CHAPTER XII

The Irrelevance of the Jury Theorem

There is a long tradition, at least since Aristotle, extolling the wisdom of groups. This, of course, exists alongside the long tradition denigrating the intelligence of common people. Aristotle gives early voice to both ideas when he says that common individuals are not very bright, but that collectively they are at least better, and possibly wise enough to rule politically. In our own time, we have volumes of sophisticated demonstrations both of the ignorance of voters in modern democracies and of the variety of ways in which collectivities can make better decisions than the individuals they comprise. In this chapter, we look at how collectivities can, under certain conditions, make better decisions than individuals, even without benefit of exchanging arguments, or perspectives, or, indeed, any communication at all. Certain mathematical facts establish just that, and democratic theorists have been intrigued. The most influential of these mathematical results is known as Condorcet’s jury theorem. After introducing it and showing briefly how it works, I wish to argue that it is too shaky a basis on which to ground the proposition that voters in democratic procedures tend to make good decisions. We will have to look elsewhere in order to support that proposition, a proposition that is crucial if the authority of democratically produced laws and policies is to be grounded partly in the epistemic value of those procedures, as epistemic proceduralism requires.

The probability that the majority will support the correct option tends toward certainty as the number of voters approaches infinity. Suppose there are two options, and suppose each voter is independently 51 percent likely to choose the correct option (and 49 percent likely to choose the incorrect option): then among a group of 1,000 voters, the probability that the majority will vote for the correct option is approximately 69 percent. If the number of voters is increased to 10,000, then that probability rises to virtual certainty: 99.97 percent. Thus, among electorates of even just moderate-sized towns, much less large nations, the majority is almost certain to choose the right option, just so long as each voter is independently just a little better than random in a two-option choice.
The proof of this striking result is fairly simple, and it is worth pre-
senting an informal version here. Begin with the fact that while a fair 
coin flipped a few times is not likely to produce a very equal head/tail 
ratio, with more tosses the ratio becomes more even. With just a few 
tosses, an outcome of, say, 70 percent heads and 30 percent tails would 
not be shocking. But with many tosses of a fair coin, a 70/30 split is 
almost out of the question. With enough tosses it becomes certain that the 
division will be almost exactly 50/50. This “law of large numbers” is 
the core of the proof of the jury theorem.

Now change the coin from a fair one to one weighted slightly in favor 
of heads, so in each toss it has a 51 percent chance of being heads. Now 
with enough tosses the percentage of heads is certain to be almost ex-
actly 51 percent. The reason is just the same as the reason a fair coin 
tossed many times produced very nearly a 50 percent split. The more 
tosses, the closer to exactly 51 percent this weighted coin is likely to be. 
Now obviously the same would be true if instead of one coin flipped re-
peatedly, we considered many coins, all weighted the same way, each 
having a 51 percent chance of coming up heads. The more coins we 
flipped, the closer the frequency of heads would come to exactly 51 
percent. Obviously, too, the same would be true if we had individual 
voters instead of coins, where each will say either “heads” or “tails,” 
but each has a 51 percent chance of saying “heads.” The more such vot-
ers, the closer the frequency of “heads” answers would come to exactly 
51 percent. Here is the payoff: if the frequency of “heads” is bound to be 
almost exactly 51 percent, then, of course, it is even more certain to be 
over 50 percent. So the chance that at least a majority will say “heads” is 
astronomical—approaching 1, or a 100 percent chance—if the group is 
large. It gets higher with the size of the group. It is also plainly higher if 
instead of 51 percent, each voter (or coin) has an even higher chance of 
saying “heads,” say 55 percent or 75 percent.

So if each voter has an individual likelihood above 50 percent (call it 
(50+n) percent) of giving the correct answer (whatever it is) to a di-
ichotomous choice (heads/tails, yes/no, true/false, better/worse, etc.), 
than in a large group the percentage giving the correct answer is bound 
to be exceedingly close to (50+n percent). Therefore, the chance that it 
will be at least 50 percent is even higher, approximating certainty as the 
group gets larger or the voters are better. In summary, concentrating on 
our starting example, if voters are all 51 percent likely to be correct, 
then in a large number of voters it is almost certain that almost exactly 
51 percent will be correct, and so even more certain that more than 50
percent will be correct. Under these assumptions, it is very likely that a proposal winning majority support is very likely to be the best or correct proposal.

The results are very much the same if we weaken the assumption that all voters have the same competence, but assume only an average competence above 50 percent, so long as the individual competences that produce this average are distributed normally around the average. Abnormal distributions change the results significantly, sometimes for better, sometimes for worse.

Independence

The mathematical result is beyond dispute, but it applies only under certain conditions. One is that enough of the votes must be statistically independent. This is often misunderstood. On the overly pessimistic side, many have said this cannot be met, since there will always be lots of influence one on another. Few will be independent of each other. What the theorem requires, though, is not causal independence, but statistical independence. Statistical independence means that the probability of one voter, say Joe, getting the right answer is exactly equal to the probability of Joe getting the right answer given that Jane did. Joe’s and Jane’s chances of being correct are independent of each other if neither of them gets a higher chance of being correct given that the other is correct.

The jury theorem’s independence requirement often inspires either too much optimism or too much pessimism. On the overly optimistic side, some have said that all that is required is that enough voters make up their own minds rather than intentionally altering their votes to follow some opinion leader. But that is clearly not enough. It is logically possible for voters each of whom makes up his or her own mind to vote identically time after time. If too many voters did that, it would radically violate the independence requirements of the jury theorem, which mathematically depends only on correlations, not intentions. Suppose, for example, that Joe and Jane each had a competence of .6, a 60 percent chance of getting the right answer. If they always vote the same way, then Jane’s getting the right answer would guarantee that Joe got the right answer. The probability of Joe doing so given that Jane did so would be 1. Since this is greater than the simple probability that Joe gets the right answer (which is Joe’s competence, or .6), independence
would be violated. It is violated whether or not Joe or Jane based their votes on those of each other. Perhaps they are just very much alike. Voters making up their own minds is not what independence means for purposes of the jury theorem.

We have seen that voters making up their own minds does not guarantee statistical independence. It is also true that voters who do not make up their own minds might yet be statistically independent. Statistical independence is compatible with some degree of deference to opinion leaders. That is, even if several voters are more inclined to vote for A if a certain pundit they all like supports A, this does not yet violate their statistical independence. The pundit is a clear common causal influence on these voters, and so in a certain sense their votes are not causally independent. Nevertheless, they can all defer to this pundit to a significant extent and remain statistically independent, so long as each has some competence with respect to knowing when to defer and when not to defer. That is, if two voters each has a competence that is higher than their fidelity to the pundit (fidelity of .6 means a voter agrees with the pundit 60 percent of the time), then the voters remain statistically independent of each other despite their both being influenced by the pundit. Moreover, since the pundit might be smart, the voters’ deference can improve their own competence, with beneficial effects on group competence.

Making up one’s own mind is not the issue. The simple fact that voters will share common influences is not fatal to the jury theorem’s applicability to democracy, and sometimes enhances it. How much influence across voters the theorem can tolerate, and how much is present in any realistic democracy, are questions that are not yet well understood. The lesson, for now, is that if there is a problem about applying the theorem to democracy, we do not yet have enough reason to think that the problem is a failure of voter independence.

**BEYOND BINARY CHOICE**

In its classic form as proven by Condorcet, the jury theorem explicitly applies only to binary choices such as yes/no, true/false, better/worse. This can look very restrictive. Political choices are complicated, and the narrowing of choices down to two is just the last stage in a process that starts with many more. On the other hand, elections and referenda do often present themselves as binary. Should the law be passed or not?
Should this person be president or that one? Should the sales tax be raised or not? Obviously, at some earlier point there were far more than two things that might be done. So the binary choice condition would not apply at those earlier times. But whatever process leads to the choices I just listed, the binary choice precondition is, starting at that point, apparently fully met.

Nevertheless, it is enormously important if the binary choice condition can be relaxed. It turns out that, in a sense, it can. There are several results along these lines, one of which is proven by Goodin and List.8 The reasoning cannot be presented here, but the main conclusion of their argument is this: when there are three or more choices, if each voter is more likely to vote for the (objectively) best alternative than she is to vote for any of the other alternatives, the chance that the best alternative will win a plurality increases with the size of the group of voters.

One underwhelming result is that the correct answer is more likely to win than any other single alternative. This is not much use when there are several other alternatives, since their probabilities of winning are cumulative. Even if the correct answer has a better chance than any other answer, the chance of it winning might be far less than the chance of some erroneous answer or other winning. Another result of questionable use for our purposes is that the chance of a plurality getting the right answer climbs quickly with the size of the group if voters’ individual competence is better than .5. But, of course, .5 is not a very interesting number when the alternatives are three or more in number.9 It would be a competence substantially better than random, and so it is a substantial assumption that would need some warrant.

The most interesting aspect of the result is the fact that the chance of the correct answer winning increases with the size of the group, approaching a group competence of 1, or infallibility. This fact seems to hold true even for the crucial case where voters are only slightly better than random, being only slightly more likely to vote for the correct answer than for any other single answer. The stunning thing about the classic binary choice theorem is that, for example, the group competence gets above .97 even where the number of voters is only the size of a small town (10,000), so long as voters have a competence of at least .51. What if there are three alternatives rather than just two? In a group of 1,001 voters with three alternatives, and voters just slightly better than random (.34 chance of the right answer, .33 for each of the wrong answers), the chance of the best alternative winning a plurality is .489, not an enormous leap from the .407 achieved by 301 voters (and the
correct answer would still lose more often than it would win). So far, there is no clear explanation of how large a group must be before voters who are just barely better than random would be virtually certain, or even more likely than not, to give the correct answer a plurality when there are more than two alternatives.

I do not want to exaggerate these limitations of the nonbinary applications of the jury theorem. First, we do know that group competence approaches infallibility at some size of the electorate even with just barely better than random individual competence, and many democratic contexts involve much larger numbers of voters than those studied so far. So group competence might well turn out to be very high in those cases. Second, some epistemic theories do not require astronomical group competence. Epistemic proceduralism, in fact, requires only that the group be better than random, and the best (or nearly so) so far as can be established in a way that does not contradict any qualified point of view. The extension to three or more alternatives is certainly progress. The irrelevance of the jury theorem, including extended versions, for our purposes, rests on the difficulty of assuming individual competence that is at least better than random. This in turn rests on two considerations, which I turn to next. One is that systematic thinkers often make systematic errors. The other is what I call the disjunction problem.

THE ILLUSIVENESS OF RANDOM INDIVIDUAL COMPETENCE

A deeper worry, one that applies in both the case of binary choice and for choices between three or more alternatives, concerns the assumption that voters are better than random. Individual voters might indeed be better than random, but this is not obvious. Factual errors, prejudice, and other factors could, for all we know, outweigh the average voter’s margin of better-than-random competence, at least on matters that are sufficiently contested that they end up being settled by a vote. Democrats and Republicans in Congress systematically vote against each other on many issues. Which party should we be sure is at least a little better than random? If they oppose each other often enough, they cannot both be better than random. If one party is, the other party’s competence is 1 minus the first party’s competence, which must be less than random. But if, as I think, either party could easily be worse than random, then it is hardly absurd to think that due to the same kinds of
biases or errors, the average congressperson could have been even worse than random.\textsuperscript{11}

Systematic individual biases and errors are, of course, very common, and they represent one kind of challenge that needs to be met before individual competence could be assumed to be at least random. There is a second kind of challenge to that assumption, which I will call the disjunction problem. Before we avail ourselves of the assumption that voters are at least a little better than random, we would need to know what random competence would be. In the Condorcetian analysis, what random competence means when there are \( k \) alternatives is getting the correct answer with a probability of \( 1/k \). Two alternatives give a random competence of \( \frac{1}{2} \), or .5; four alternatives, \( \frac{1}{4} \) or .25; and so on. Consider a choice among three alternatives: A, B, and C. If we suppose, a priori, that voters are a little better than random, we might let them have, say, a .34 chance of getting the right answer and a .33 chance of each of the wrong answers. But suppose we presented the choice differently: alternative A versus the disjunction of B or C. By leaving the choice between B and C for later, the choice is now binary. Since the choice is now a binary one, are we suddenly entitled to suppose voters must be at least a little better than .5? Is the minimal, modest assumption that they are more likely than .5 to choose A, the right answer? Quite a promotion.

To put the point more precisely, for a set of \( k \) alternatives, assuming a competence of \( 1/k \) implies that if any of the alternatives were disaggregated (showing that it was actually disjunctive) to create \( k+n \) alternatives, competence would be somewhat greater than \( 1/(k+n) \), that is, somewhat better than random for that choice set. It is as if the assumption that looked weak has just turned out to be stronger. An assumption of random competence over \( k \) alternatives is, in effect, also an assumption of better than random competence for the embedded \( k+n \) alternatives. Indeed, if the number of disjuncts is significant as compared with \( k \), then a competence of \( 1/k \) is much better than random for the set of \( k+n \).

What this shows is that, since some of a set of alternatives are often really disjuncts, there is no principled sense in which it is a weak or obvious assumption to suppose individuals have better than random competence over a given set of alternatives. Consider the proverbial blind men and the elephant. Each can touch a different part, but this is not enough to identify the kind of animal before them. If they are asked whether the animal before them is an elephant, they are given a binary
choice: yes/no. But “no” is the answer they should give if they think it is any animal other than an elephant. “No” means “hippo, or rhino, or mule, or horse...” To choose “yes” at the “random” rate of .5 they would need some strong suspicion that it is an elephant rather than any of the other possibilities. A competence of .5 would be quite high given that there might be dozens of animals it could be given the little each of them knows. Would an assumption of .5 competence be a blind, dumb, random competence because there are two choices, and even a random device would perform at .5? Or would it be a rather high competence in light of all the possible animals they might be faced with? A random device would perform at .5, but a thinking person might well perform well below that, and for good reasons. Odds are (or might be), given what they know, that it is something other than an elephant. Knowing that the men have a binary choice does not automatically allow us to assume that unless something has gone badly wrong they should have a competence of .5.

This problem might seem limited to the special case in which at least one of the alternatives is disjunctive. But the selection of almost any law or policy leaves significantly different possible ways of instantiating it, not just in the means employed, but also the ends. Should we build a bridge over the channel or not? If so, should it be a four-lane, a two-lane, built now, or later? And so on. So many political alternatives, as presented for social choice, are disjunctive, and so the disjuncts could have been presented as separate choices, giving rise to the difficulty I have pointed to. This difficulty about how to count alternatives raises questions about the a priori assumption that voters can be assumed, for Condorcetian purposes, to be at least a little better than random. And without that assumption, or some substantive support for the competence assumption, the jury theorem gets us nothing.

Bayes in Brief

There is another mathematical approach to group competence, relying on Bayes’ theorem. The two main problems I have counted against the jury theorem apply there as well. On the Bayesian approach, each voter takes the fact that many others voted for \( p \) as evidence in favor of \( p \). As each revises her own estimation of the probability of \( p \) upward in accordance with this evidence, voters bootstrap each other up to very high levels of confidence that \( p \) is that case. This Bayesian approach requires
that participants take each other to have a competence higher than random. But just as before, the idea of random competence depends on there being some privileged way to count alternatives, and this is the first problem. As we saw earlier, political choices are often disjunctive, and that provides many different ways the choice might be presented. For this reason, there is no sense in which a “random” competence over some particular way of presenting the alternatives can be counted as a weak assumption, as I argued earlier. Also, of course, there are many sources of systematic error or bias that allow for the possibility that voters are very often pervasively worse than random. Again, the assumption that voters are better than random is not freely available, but would need some argument. The stance I have argued for here with respect to the jury theorem applies without alteration to the Bayesian approach to democratic group competence.

COMMUNICATION

The jury theorem makes no use of interpersonal communication. The Bayesian model has a small social element: participants must be able to revise their opinions in light of information about how many people had certain opinions in the last round. Still, knowing the results of a poll or a vote and using it as data—as Bayesian participants do—is still nothing like hearing people explain their opinions. Discussion is still utterly absent from the model. If the blind men can talk with each other, there is some hope that they can figure out that the object is an elephant, though none could do this alone. But neither the jury theorem nor Bayes’ theorem actually models the blind men sharing their perspectives. Under majority rule if they were better than random individually, the group (especially if it is large) will have a surprisingly high chance of being correct. But, of course, they will not be individually very competent, since we know they are each inclined to say that, no, it is not an elephant. That is the end of that story from the jury theorem’s point of view. Bayes’ theorem adds a layer. After the first round of opinions, each should revise his opinion in the direction of the majority. But this is clearly not going to help anything. If anyone had been suspecting it was an elephant, the large majority against them would disabuse them of that notion, on Bayesian reasoning. Bayesian blind men would not figure out that it was an elephant simply by wondering whether it is, looking at the results, and updating their probabilities. They would
need to talk to each other, something absent from both the Bayesian and the Condorcetian model.

Aristotle makes several remarks that sound very much like the elephant analogy:

The many, of whom each individual is but an ordinary person, when they meet together may very likely be better than the few good, if regarded not individually but collectively, just as a feast to which many contribute is better than a dinner provided out of a single purse. For each individual among the many has a share of virtue and prudence, and when they meet together, they become in a manner one man, who has many feet, and hands, and senses. ¹³

At a feast the dishes are publicly shared and appreciated in combination. This is an utterly different epistemic model from the jury theorem. The mathematics of the jury theorem is not really driven by the bringing together of different parts of a puzzle, but simply by the statistical fact that the fraction of a large group that will vote yes will come very close to the probability the individuals have of voting yes. It is a mathematical fact that applies to coin flips in exactly the same way it applies to votes. A large number of weighted coins, each of which has, say, a 51 percent chance of turning up heads, will produce very close to 51 percent heads. Should we say that the coins bring their different perspectives together? Is each coin like one of the blind men in the elephant story? This is clearly a mistake. The reason this is important is that it is very natural and plausible to think that if democracy has any epistemic value it is partly to do with the sharing of diverse perspectives. Many have suggested that the jury theorem is a mathematical formalization of that very mechanism. The coin-flip example shows, I hope, that it simply is not. And this, in turn, is important if I am right that the jury theorem is really not available for these democratic uses owing to the disjunction problem and other problems about the assumption of better-than-random voter competence. We should not conclude from this that the idea of sharing perspectives turns out to be unavailable. The jury theorem, which is unavailable, is a different idea altogether.

**The Disjunction Problem and Epistemic Proceduralism**

According to epistemic proceduralism, under the right conditions, democratic decisions have their legitimacy and authority partly because
of a publicly recognizable tendency to make good decisions, at least better than a random procedure. The case for epistemic proceduralism escapes the critique I have laid out against the statistical approaches for the following reasons.

The disjunction problem shows that there is no such thing as an obvious or trivial assumption that voters are better than random. Epistemic proceduralism, however, never makes such an assumption. Rather, argument is offered for the quality of group competence directly, and not in a way that first assumes any particular relation between individual competence and random competence. We do still need to use the idea of better-than-random competence in one way, because that is what epistemic proceduralism requires of group competence. After all, if group competence was not even better than random, then why not choose randomly? So, does the disjunction problem apply again at this level?

First, notice that the disjunction problem does not show that there is no way to define random competence, but only that there is no privileged way to count alternatives that would warrant the intuitive thought that individuals must be at least better than random. Now, we could make the same charge against the idea that the group must be at least better than random. If there are four alternatives presented, then “better than random” means better than .25. But it is odd to think that we can freely assume that if three of the four alternatives are grouped into one disjunctive alternative, leaving only two alternatives, the group competence automatically goes up to .5. That is the challenge of the disjunction problem posed at the group level.

Recall, though, that the lesson of the disjunction problem is that no competence assumption is available, so to speak, for free. We can choose one way of defining random competence, such as the performance of a device randomly selecting from the alternatives however they are actually presented. What we cannot do is go on to assume that people or groups must obviously be at least that good. It is necessary to point to actual mechanisms or other reasons to believe that individuals or groups will actually perform that well. The mechanism I appeal to is interpersonal communication and reasoning about the question at hand. So, consider the blind men and the elephant. On the question “Is it an elephant?” they would individually be no better than random (or worse). Neither the jury theorem nor a Bayesian process of updating beliefs in light of the beliefs of others would yield a group competence above random. But if they communicate with each other, it is highly likely that they would figure out that it was an elephant.
Are we sneaking the jury theorem in by the back door? Suppose we accepted this and said that under proper conditions of communication the group competence on the elephant question, after communication, would be better than random. It follows by the jury theorem that under those same conditions, after communication, individual competence would be better than random. If it were not, then the group competence could not be, as the theorem shows. Someone might say that the jury theorem is playing a crucial role here, since the communication pushes the group competence above random only by pushing individual competence above random and aggregating individual judgments. The jury theorem, though, is about amplifying individual competence to a much higher group competence, and that kind of amplification is no part of the story of the communicating blind men. All the jury theorem adds to our story is the fairly uninteresting fact that if the group competence is above random, then individual competence will also be above random, albeit significantly lower than the group competence. The disjunction problem is avoided because at no point do we avail ourselves of the intuition that the group or the individuals must naturally be better than random. The blind men trying to identify the elephant are individually hopeless. Even so, we expect communication (under the right conditions) to tend to make the individuals and the group better than random (the individuals less so than the group). We define one interpretation for random competence: as good as a random device selecting from the alternatives however they are actually presented. But we do not simply assume individuals or groups are this good. We argue for it on the basis of the epistemic value of communication. Since the disjunction problem only counts against the free assumption of competence better than random, it is not a problem for our approach.

Obviously, I have not given any detailed account of how and when reasoning together will improve group competence. In many settings there are dynamics such as “groupthink,” and polarization effects that can undo the epistemic potential of thinking together. Letting the blind men communicate is only meant to illustrate how an account of this kind could provide a basis for the epistemic value of group deliberation, without in any way relying on the jury theorem’s competence-amplifying effects, and without making the assumption—discredited by the disjunction problem—that competence above random can be taken for granted.

We should remember that epistemic proceduralism would need it to be the case that group competence was better than random in a certain
way, though not simply by getting more correct answers than random. Rather, good group competence on the most important issues could outweigh poor performance on less important matters. So what we would need is not better-than-random competence overall, but an above-random score when the value of decisions is given weight proportional to their importance. As I have discussed in chapter 9 ("How Would Democracy Know?"), we can make this model relatively tractable by listing a set of especially important matters, what I will call primary bads to be avoided, and then by a mixture of argument and conjecture suppose that sufficiently good competence here will translate to better-than-random weighted score overall. The upshot is that even though epistemic proceduralism does not need the same assumption of above-random competence as the jury theorem, it does still need above-random competence, somehow interpreted, when it comes to avoiding primary bads. It is not yet clear, then, how epistemic proceduralism escapes the force of the critique I have mounted of the jury theorem approach.

In order to extrapolate from good group performance on primary bads to an above-random weighted group score generally, either we need to assume that group performance on other less important matters is no worse than random, or we need to suppose that group performance on primary bads is enough better than random that even poor performance elsewhere would not prevent the weighted score from being above random. I think there is nothing trivial or minimal about assuming that competence on other matters is at least random. One reason is the disjunction problem, and I will not repeat it here. But there is also the less technical point that it is easy to be worse than random if one has a systematic bias, or one’s information is faulty in a crucial way. A person performing worse than random on some type of cognitive task is not really much more mysterious than a coin persistently getting heads less than half the time. It must not be a fair coin, and the person must be biased or misinformed or some such thing. So, we are left needing to maintain that group performance on important issues (avoiding primary bads) is, under the favorable but possible conditions that epistemic proceduralism needs, and after public deliberation of certain kinds, enough better than random to outweigh, in the weighted score, any especially poor performance by the group in other areas.

It is true, of course, that if group performance on primary bads is held to be not just above random, but some significant amount above random, then the jury theorem would entail that individual competence is not just above random, but significantly above random (though
by a smaller margin than group competence is above random). But it is important to see that this does not somehow make the jury theorem available after all. This still does not give the jury theorem any way to start with information about individual competence and get new information about high group competence. Our reasoning has gone in the reverse order.

Conclusion

There is good reason to turn our focus away from aggregating votes and toward the formation of the attitudes that go into voting. This is an old refrain that deliberative democrats use against models that understand voting as expressing preferences, but it turns out to be appropriate where votes are understood as expressing judgments, too. The leading models that take an aggregative approach to judgments, in hopes of showing they produce a collective wisdom—the jury theorem, Bayes’ theorem, and so on—are simply not entitled to the assumptions they need about individual competence. The epistemic engine of democracy will have to lie elsewhere, somewhere that explains how individual judgments come to have the requisite quality.

Another influential explanation of how democracy might have epistemic value draws on an analogy between democratic procedures and a contractualist theory of rightness or justice. In the next chapter we will see that this, too, falls short.