Trade and Location with Horizontal and Vertical Multi-Region Firms*

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Abstract

This paper analyses the effect on agglomeration tendencies of allowing multi-region firms in a standard trade and location model, the core-periphery (CP) model developed by Krugman (1991). The introduction of horizontal multi-region firms dampens the agglomeration effects found in the CP model by reducing the range of trade costs for which the core-periphery equilibrium occurs. The introduction of vertical multi-region firms that separate the location of headquarters and plants has two counteracting effects. While headquarters exhibit a strong tendency to concentrate, plants tend to spread out. The equilibrium is always asymmetric in spite of the underlying symmetry of the model.

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1. Introduction

The recent literature on economic geography offers a theoretical basis for one common worry of politicians in peripheral regions, namely that industrial production has a tendency to agglomerate in central regions. For instance, in Krugman (1991) and Krugman and Venables (1995), the interaction between scale economies and trade costs gives rise to linkages between firms and labour/customers or between the firms themselves via their demand for intermediate products. This may produce a core-periphery structure where industrial production is concentrated to the core region.[1]

Models of this kind have been used in several studies to analyse the effects of regional integration on the location of industrial activity and regional income inequality.[2] Typically, a production structure with dispersed industrial activity can be sustained with high trade costs, while lower trade costs may induce complete agglomeration of industrial activity and, under certain circumstances, increased inequality in real incomes.

However, the strong effects on the location of industrial activity in these models appear to some extent to be a consequence of the rather restrictive assumptions about the firm's production function. Scale economies are assumed only to be present at plant level, which implies that each variety of a differentiated product will be produced at a single location. This makes the firm's locational decision a very simple one; the firm can choose to produce in either one of two regions, but never in both. In reality, however, many firms do operate in more than one region.

In this paper we study the effects on agglomeration tendencies of allowing such firms. To this end, we modify the core-periphery (CP) model developed by Krugman (1991). In the CP model, there are two linkages that tend to lead to agglomeration. First, a large region tends to have a low price index because many products are sold at prices that do not include trade costs. This implies a higher real wage in a larger region, which attracts more workers. This is a so-called supply

[1] See the survey by Ottaviano and Puga (1998).
[2] For surveys in this area, see Baldwin and Venables (1996) and Ottaviano and Puga (1998).

linkage. Second, a large region constitutes a large home market. A firm with a large home market tends to be more profitable than a firm with a small home market in a world where goods are traded at a cost. Therefore, firms in a larger region are able to offer higher nominal wages than are firms located in a smaller one. Again, this attracts more workers to the larger region. This is a so-called demand linkage.

We focus on two cases. First, we introduce horizontal multi-region firms, which are firms that produce the same good in both regions. These firms are assumed to have multi-plant economies of scale, which makes it less costly to set up a second plant in a new region than to set up a whole new firm. Examples of such firms are softdrink and beer manufacturers. For instance, Coca-Cola has head offices at the national level that coordinates marketing and sales. Normally, however, the company operates several bottling plants within each country.[3] These bottling plants are usually located close to large regional markets, which indicates the importance of market access and transportation costs.[4]

Second, we introduce vertical multi-region firms, which are firms that separate geographically different kinds of activities. In our model, we allow firms to separate headquarters from actual production; the former taken to be reflected in the fixed costs and the latter in the variable costs of the firm. In this context, headquarter activities may include pure back-office facilities as well as R&D. There are many examples of firms that separate their headquarters from their production plants. For instance, the Swedish telecom company Ericsson locates headquarters, production and R&D in different places within Sweden.[5] Often firms locate their headquarters in or close to the capital city and production plants in other locations. In particular this seems to be common for large firms.

Our results indicate that as we make the firm's decision whether to become a single-region or multi-region firm endogenous, we get outcomes that are more

[3] Worldwide, Coca-Cola is bottled in more than 1.100 locations.
[4] In Germany, for instance, Coca-Cola operates bottling plants in many of the large cities such as Berlin and Hamburg.
[5] The main corporate headquarters are situated in Stockholm, a major R&D center is situated in Lund, whereas mobile phones are manufactured in Kumla.

consistent with observations on locational patterns. The introduction of horizontal multi-region firms leads to less agglomeration in the sense that production will spread to both regions for a larger range of trade costs. This result is similar to the finding of Markusen and Venables (1996) in a model of multinational firms. They show that the tendency for concentration of industrial activity may be weaker when we allow firms to set up production plants both at home and abroad.[6] Our analysis, however, differs from theirs by explicitly analysing the role of economic integration for the stability of possible long-run equilibria.

When firms can become vertical multi-region firms, changes in trade costs produce more gradual changes in the location of industrial activity and the long-run equilibrium will be asymmetric in spite of underlying symmetry in the parameters of the model. Furthermore, the headquarters that essentially produce services that are freely traded within the firm exhibit stronger tendencies to concentrate than the production plants.[7]

The rest of the paper is organised as follows: In section 2, the model with horizontal multi-region firms is presented and the stability of an equilibrium where manufacturing production is divided symmetrically between two regions is analysed. In section 3, vertical multi-region firms are introduced and we analyse how this affects the stability of equilibria for different levels of trade costs. Finally, in section 4, we summarise our conclusions.

2. Horizontal Multi-Region Firms

2.1 The Model

There are two regions in the economy; region 1 and 2; and two types of labour; farmers and workers. Farmers are immobile and produce a freely traded homogenous

[6] More specifically, they show that the set of factor allocations consistent with equilibria without complete agglomeration is larger when firms are allowed to operate both at home and abroad.
[7] That R&D activities tend to be more concentrated than production activities is documented empirically by Audretsch and Feldman (1996). Theoretically, the possibility of such an outcome has been shown by Martin and Ottaviano (1996) in a growth model which bears some resemblance to our model in the sense that R&D is traded without costs while goods are not.

good with a unit labour input coefficient equal to one. We take this good as numeraire so that wages and prices in this sector equal one in both regions. Workers produce differentiated products and can move between the regions.

Individuals derive utility from a utility function of the following form:

$$U = C_M^{\mu} C_A^{1-\mu}; \quad C_M = \left(\sum_{i=1}^{N} c_i^{1-1/\sigma} \right)^{\frac{1}{1-1/\sigma}};$$

where C_A is the consumed quantity of the homogenous agricultural good, C_M is the consumed quantity of a CES-index of manufactured products, N is a large number of products and σ>1 the elasticity of substitution between each pair of manufactured products. The parameter μ is the constant share of expenditure on manufactures. The total population is normalised to one and we follow Krugman (1991) in assuming that there are μ workers and 1-μ farmers. Furthermore, farmers are equally divided between the two regions, ensuring a minimum size of demand for manufactures in each region.

Setting up a single-plant firm entails a fixed cost that consists partly of firm-specific fixed costs such as costs for R&D, management and marketing and partly of plant-specific fixed costs such as buildings and machinery. However, the firm may choose to set up a second plant as well, in which case it will have to incur an additional fixed cost. If it does choose to set up two plants, each located in different regions, the total fixed costs are divided equally between the two regions in which the firm operates.[8] The cost function of a horizontal multi-region firm can then be expressed as:

$$w_j L_{ij} + w_k L_{ik} = \frac{(\alpha + \gamma^h)}{2}(w_j + w_k) + \beta(x_{ij} w_j + x_{ik} w_k) \qquad (1)$$

[8] This assumption is made to isolate the horizontal element of the firm's structure. If the fixed costs had been assumed to be incurred asymmetrically between the two regions, a vertical element would have been introduced.

where j=1, 2, k=1, 2, $j \neq k$, w_j and w_k are the wage rates in region j and k, respectively, L_{ij} and L_{ik} are the amounts of workers used by firm i in region j and k, respectively, α is the fixed cost in terms of workers for setting up a single-plant firm while γ^h is the additional fixed cost for setting up a second plant, β is the marginal cost, and x_{ij} and x_{ij} are the output levels of firm i in region j and k, respectively. We assume that $\gamma^h < \alpha$, which implies that there are multiplant economies of scale.

Shipments of manufactured goods between regions are assumed to require trade costs of the iceberg type. To deliver one unit of x to the other region, τ>1 units have to be shipped.

The manufacturing firms are assumed to operate under Chamberlinian large group competition, and a typical firm producing in region j will set price as a mark-up on marginal cost according to:

$$p_k = \left(\frac{\sigma}{\sigma - 1}\right)\beta w_j \tag{2}$$

where p_k is the producer price of products sold in region k. Choosing units so that β=(1-1/σ) implies that $p_k = w_j$. An exporting firm that charges the price p_k on its deliveries to the home market will charge τp_k on its exports to the other region, since the price that faces the consumer has to include any trade costs involved.

The choice between becoming a single-region and a multi-region firm is modelled as a complementary slackness problem. In equilibrium, profits will be less than or equal to zero, with equality applying for any firm type active (cf. Markusen and Venables, 1998). Formally, we assume that the following inequalities hold:

$$\frac{\left(x_{jj} + \tau x_{jk}\right)}{\sigma} - \alpha \leq 0 \tag{3}$$

$$\left(\frac{x_{jj}}{\sigma}-\frac{\alpha+\gamma^{h}}{2}\right)w_{j}+\left(\frac{x_{kk}}{\sigma}-\frac{\alpha+\gamma^{h}}{2}\right)w_{k}\leq 0 \tag{4}$$

where x_{jj} (x_{kk}) is the quantity sold by a firm producing in j (k) in its domestic market, while x_{jk} (x_{kj}) is the quantity sold by a firm producing in j (k) in region k's (j's) market.[9] The complementary variable for equation (3) is n_j, the number of single-region exporting firms in region j, and the complementary variable for equation (4) is m, the number of multi-region firms active.

Market-clearing for the manufacturing sector implies that delivered quantities equal demand:

$$x_{jj}=\frac{p_{j}^{-\sigma}\mu Y_{j}}{P_{j}^{1-\sigma}} \qquad\qquad x_{jk}=\frac{(\tau p_{j})^{-\sigma}\mu Y_{k}}{P_{k}^{1-\sigma}} \tag{5}$$

where j=1, 2; k= 1, 2; $j\neq k$; Y_j and Y_k are total incomes in region j and k, respectively; and P_j is the CES price index of manufactured goods for consumers in region j. The income of region j is given by:

$$Y_{j}=\frac{1-\mu}{2}+w_{j}\lambda_{j}\mu \tag{6}$$

where λ_j is the share of the total number of workers that can be found in region j, i.e., $\lambda_{j}=L_{j}/(L_{1}+L_{2})$. The price index is defined as:

$$P_{j}\equiv\left((n_{j}+m)p_{j}^{1-\sigma}+n_{k}(\tau p_{k})^{1-\sigma}\right)^{1/(1-\sigma)} \tag{7}$$

Factor-market clearing implies that the total demand for workers has to equate total supply in each region j:

[9] In deriving (3) and (4) we have utilised the equality between producer prices and wages.

$$L_j = n_j \left(\alpha + \beta (x_{jj} + \tau x_{jk})\right) + m\left(\frac{\alpha + \gamma^h}{2} + \beta x_{jj}\right) \quad (8)$$

For a given supply of workers in the two regions, the system of equations and inequalities specified by (2) – (8) determine equilibrium values of the number of active firms of each type, wages, prices, delivered quantities to different regions and total incomes.

2.2 Condition for Entry of Horizontal Multi-Region Firms

A manufacturing firm has incentive to locate production in both regions in order to avoid trade costs. However, the gain from saving on trade costs has to be weighed against the additional fixed costs that have to be incurred in order to become a multi-region firm.

When trade costs are sufficiently high and/or the additional fixed costs for setting up a second plant sufficiently low, there will only be multi-region firms in an equilibrium where manufacturing workers are divided symmetrically between the two regions (cf. Horstmann and Markusen, 1992, Brainard, 1993). In such a symmetric equilibrium, the endogenous variables will take identical values for the two regions, and, thus, we can drop the region indices. Furthermore, it is easily verified that in this equilibrium $w_1=w_2=1$. Using this reveal that the multi-region firms have higher variable profits, on the one hand, but higher fixed costs, on the other. Using expressions for profits and the factor-market clearing condition we find that profits for multi-region firms are higher than profits for single-region firms if:

$$\frac{\gamma^h}{\alpha} < \frac{(1-\tau^{1-\sigma})}{(1+\tau^{1-\sigma})}. \quad (9)$$

The right hand side of (9) is bounded between zero and one and increasing in τ. This implies that as the additional fixed cost of becoming a multi-region firm approaches the fixed cost of a single-region firm, α, condition (9) is unlikely to be satisfied and there will be only single-region firms. On the other hand, as τ goes to infinity (i.e., the expression on the right hand side approaches one), (9) is likely to be satisfied and there will be only multi-region firms.

Is it possible that even if there are no multi-region firms in the symmetric equilibrium, they will enter into the analysis as regions become dissimilar in size? No, the multi-region strategy will always be more advantageous when the regions are of similar size. Because a fairly large fraction of output would be exported in both directions if there were single-region firms, the saving on trade costs for a multi-region firm is the largest in this situation. Therefore, if no multi-region firms enter in the symmetric equilibrium, they will never enter and the analysis becomes identical to the one generated by the original CP-model. In other words, violation of condition (9) implies that the model behaves exactly like the original CP-model. Consequently, in the subsequent analysis we shall assume that condition (9) is satisfied. In the symmetric equilibrium, we then have the following:

$$p = w = 1, \quad Y = \frac{1}{2}, \quad x = \sigma(\alpha + \gamma^h), \quad m = \mu / \sigma(\alpha + \gamma^h), \quad P = \left[\frac{\mu}{\sigma(\alpha + \gamma^h)}\right]^{\frac{1}{1-\sigma}}$$

2.3 Stability Analysis

A well-known property of the CP-model is that it exhibits multiple equilibria. Our point of departure is the symmetric equilibrium, which because of the symmetry of the parameters must be an equilibrium. However this equilibrium may or may not be stable. In this section, we analyse the stability of the symmetric equilibrium as trade costs are lowered.

The standard way of analysing stability of the symmetric equilibrium is to move one unit of manufacturing labour to the other region, let firms enter and exit, and thereafter compare the real wage in the two regions (e.g., Krugman, 1991). If the real wage in the receiving region is lower, the initial equilibrium is considered stable.

However, if the real wage in the receiving region is higher, even more workers will want to move there, and the initial equilibrium is taken to be unstable.

The real wage ω of each region *j* is given by:

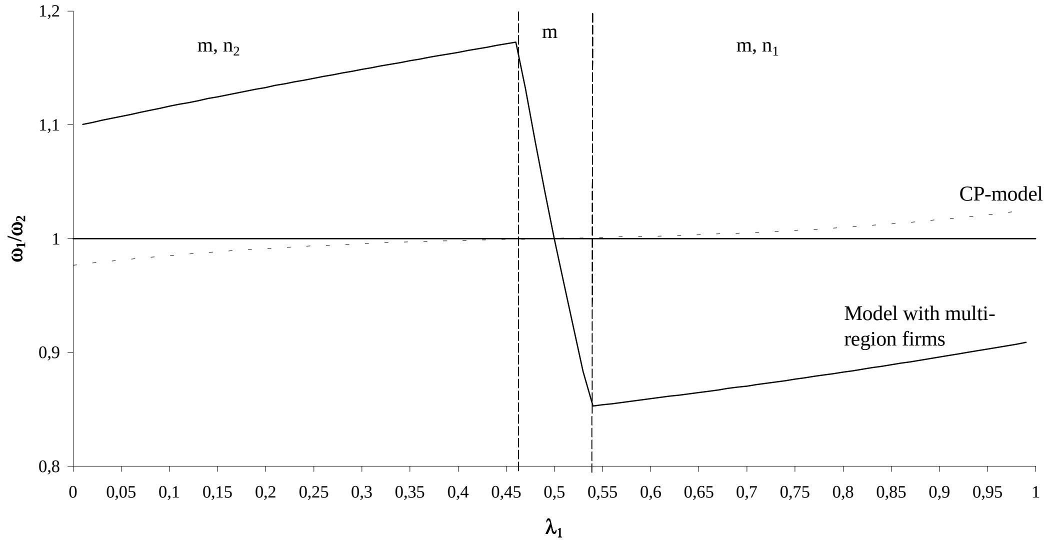
$$\omega_j \equiv w_j P_j^{-\mu} \tag{10}$$

One important way in which the analysis in the case with multi-region firms differs from the one generated by the original CP-model is that in the former there is no trade in the symmetric equilibrium. All manufacturing varieties are produced in both regions. This implies that there are no agglomeration forces at work as long as single-region firms do not enter. The reason for this is that the usual advantages of a lower price index and a larger home market in a larger region disappear when firms produce in both regions. In other words, the horizontal multi-region firms cut off the supply and demand linkages in the CP-model.

Figure 1 shows how relative real wages are affected by the allocation of workers between the regions using the following parameter values: τ=1.6, σ=4, μ=0.3, γ^h = 0.2α.[10] For comparison we also display (as a dotted line) the relative real wage for the original CP-model with the same parameter values. Since parameter values have be chosen so that condition in (9) is satisfied, only multi-region firms are operating in the symmetric equilibrium.

[10] The simulations have been run in GAMS which is able to handle complementarity problems (see Rutherford, 1994).

Figure 1. Horizontal Firms, τ=1.6



Starting at the symmetric equilibrium, a reallocation of workers from one region to another will reduce the relative real wage of the region that becomes larger. Thus, the symmetric equilibrium with multi-region firms is stable, whereas it would be unstable in the original CP-model (since the dotted line is upward sloping). For allocations of workers close to the symmetric one, multi-region firms continue to be the only firm-type active. Here, a reallocation of workers decreases the relative real wage of the larger region. Since all varieties are produced in both regions the price index will only depend on the nominal wage in the region and the number of varieties that are produced. This means that the relative real wage is given by

$$\frac{\omega_1}{\omega_2}=\left(\frac{w_1}{w_2}\right)^{1-\mu},$$

i.e., the relative real wage will be determined by the relative nominal wage. Without any linkages affecting the demand for labour, an increase in the supply of workers will lead to a decrease in nominal wages. Therefore, the relative nominal wage will be lower in the region with the larger supply of workers.

To establish the effect on nominal wages of a reallocation of workers, we solve for w_1, using the factor market clearing condition in (8):

$$w_1 = \frac{\mu(1-\mu)(\sigma-1)}{2\lambda_1\mu(\sigma-\mu(\sigma-1)) - m\sigma(\alpha+\gamma^h)}. \quad (11)$$

The derivative of this expression with respect to λ_1 is negative, which implies that the nominal wage in region 1 falls as region 1 holds a larger fraction of the total supply of workers.[11]

Next consider what happens outside the range around the symmetric equilibrium where only multi-regional firms exists. As we make regions more dissimilar in size, the proportion of output that firms sell in the small region becomes smaller. This means that the multi-region strategy becomes less advantageous and, eventually, there will be entry of single-region firms in the larger region.

The entry of single-region firms introduces agglomeration forces into the model. The single-region firms have better access to their domestic market than to the other one. This means that as domestic demand increases relative to foreign demand, the firms can afford to pay higher nominal wages (the demand linkage). At the same time, the induced increase in the number of locally produced varieties tends to lower the price index on manufacturing products (the supply linkage). This tends to raise the relative real wage of the expanding region. As can be verified from Figure 1, this is what happens as we successively reallocate workers from the smaller to the larger region.

[11] This can be shown by noting that

$$\text{sign}\left(\frac{\partial w_1}{\partial \lambda_1}\right) = \text{sign}\big(\mu(\sigma-1)-\sigma\big).$$

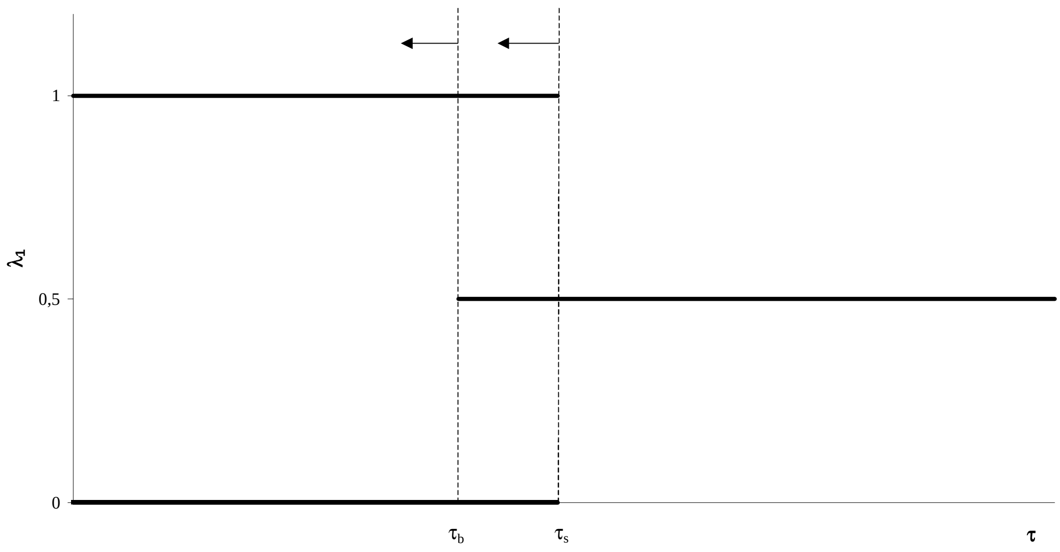
For $\sigma>1$ and $0<\mu<1$, the right hand side of this expression will always be negative. An expression corresponding to (11) for w_2 shows that the wage rate in region 2 rises with λ_1.

In the particular case shown in Figure 1, the increase in the relative real wage of the larger region is not large enough to bring the model to a new equilibrium where real wages are fully equalised. However, if potential agglomeration forces were stronger, we could get unstable equilibria in between the symmetric equilibrium and the core-periphery outcome. With the parameter values chosen in Figure 1, such unstable equilbria arise when τ lies in the range between 1.15 and 1.27. As we lower τ below 1.15, trade costs are too low for any firms to find it advantageous to produce in both regions, so there will only be single-region firms active. We then get the same long-run outcome as in the original CP-model, i.e. the core-periphery solution.

2.4 Long-run equilibria

The effect on long-run equilibria is shown in Figure 2. To the left of τ_s (the so-called sustain point), the core-periphery outcome is supported by the original CP-model. To the left of τ_b (the so-called break point), the symmetric equilibrium is unstable in the CP-model. In between τ_b and τ_s, the symmetric equilibrium is locally, but not globally, stable. The effect of introducing horizontal multi-region firms is to push both τ_b and τ_s to the left. Thus, the symmetric equilibrium, now dominated by multi-region firms, is stable for a larger range of trade costs.

Figure 2. The Effect of Introducing Horizontal Firms on Long-Term Equilibra



We can also note that, for a given level of trade costs, changes in the degree of multiplant economies of scale will have a similar effect as changes in trade costs on the location of production. For a high degree of multiplant economies of scale, the symmetric equilibrium will be stable because firms tend to be multi-region firms. For a low degree of multiplant economies, on the other hand, we may get the core-periphery outcome because firms tend to be single-region firms.

The fact that trade costs and the degree of multiplant economies of scale may change simultaneously has important implications. Reduced trade costs due to regional integration may increase the tendency of agglomeration. However, technical improvements that make it less costly to set up and maintain production units in several geographical locations are likely to have the opposite effect. The present trend with rapid technical progress in information technology may therefore be a force that counteracts agglomeration tendencies created by regional integration. With technological changes that lead to a successive increase in the degree of multiplant of scale, a dispersed production structure may thus very well continue to be supported in the face of far-reaching trade liberalisation.

3. Vertical Multi-Region Firms

We now turn to a model where we allow firms to become vertical multi-region firms, i.e., firms that can split their activities and locate different types of activities in different regions. The variable costs are assumed to be associated with production of the final products, while the fixed cost of setting up a firm is assumed to be associated with headquarter services, defined as services that are freely traded within the firm.[12] Headquarter services may include activities such as back-office activities, management, marketing and R&D.[13]

The firm has to decide whether to produce as a local firm, exporting to the other region, or to become a vertical firm that is headquartered in one region but produces in the other. If the firm decides to become a vertical firm, it also has to decide where to locate production and where to locate the headquarters. Furthermore, if it decides to become a vertical firm, it will export some of its output back to its home region (i.e., the region in which it has its headquarters).

3.1 The Model

The fixed cost α is now interpreted as fixed costs for headquarter services only and we assume the following cost function for vertical multi-region firms:

$$w_j L_{ij} + w_k L_{ik} = \alpha w_j + \beta x_{ik} w_k \,; \; j = 1, 2; \; k = 1, 2; j \neq k \qquad (12)$$

where region j is assumed to be the region where headquarters are located, while k is the region where actual production takes place.

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[12] The assumption that headquarter services are freely traded implies that headquarters can be located solely on the basis of production costs. As far as R&D activities are concerned, this assumption is the most appropriate for codified knowledge such as patents and blueprints. It is, however, easy to imagine activities where this assumption is less suitable. For instance, some R&D activities consist of developing and adapting the production process. This type of activity would be costly to undertake at a different location than the actual production. For our purposes we will simply assume that this type of R&D is part of the production process.

[13] It could be argued that separating geographically headquarters from production plants should involve additional costs, for instance, because communication becomes more costly. However, adding such an extra fixed cost does not alter the substance of the analysis in this section (see Ekholm & Forslid, 1998).

The complementary slackness equation for m_j, the number of multi-region firms headquartered in region j, can be written as:

$$\frac{\left(x_{kk} + \tau x_{kj}\right)}{\sigma} w_k - \alpha w_j \le 0 \qquad (13)$$

and the factor-market clearing condition as:

$$L_j = n_j\left(\alpha + \beta(x_{jj} + \tau x_{jk})\right) + m_j \alpha + m_k \beta(x_{jj} + \tau x_{jk}) \qquad (14)$$

Furthermore, the price index has to be redefined as

$$P_j \equiv \left((n_j + m_k) p_j^{1-\sigma} + (n_k + m_j)(\tau p_k)^{1-\sigma}\right)^{1/(1-\sigma)}. \qquad (15)$$

Equilibrium values of the number of active firms, wages, prices, delivered quantities and incomes are now given by the system of equations and inequalities specified by (2)-(3), (5)-(6), and (13)-(15).

Because headquarter services are freely traded, the firms will choose to locate headquarters where production costs are the lowest, i.e., where nominal wages are the lowest.[14] We therefore now turn to determining which region has the lower nominal wage.

[14] Note that as manufacturing firms only use inputs of homogenous workers in this model, this does not imply that firms will locate headquarters in typical low-wage countries, meaning to be countries relatively abundant in unskilled labour. It only implies that, all else equal, the firm will choose to locate its headquarters where it is cheapest to produce headquarter services of a given quality. If headquarters were more intensive in skilled labour than production plants (which seems to be the empirically relevant assumption), firms would tend to locate their headquarters in the relatively skill-abundant region. The location of production plants, on the other hand, would be affected by market access considerations, i.e. relative size. However, allowing for factor mobility in such a model, where real factor returns are affected by both relative size and relative factor endowments is beyond the scope of this paper.

3.2 The relationship between region size and nominal wage

The relationship between region size and nominal wage in the absence of vertical multi-region firms can be derived in the following way: Start out in the symmetric equilibrium where $w_1 = w_2 = 1$. From the zero-profit condition follows that output levels per firm are constant. By substituting the demand functions we get the so-called wage equation for firms producing final goods in region *j* (Fujita *et al*., 1999, p. 52-53):

$$w_j = \left(\frac{Y_j}{P_j^{1-\sigma}} + \frac{\tau^{1-\sigma} Y_k}{P_k^{1-\sigma}} \right)^{\frac{1}{\sigma}} \tag{16}$$

Differentiation of (16), the nominal income equation (6), and the price index (15) on the assumption that there are single-region firms only yields after substitution:

$$\frac{\mathrm{d}w_j}{\mathrm{d}\lambda_j} = \frac{2z(\mu - z)}{\sigma(1-z^2) - z(\mu - z)} \tag{17}$$

where $z = (1-\tau^{1-\sigma})/(1+\tau^{1-\sigma})$. Since the denominator of the right hand side of (17) is always positive, a necessary and sufficient condition for the whole expression to be negative, and thus for the nominal wage to decrease in the region that becomes slightly larger, is that $z>\mu$.[15] Using the definition of *z* the condition can also be written as:

$$\tau > \left(\frac{1+\mu}{1-\mu} \right)^{\frac{1}{\sigma-1}} \tag{18}$$

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[15] Note that the denominator in (17) is increasing in σ. If we evaluate the denominator at $\sigma=1$, we get $1-z\mu$. This expression is positive since $z<1$ and $\mu<1$. Consequently, the denominator must be positive for all possible values of $\sigma>1$.

Condition (18) reveals that if trade costs are low and/or the share of manufacturing products in total spending high, nominal wages may increase rather than decrease in the larger region. Agglomeration forces are in this case particularly strong. Therefore, the reallocation of workers not only increases the real wage in the receiving region, but the nominal wage as well. In this case, the symmetric equilibrium will always be unstable and, just as it would be in the original CP-model, we always get the core-periphery outcome in the long-run equilibrium.[16] In order to focus on novel results, we therefore restrict our analysis to the case where $z>\mu$.

3.3 Stability Analysis

Figure 3 shows, using the same values of σ, μ and τ as in Figure 1, how relative real wages are affected by the allocation of workers.[17] In the symmetric equilibrium, the number of headquarters and plants will be equal in each of the two regions.[18]

Consider what happens as we make one region slightly larger in terms of the supply of workers. The location of headquarters and production is simultaneously determined by free entry and exit of firms and by headquarters moving to the region with lower nominal wage. Headquarters will relocate until nominal wages are equalised between the regions (or until they become completely concentrated in the larger region). The analysis in the previous section shows that entry and exit of headquarters and plants in fixed proportions, as in the CP-model, would not yield a new equilibrium, since the nominal wage would become lower in the larger region. Consequently, the proportion of headquarters must be higher in the larger region. Is it possible that only headquarters will move to the larger region while the total number of production units is unchanged? The answer is no. Because production costs are the same while sales are higher in the larger market, profits will be higher for firms

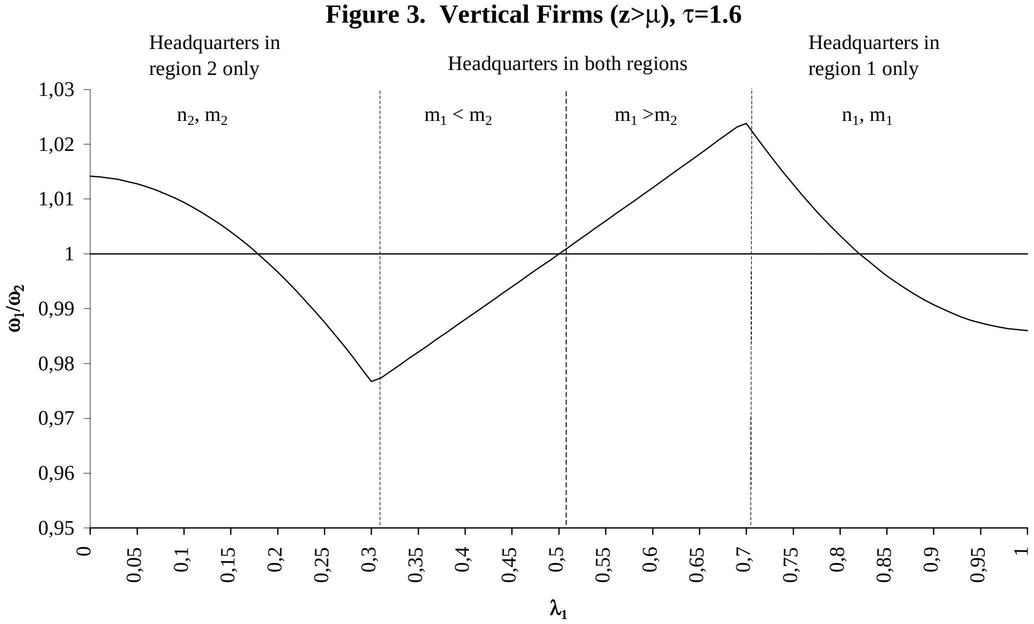
[16] This case produces the interesting possibility that the smaller region becomes specialised in headquarter activities for given factor allocations. For a more detailed analysis of this case, see Ekholm and Forslid (1998).

[17] These parameter values satisfy the condition $z>\mu$.

[18] The number of headquarters and plants in each region will be equal to $\mu/2\alpha\sigma$ and $x = \alpha\sigma$.

producing in the larger region. This gives incentives to relocate production as well to the larger region.

Figure 3. Vertical Firms (z>μ), τ=1.6



Consider now the stability properties of the model around the symmetric equilibrium. The relative nominal wage will be unaffected by reallocations of workers between the two regions as long as headquarter activities are undertaken in both regions. However, the price index will be lower in the larger region since more varieties are produced there. This implies a higher relative real wage in the larger region and an unstable symmetric equilibrium.

However, when we reach the point where all headquarters are located in the larger region, marked by a kink in the curve in Figure 3, further reallocations of workers will lead to a fall in the relative nominal wage of the larger region. We are then in a situation where only production activities move between the regions. Just as in the CP-model, the nominal wage now falls in the larger region and rises in the smaller region. For the chosen level of trade costs, the nominal wage decreases (increases) more than the price index in the larger (smaller) region, which implies that the relative real wage of the larger region is now decreasing. In Figure 3, the relative real wage curve crosses the horizontal line where real wages are equalised at allocations in between those of complete symmetry and core-periphery. Since the curves cut the horizontal line from above, these equilibria are stable.

From this we may conclude that the introduction of vertical firms have a destabilising effect on the symmetric equilibrium in the sense that it becomes unstable also for very high trade costs. However, the introduction of vertical firms will at the same time have an effect in the direction of weakening agglomeration tendencies in the sense that the parameter space for which we get the core-periphery outcome is reduced. For instance, while, as was shown in Figure 1, the CP-model leads to complete agglomeration for τ=1.6, the model with vertical multi-region firms produces a stable equilibrium with 17-18 percent of manufacturing production in the smaller region. In order for complete agglomeration to occur, trade costs again have to be lower than in the original CP model.

The intuition for this result is that as headquarters tend to concentrate in the region with lower wages, the symmetric equilibrium becomes destabilised. However, the introduction of vertical firms also has another effect. Normally, in order to produce in the smaller region firms have to attract workers by paying a higher nominal wage to compensate for a higher price index in that region. However, when firms can split production and headquarters it becomes less costly to locate production in the smaller region, since the firms then can go on paying the lower nominal wage to headquarter workers employed in the larger region. This also means that the potential cost saving from moving production into the larger region is less with vertical multi-region firms than with single-region firms only. Because of this, full agglomeration will occur at a lower trade cost than in the CP-model.

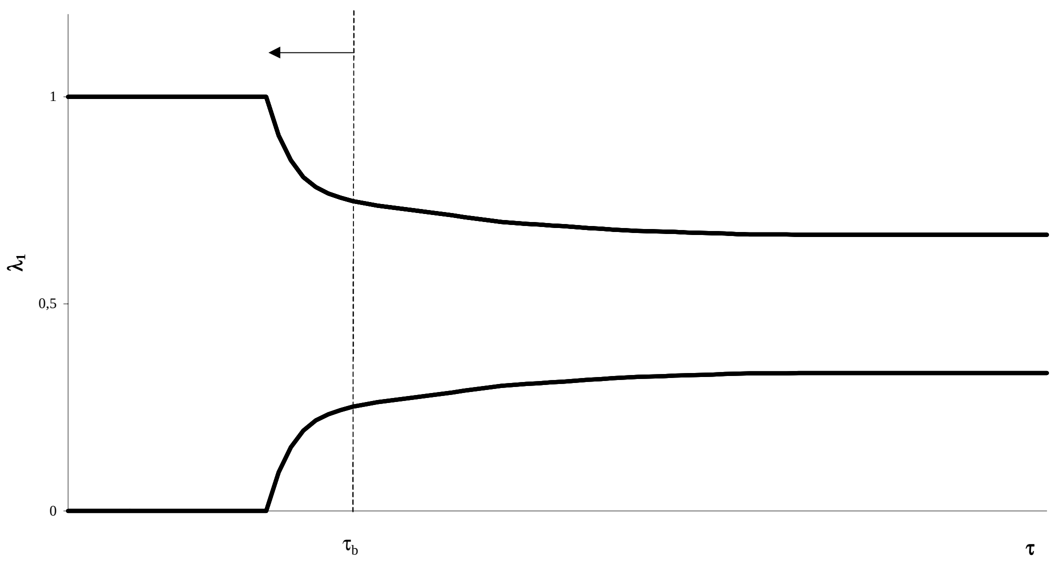
Finally, consider the effect of a gradual trade liberalisation. Beyond the kink in Figure 3 the model behaves like the CP-model. Lower trade costs therefore rotate the curve anti-clock wise, which implies that the stable equilibrium will become more and more asymmetric. Finally, it will slide all the way to complete agglomeration.

Agglomeration is thus here a gradual process rather than the catastrophical event that occurs in the original CP-model as soon as trade costs fall below some threshold level.

3.4 Long-run equilibria

The long-run equilibria are shown in Figure 4. The symmetric equilibrium will not be stable even for high trade costs. Instead, there will be stable equilibria in between complete symmetry and the core-periphery structure. As trade costs are lowered, the stable equilibria will become more and more asymmetric, until, eventually, we get the core-periphery outcome. This occurs for a lower trade cost than in the original CP-model, which means that the sustain point, τ_s, is pushed to the left.

Figure 4. Long-Term Equilibria with Vertical Firms



4. Conclusions

Our analysis shows that the introduction of horizontal multi-region firms tends to weaken the tendencies for agglomeration in the sense that it decreases the range of trade costs for which the core-periphery outcome occurs. This effect is more pronounced the higher the degree of multiplant economies of scale.

The case of vertical multi-region firms is less clear-cut. In our analysis we have assumed that headquarter services are freely traded within the firm, so that only relative production costs matter for the location of headquarters. For final production, on the other hand, both relative production costs and market access are important factors determining the location. Just as with horizontal multi-region firms, the range of trade costs for which full agglomeration occurs decreases. However, because headquarters tend to agglomerate in one region, the symmetric equilibrium becomes unstable. Taken together, this implies that the introduction of vertical multi-region firms generates a tendency for outcomes that lie in between the two extremes of complete symmetry and complete agglomeration. Changes in trade costs produce more gradual changes in the location of industrial activity compared to the original CP-model. Furthermore, long-term equilibria may very well be asymmetric in spite of underlying symmetry in the parameters of the model.

References

Audretsch, D. B. and M. P. Feldmann (1996), "R&D Spillovers and the Geography of Innovation and Production", *American Economic Review*, 86, 630-640.

Baldwin R. and A. J. Venables (1996) "Regional Economic Integration". In *Handbook of International Trade, Vol. III*. Amsterdam: North-Holland.

Brainard, S. L. (1993), "A simple theory of multinational corporations and trade with a trade-off between proximity and concentration", NBER Working Paper No. 4269.

Ekholm K. and R. Forslid (1998) "Trade and Location with Horizontal and Vertical Multi-Region Firms", IUI Working Paper No. 504.

Horstmann, I. J. and J. R. Markusen (1992), "Endogenous market structure structures in international trade (natura facit saltum)", *Journal of International Economics*, 32, 109-129.

Fujita, M., P. R. Krugman, and A. J. Venables (1998), *The Spatial Economy: Cities, Regions, and International Trade*, mimeo.

Krugman P. R. (1991) "Increasing Returns and Economic Geography", *Journal of Political Economy* 99, 483-99.

Krugman P. R. and A. Venables (1995) "Globalization and the Inequality of Nations", *Quarterly Journal of Economics* 60, 857-80.

Markusen J. R. and A. J. Venables (1996) "The Theory of Endowment, Intra-Industry and Multinational Trade", *CEPR* Working Paper No.1341.

Markusen J. R. and A. J. Venables (1998) "Multinational Firms and the New Trade Theory", *Journal of International Economics* 42, 183-203.

Martin, P. and G. I. P. Ottaviano (1996), "Growth and Agglomeration", CEPR Discussion Paper No. 1529.

Ottaviano, G. I. P. and D. Puga (1998), "Agglomeration in the Global Economy: A Survey of the 'New Economic Geography'", *World Economy* 21, 707-731.

Puga, D. (1997), *Regional Integration and the Location of Economic Activity*, Ph.D. dissertation, London School of Economics.

Rutherford, T. F. (1994), "Extensions of GAMS for Complementarity Problems Arising in Applied Economic Analysis", mimeo, Department of Economics, University of Colorado at Boulder.