2 \frac{dx \cdot dx + dz^2}{z^2} + R^2 d\Omega_5^2. \tag{3.3}$$

The second part of the metric is 5-dimensional sphere S_5 in transverse directions to the stack of the branes, while the first part is a metric of the 5-dimensional anti-de-Sitter space, the multidimensional analogue of a hyperbolic plane. The $z = 0$ is the asymptotic boundary of this hyperbolic space, which is isomorphic to 3+1 dimensional Minkowski space where the gauge theory is defined. It is usually stated that the gauge theory lives "on the boundary", while gravity lives in the "bulk" of the AdS.

Now we recall that the gravitational background is the coherent collection of the gravitons – the low energy escitations of the *closed* strings. At this point we arrived to the second part of the correspondence: **the stack of the D3 branes can be seen from the point of view of the closed strings as the source producing the gravitational $\mathbf{AdS_5 \times S_5}$ background.**

4 AdS/CFT

We just showed that the same object: the stack of N D3-branes can be described either from the point of view of the closed strings as the $SU(N)$ Yang-Mills theory, or, from the point of view of the closed strings, as the theory of gravity in the $\mathrm{AdS}_5 \times \mathrm{S}_5$ background. The AdS/CFT correspondence states that these theories are equivalent, since they describe the same object in string theory. Let us take a look on how this correspondence work.

4.1 Symmetries

The most powerful matching principle behind the correspondence is again the principle of the symmetries. The global symmetries in the gauge field theory correspond to the isometries of the theory of gravity [6].

So far we skipped completely the subject of a supersymmetry, which is beyond the scope of the current overview, however for completeness we have to mention that all the theories participating in the correspondence are actually extended by the supersymmetry. In case of the Yang Mills theory of the branes, is has 4 supercharges. These 4 supercharges form $SU(4)$ global symmetry group, which is isometric to $SO(6)$ – the group of rotations in 6-dimensional space. The rotations of a unit vector in 6D is in turn equivalent to translations of its endpoint along the 5-dimensional sphere. Therefore we arrive to the conclusion that **the supersymmetry of the gauge theory is reflected in the $\mathbf{S_5}$ part of the geometry on the gravity side of the correspondence.**

Another feature of the gauge theory on the boundary is its scale invariance. The gauge symmetry and the supersymmetry turn out do be so restrictive, that it is impossible to introduce any dimensionful parameter in the theory, without breaking the symmetries. The scale invariance together with the Lorentz invariance on the brane form conformal symmetry group (hence the name: conformal field theory). The scale invariance is also reflected in the geometry of the AdS space. Consider the metric

$$ds^2 = \frac{1}{z^2}\left(dx^\mu dx_\mu + dz^2\right). \tag{4.1}$$

The scale transformation in the boundary consists of rescaling all the coordinates

$$x_\mu \to \lambda x_\mu \tag{4.2}$$

However, this can be *undone* in the bulk, by rescaling the holographic coordinate $z \to \lambda z$, due to the denominator in the metric. After this rescaling the metric returns to its original shape, therefore **the scale (conformal) invariance is the isometry of the AdS space**.

4.2 Coupling constant: gauge/gravity duality

The crucial and the most useful feature of the correspondence is that it allows to treat the strongly coupled gauge theory by means of the weakly curved gravity and vice versa the weakly coupled field theory allows one to get insight over the quantum gravity [6, 7]. The coupling constants in the Yang-Mills and string theory are proportional:

$$g_{YM}^2 = 4\pi g_s, \tag{4.3}$$

however the effective coupling constant, which enters the leading diagrams of the Yang-Mills is the so called 't Hooft coupling

$$\lambda = g_{YM} N^2. \tag{4.4}$$

When N is large, the 't Hooft constant can be large even if g_{YM} is small, giving the weakly coupled string theory on the dual side.

The curvature radius of both sphere and AdS in (3.3) is also related to N, roughly because the curvature is supported by the number of D3 branes:

$$R = \lambda^{1/4} l_s, \tag{4.5}$$

where l_s is the quantum string length scale. **The gravity in the bulk is weakly curved as long as $R \gg l_s$, meaning $\lambda \gg 1$. On the other hand the quantum corrections to the gravity treatment are suppressed as long as $g_s \sim \lambda^2/N^4 \ll 1$, meaning $N \gg 1$.**

4.3 Breaking symmetries: applications

Having this powerful tool at hand one would like to apply the gauge/gravity duality to many unsilved problems in physics, which were previously unaccessible due to the strong coupling constant in the corresponding gauge field theory. One example is QCD, which was mentioned already, the other one in the strongly correlated quantum systems in Condensed matter [8]. In both case before applying the duality one has to get rid of the excessive amount of symmetry, which is present in the stringy construction, but will be broken in the applications. For instance QCD is not supersymmetric and Condensed Matter systems break Lorentz invariance by finite temperature, chemical potential and the crystal lattice. Construction of such type of models is extremely hard from the string theory "top-down" point of view. However keeping in mind the general principles, one can try to construct the dual backgrounds "bottom-up" with desired features and produce the duality inspired phenomenological models in this way. Let's discuss briefly how these holgraphic models can be constructed.

In the previous section we identified the S_5 part of the gravity background as corresponding to the supersymmetry. Therefore in order to break it in the holographic construction it is enough to choose a point on this sphere and completely suppress the dynamics along these directions. Therefore for the non-supersymmetric boundary theories in 3+1 dimensional Minkowsky space-time one is left with 4+1 dimensional bulk AdS spacetime with only one extra dimension (4.1). It gets particularly clear now why these models are called holographic.

The other generalization is the introduction of finite temperature in the boundary. From the geometrical point of view the finite temperature can be seen as the compactification of the Euclidean time direction. Since the bulk and boundary theories share the same time coordinate, the gravitational dual to the thermal field theory would have a compactified time at asymptotic infinity as well. Those familiar with the Hawking radiation would immediately recognize the gravitational solution which has a compact Euclidean time direction at infinity: this is a black hole. In complete analogy **the gravitational dual to the thermal field theory is a black hole in AdS space-time**

$$ds^2 = \frac{1}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + dx_i^2\right), \qquad f(z) = 1 - (z/z_0)^4 \tag{4.6}$$

Here the temperature is related to the horizon radius as

$$\frac{1}{T} = \pi z_0. \tag{4.7}$$

One can show furthermore[8] that the finite chemical potential on the boundary would correspond to the charge of the dual black hole, turning it into AdS–Reissner-Nordstrom solution. In case one wishes to introduce the effects of the crystal lattice, one can consider the spatially modulated chemical potential, which produces the spatially modulated black holes as the solutions to the corresponding Einstein equations. In this way **the AdS/CFT duality relates the physics of the strongly correlated quantum materials to the physics of black holes in the curved auxiliary spacetime.**

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