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Monetary Policy Trade-Offs in an Estimated Open-Economy DSGE Model*

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Abstract

This paper studies the trade-offs between stabilizing CPI inflation and alternative measures of the output gap in Ramses, the Riksbank's estimated dynamic stochastic general equilibrium (DSGE) model of a small open economy. Our main finding is that the trade-off between stabilizing CPI inflation and the output gap strongly depends on which concept of potential output in the output gap between output and potential output is used in the loss function. If potential output is defined as a smooth trend this trade-off is much more pronounced compared to the case when potential output is defined as the output level that would prevail if prices and wages were flexible.

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Keywords: Optimal monetary policy, instrument rules, open-economy DSGE models, propagation of shocks, impulse responses, output gap, potential output

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1. Introduction

In this paper, we use an estimated open economy model to study the trade-off between stabilizing CPI inflation and the output gap, and how this trade-off depends on alternative definitions of the output gap. Specifically, we compare variance trade-offs under optimal monetary policy and under an estimated instrument rule. We do this analysis in Ramses, the main model used at Sveriges Riksbank for forecasting and policy analysis. Ramses is a small open-economy dynamic stochastic general equilibrium (DSGE) model estimated with Bayesian techniques and is described in Adolfson, Laséen, Lindé, and Villani (ALLV) [4] and [5].

The notion that alternative definitions of the output gap can have important implications for the conduct of monetary policy is visualized in figure 1.1, which depicts one statistical and three model-based output gaps in Sweden 1997-2007.[1] As expected, the correlation is highest between the statistical HP-filtered output gap and the model trend output gap (where the trend is the model's unit-root technology shock). Even so, the upper panel of the figure demonstrates that the correlation between the routinely-used statistical HP-filtered output gap and all three model based gaps is well below unity, and that their variances are also clearly different.[2] By implication, adhering to one of these measures should have non-trivial implications for monetary policy.

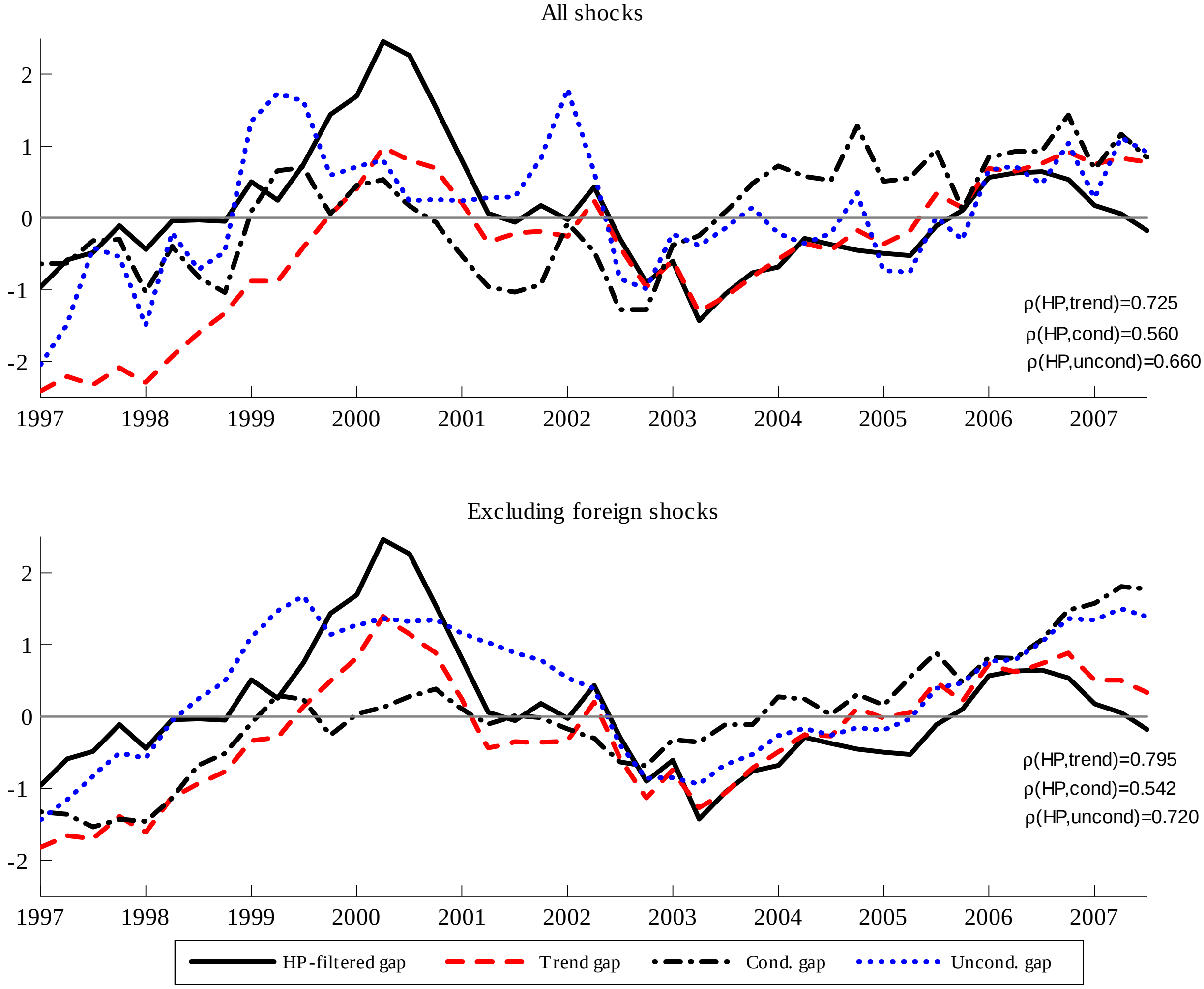
We define optimal monetary policy as a central bank that minimizes an intertemporal loss function under commitment. We assume the central bank adopts a quadratic loss function that corresponds to flexible inflation targeting and is the weighted sum of three terms: the squared inflation gap between 4-quarter CPI inflation and the inflation target, the squared output gap (measured as the deviation between output and potential output), and the squared quarterly change in the central banks policy rate. To get an idea about how inefficient the empirically estimated rule is compared with optimal policy and about the policy preferences implied by the estimated rule, we compare the optimal policy with policy following the estimated instrument rule.

The definition of potential output is important since this latent variable is used to compute the output gap (the difference between output and potential output) in the loss function. A conventional measure of potential output is a smooth trend, such as the result of a Hodrick-Prescott (HP) filter.

[1] We use Swedish data on seasonally adjusted GDP per capita 1980Q2 -2007Q3 as our measure of output. Potential output computed with the HP-filter uses a smoothing coefficient of $\lambda = 1600$ on actual data, whereas the trend, flexible price conditional and unconditional potential output is computed via Kalman filtering techniques using the estimated model in section 2. Exact definitions of the various concepts of potential output in the model are provided in section 2.1.5.

[2] The correlation coefficients between the HP-filtered output gap and the estimated DSGE model's output gaps are not computed on data after 2005Q4 to avoid the well-known endpoint problems of the HP-filter (which causes the HP-filtered gap to drop notably towards the end of the sample in Figure 1.1).

Figure 1.1: Output gaps for Sweden 1997Q1 - 2007Q3 using different measures of potential output.



A second definition of potential output, promoted in the recent academic literature, is defined as the level of output that would prevail if prices and wages were flexible, see for instance Woodford [25] and Galí [14]. This latter measure of potential output is in line with the work of Kydland and Prescott [19], since it incorporates efficient fluctuations of output due to technology shocks.

Using an approach similar to ours, subsequent work by Justiniano, Primiceri and Tambalotti [17], and Edge, Kiley and Laforte [13] present measures of potential output for the US economy within closed-economy frameworks. Justiniano, Primiceri and Tambalotti [17] study the inflation and output stabilization trade-off in the US using an estimated DSGE model. They find that the gap between optimal output (maximizing the household's utility function) and potential output (the fully competitive equilibrium) is virtually zero once they treat the observed high-frequency movement in wages as measurement errors rather than variations in workers' market power. Therefore, they conclude that inefficient movements in US output could have been eliminated without increasing price and wage inflation. To the extent that the welfare function is a good representation of the actual monetary policy objectives, they find that the historical conduct of monetary policy - as described by an estimated interest rate rule - has not performed well. We extend their analysis to an open-economy setting by using an estimated DSGE model with trade channels. By comparing the upper and lower panels in figure 1.1 above, we see that open economy aspects matter importantly for the computed output gaps.[3] Another important difference is that we build on the recent empirical results in Gali, Smets and Wouters [15], and assume that observed movements in real wage represent variations in workers' market power. Finally, and as mentioned above, we do not use the model's welfare function, but model the monetary policy objectives directly.

To build intuition behind the results, we start out by discussing how alternative monetary policies affect the transmission of **two** key shocks in the model. According to our estimated model, shocks to total factor productivity is a dominant driver of business cycles in Sweden (at least for policy under a simple instrument rule), why these are particularly interesting to study. The estimated model assigns a dominant role to productivity shocks in order to explain the fact that the correlation between GDP growth and CPI inflation is about -0.5 for the years 1950–2007**.** Productivity shocks have also been shown by ALLV [6] to play a key role for understanding the episode with low inflation and high output growth in Sweden 2003–2006. We therefore examine how monetary policy may affect the propagation of very persistent but stationary technology shocks. By comparing the impulse-response functions for stationary technology shocks conditional on optimal policy and conditional on the estimated instrument rule, we find that monetary policy indeed has an important role in the transmission of these shocks into the economy. Moreover, the specification of potential output in the output-gap definition is important for the transmission of technology shocks. If potential output is defined as trend output, the output response after a technology shock will be substantially smaller than if potential output is specified as the level of output under flexible prices and wages.[4] Furthermore, to examine the effects of a shock that creates a trade-off between

[3] The foreign shocks are those defined in Section 4, with the exception that the unit root technology shock (which is common to both the domestic and foreign economies to ensure balanced growth in the model) is here treated as a domestic shock for comparison with the closed economy literature. If we include the permanent technology shock among the set of foreign shocks, we would see more noticeable differences in the low frequency components of the output gaps in Figure 1.1. Finally, notice that the statistical HP-filtered gap is kept unchanged in both panels to provide a basis of reference.

[4] As in ALLS [2], we consider both a conditional and an unconditional measure of potential output under flexible prices and wages. Conditional potential output is contingent upon the existing current predetermined variables, whereas unconditional potential output is computed assuming the flexible price equilibrium has lasted forever, see Section 2.1.5 for further details.

inflation and output-gap stabilization regardless of which output-gap definition is used in the loss function, we also analyse the impulse responses to a labor supply shock.

We then examine the variance trade-off the central bank is facing under various specifications of the loss function, comparing the different output-gap definitions. Results for the estimated instrument rule are also reported. The efficient variance frontiers are computed with a given weight on interest-rate smoothing. As a benchmark, we use a weight of 0.37 on the squared changes in the nominal interest rate in the loss function.[5] However, it turns out that the volatility of the nominal interest rate in this case heavily violates the zero lower bound (ZLB) of the interest rate. Therefore, we also follow the suggestion by Woodford [25] and Levine, Pearlman, and Yang [20] and investigate to what extent the efficient variance frontier is affected by increasing the weight on the squared interest rate in the loss function, in order to ensure a low probability of the nominal interest rate falling below zero. In addition, we quantify to what extent the estimated instrument rule can be improved by optimizing the response coefficients of the simple instrument rule to minimize the loss function. Finally, we examine how different sets of shocks (technology, markup, preference, and foreign shocks) affect the variance trade-offs faced by the central bank for different definitions of the output gap in the loss function.

Our main findings are as follows. First, the stationary productivity shocks create a sharp trade-off between stabilizing CPI inflation and stabilizing the output gap when trend output is computed with a smooth trend. Second, using an output gap in the loss function where potential output is defined as the level of output under flexible prices and wages improves the policy trade-off, but the trade-off still remains significant, in particular for labor supply shocks (which are isomorphic to wage markup shocks) and price markup shocks. Third, the estimated instrument rule is clearly inefficient relative to optimal policy. Most of this ineffectiveness is driven by the fact that the estimated policy rule responds very inefficiently to fluctuations induced by foreign shocks. Fourth, optimizing the coefficients in the simple instrument rule closes about half the distance relative to optimal policy. Finally, imposing the ZLB constraint for the nominal interest rate shifts out the variance frontiers somewhat, but the conclusions regarding the trend output gap and the flexible price-wage output gap are − at least in our approximative approach − robust to introducing this constraint.[6]

[5] This number stems from estimating the model on Swedish data under the assumption that the Riksbank conducted monetary policy according to the loss function with the trend output gap, see ALLS [2].

[6] This conclusion is supported by the findings in Hebden, Lindé and Svensson [16] which show, by means of stochastic simulations in the standard hybrid New Keynesian model, that the difference between unconstrained (no zero bound constraint) and constrained (respecting the non-linear zero lower bound constraint) optimal monetary policy under commitment differs very little for empirically plausible probabilities of hitting the zero lower bound.

The outline of the paper is as follows: Section 2 presents the model and very briefly discusses the data and the estimation of the model. Section 3 discusses the impulse responses to a stationary technology shock and a labor supply shock and their dependency on the policy assumption made. Section 4 illustrates the variance trade-offs the central bank is facing under different output-gap definitions. Finally, section 5 presents a summary and some conclusions. An appendix contains some technical details. More technical details are reported in Adolfson, Laséen, Lindé and Svensson (ALLS) [3].

2. Model and parameters

2.1. Model overview

Ramses is a small open-economy DSGE model developed in a series of papers by ALLV [4] and [5], and shares its basic closed economy features with many new Keynesian models, including the benchmark models of Christiano, Eichenbaum and Evans [9], Altig, Christiano, Eichenbaum and Lindé [7], and Smets and Wouters [23]. The model economy consists of households, domestic goods firms, importing consumption and importing investment firms, exporting firms, a government, a central bank, and an exogenous foreign economy. Within each manufacturing sector there is a continuum of firms that each produces a differentiated good and sets prices according to an indexation variant of the Calvo model. Domestic as well as global production grows with technology that contains a stochastic unit-root, see Altig et al. [7]. In what follows we provide the optimization problems of the different firms and the households, and describe the behavior of the central bank.[7]

2.1.1. Domestic goods firms

The domestic goods firms produce their goods using capital and labor inputs, and sell them to a retailer which transforms the intermediate products into a homogenous final good that in turn is sold to the households.

The final domestic good is a composite of a continuum of differentiated intermediate goods, each supplied by a different firm. Output, Y_t, of the final domestic good is produced with the constant elasticity of substitution (CES) function

$$Y_t = \left[\int_0^1 (Y_{it})^{\frac{1}{\lambda_t^d}} \, di \right]^{\lambda_t^d}, \quad 1 \leq \lambda_t^d < \infty, \tag{2.1}$$

[7] For a complete list of the log-linearized equations in the model we refer to ALLS [2].

where Y_{it}, $0 \leq i \leq 1$, is the input of intermediate good i and λ_t^d is a stochastic process that determines the time-varying flexible-price markup in the domestic goods market. The production of the intermediate good i by intermediate-good firm i is given by

$$Y_{it} = z_t^{1-\alpha} \epsilon_t K_{it}^{\alpha} H_{it}^{1-\alpha} - z_t \phi, \tag{2.2}$$

where z_t is a unit-root technology shock common to the domestic and foreign economies, ϵ_t is a domestic covariance stationary technology shock, K_{it} the capital stock and H_{it} denotes homogeneous labor hired by the i^{th} firm. A fixed cost $z_t \phi$ is included in the production function. We set this parameter so that profits are zero in steady state, following Christiano et al. [9].

We allow for working capital by assuming that a fraction ν of the intermediate firms' wage bill has to be financed in advance through loans from a financial intermediary. Cost minimization yields the following nominal marginal cost for intermediate firm i:

$$\mathrm{MC}_{it}^d = \frac{1}{(1-\alpha)^{1-\alpha}} \frac{1}{\alpha^\alpha} (R_t^k)^\alpha \left[W_t(1+\nu(R_{t-1}-1))\right]^{1-\alpha} \frac{1}{z_t^{1-\alpha}} \frac{1}{\epsilon_t}, \tag{2.3}$$

where R_t^k is the gross nominal rental rate per unit of capital, R_{t-1} the gross nominal (economy wide) interest rate, and W_t the nominal wage rate per unit of aggregate, homogeneous, labor H_{it}.

Each of the domestic goods firms is subject to price stickiness through an indexation variant of the Calvo [8] model. Each intermediate firm faces in any period a probability $1 - \xi_d$ that it can reoptimize its price. The reoptimized price is denoted $P_t^{d,new}$. For the firms that are not allowed to reoptimize their price, we adopt an indexation scheme with partial indexation to the current inflation target, $\bar{\pi}_{t+1}^c$, since there is a perceived (time-varying) CPI inflation target in the model , and partial indexation to last period's inflation rate in order to allow for a lagged pricing term in the Phillips curve

$$P_{t+1}^d = \left(\pi_t^d\right)^{\kappa_d} \left(\bar{\pi}_{t+1}^c\right)^{1-\kappa_d} P_t^d, \tag{2.4}$$

where P_t^d is the price level, $\pi_t^d = P_{t+1}^d / P_t^d$ is gross inflation in the domestic sector, and κ_d is an indexation parameter. The different firms maximize profits taking into account that there might not be a chance to optimally change the price in the future. Firm i therefore faces the following optimization problem when setting its price

$$\max_{P_t^{d,new}} \mathrm{E}_t \sum_{s=0}^{\infty} (\beta \xi_d)^s \, \upsilon_{t+s} [(\left(\pi_t^d \pi_{t+1}^d ... \pi_{t+s-1}^d\right)^{\kappa_d} \left(\bar{\pi}_{t+1}^c \bar{\pi}_{t+2}^c ... \bar{\pi}_{t+s}^c\right)^{1-\kappa_d} P_t^{d,new}) Y_{i,t+s} \\ - \mathrm{MC}_{i,t+s}^d (Y_{i,t+s} + z_{t+s} \phi^j)], \tag{2.5}$$

where the firm is using the stochastic household discount factor $(\beta\xi_d)^s \upsilon_{t+s}$ to make profits conditional upon utility. β is the discount factor, and υ_{t+s} the marginal utility of the households' nominal income in period $t+s$, which is exogenous to the intermediate firms.

2.1.2. Importing and exporting firms

The importing consumption and importing investment firms buy a homogenous good at price P_t^* in the world market, and convert it into a differentiated good through a brand naming technology. The exporting firms buy the (homogenous) domestic final good at price P_t^d and turn this into a differentiated export good through the same type of brand naming. The nominal marginal cost of the importing and exporting firms are thus $S_t P_t^*$ and P_t^d/S_t, respectively, where S_t is the nominal exchange rate (domestic currency per unit of foreign currency). The differentiated import and export goods are subsequently aggregated by an import consumption, import investment and export packer, respectively, so that the final import consumption, import investment, and export good is each a CES composite according to the following:

$$C_t^m = \left[\int_0^1 (C_{it}^m)^{\frac{1}{\lambda_t^{mc}}} di\right]^{\lambda_t^{mc}}, \quad I_t^m = \left[\int_0^1 (I_{it}^m)^{\frac{1}{\lambda_t^{mi}}} di\right]^{\lambda_t^{mi}}, \quad X_t = \left[\int_0^1 (X_{it})^{\frac{1}{\lambda_t^{x}}} di\right]^{\lambda_t^{x}}, \tag{2.6}$$

where $1 \leq \lambda_t^j < \infty$ for $j = \{mc, mi, x\}$ is the time-varying flexible-price markup in the import consumption (mc), import investment (mi) and export (x) sector. By assumption the continuum of consumption and investment importers invoice in the domestic currency and exporters in the foreign currency. To allow for short-run incomplete exchange rate pass-through to import as well as export prices we introduce nominal rigidities in the local currency price. This is modeled through the same type of Calvo setup as above. The price setting problems of the importing and exporting firms are completely analogous to that of the domestic firms in equation (2.5).[8] In total there are thus four specific Phillips curve relations determining inflation in the domestic, import consumption, import investment and export sectors.

[8] Total export demand satisfies $C_t^x + I_t^x = \left[\frac{P_t^x}{P_t^*}\right]^{-\eta_f} Y_t^*$, where C_t^x and I_t^x is demand for consumption and investment goods, respectively; P_t^x the export price; P_t^* the foreign price level; Y_t^* foreign output and η_f the elasticity of substitution across foreign goods.

2.1.3. Households

There is a continuum of households which attain utility from consumption, leisure and real cash balances. The preferences of household j are given by

$$\mathrm{E}_0^j \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \ln\left(C_{jt} - bC_{j,t-1}\right) - \zeta_t^h A_L \frac{(h_{jt})^{1+\sigma_L}}{1+\sigma_L} + A_q \frac{\left(\frac{Q_{jt}}{z_t P_t^d}\right)^{1-\sigma_q}}{1-\sigma_q} \right], \tag{2.7}$$

where C_{jt}, h_{jt} and Q_{jt}/P_t^d denote the j^{th} household's levels of aggregate consumption, labor supply and real cash holdings, respectively. Consumption is subject to habit formation through $bC_{j,t-1}$, such that the household's marginal utility of consumption is increasing in the quantity of goods consumed last period. ζ_t^c and ζ_t^h are persistent preference shocks to consumption and labor supply, respectively. Households consume a basket of domestically produced goods (C_t^d) and imported products (C_t^m) which are supplied by the domestic and importing consumption firms, respectively. Aggregate consumption is assumed to be given by the following CES function:

$$C_t = \left[(1-\omega_c)^{1/\eta_c}(C_t^d)^{(\eta_c-1)/\eta_c} + \omega_c^{1/\eta_c}\left(C_t^m\right)^{(\eta_c-1)/\eta_c}\right]^{\eta_c/(\eta_c-1)},$$

where ω_c is the share of imports in consumption, and η_c is the elasticity of substitution across consumption goods.

The households can invest in their stock of capital, save in domestic bonds and/or foreign bonds and hold cash. The households invest in a basket of domestic and imported investment goods to form the capital stock, and decide how much capital to rent to the domestic firms given costs of adjusting the investment rate. The households can increase their capital stock by investing in additional physical capital (I_t), taking one period to come in action. The capital accumulation equation is given by

$$K_{t+1} = (1-\delta)K_t + \Upsilon_t[1 - \tilde{S}\left(I_t/I_{t-1}\right)]I_t, \tag{2.8}$$

where $\tilde{S}\left(I_t/I_{t-1}\right)$ determines the investment adjustment costs through the estimated parameter $\tilde{S}''$, and Υ_t is a stationary investment-specific technology shock. Total investment is assumed to be given by a CES aggregate of domestic and imported investment goods (I_t^d and I_t^m, respectively) according to

$$I_t = \left[(1-\omega_i)^{1/\eta_i}\left(I_t^d\right)^{(\eta_i-1)/\eta_i} + \omega_i^{1/\eta_i}\left(I_t^m\right)^{(\eta_i-1)/\eta_i}\right]^{\eta_i/(\eta_i-1)}, \tag{2.9}$$

where ω_i is the share of imports in investment, and η_i is the elasticity of substitution across investment goods.

Each household is a monopoly supplier of a differentiated labor service which implies that they can set their own wage, see Erceg, Henderson and Levin [12]. After having set their wage, households supply the firms' demand for labor,

$$h_{jt} = \left[\frac{W_{jt}}{W_t}\right]^{\frac{\lambda_w}{1-\lambda_w}} H_t,$$

at the going wage rate. Each household sells its labor to a firm which transforms household labor into a homogenous good that is demanded by each of the domestic goods producing firms. Wage stickiness is introduced through the Calvo [8] setup, where household j reoptimizes its nominal wage rate W_{jt}^{new} according to the following[9]

$$\begin{array}{c} \max\limits_{W_{jt}^{new}} \quad \mathrm{E}_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \left[-\zeta_{t+s}^h A_L \frac{(h_{j,t+s})^{1+\sigma_L}}{1+\sigma_L} + \right. \\ \left. \upsilon_{t+s} \frac{(1-\tau_{t+s}^y)}{(1+\tau_{t+s}^w)} \left(\left(\pi_t^c ... \pi_{t+s-1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c ... \bar{\pi}_{t+s}^c\right)^{(1-\kappa_w)} \left(\mu_{z,t+1} ... \mu_{z,t+s}\right) W_{jt}^{new} \right) h_{j,t+s} \right], \end{array} \tag{2.10}$$

where ξ_w is the probability that a household is not allowed to reoptimize its wage, τ_t^y a labor income tax, τ_t^w a pay-roll tax (paid for simplicity by the households), and $\mu_{zt} = z_t/z_{t-1}$ is the growth rate of the unit-root technology shock.

The choice between domestic and foreign bond holdings balances into an arbitrage condition pinning down expected exchange rate changes (that is, an uncovered interest rate parity condition). To ensure a well-defined steady-state in the model, we assume that there is premium on the foreign bond holdings which depends on the aggregate net foreign asset position of the domestic households, see, for instance, Schmitt-Grohé and Uribe [22]. Compared to a standard setting the risk premium is allowed to be negatively correlated with the expected change in the exchange rate (that is, the expected depreciation), following the evidence discussed in for example Duarte and Stockman [11]. For a detailed discussion and evaluation of this modification see ALLV [5]. The risk premium is given by:

$$\Phi(a_t, S_t, \tilde{\phi}_t) = \exp\left(-\tilde{\phi}_a(a_t - \bar{a}) - \tilde{\phi}_s\left(\frac{E_t S_{t+1}}{S_t}\frac{S_t}{S_{t-1}} - 1\right) + \tilde{\phi}_t\right), \tag{2.11}$$

where $a_t \equiv (S_t B_t^*)/(P_t z_t)$ is the net foreign asset position, S_t the nominal exchange rate, and $\tilde{\phi}_t$ is a shock to the risk premium.

To clear the final goods market, the foreign bond market, and the loan market for working capital, the following three constraints must hold in equilibrium:

$$C_t^d + I_t^d + G_t + C_t^x + I_t^x \leq z_t^{1-\alpha}\epsilon_t K_t^\alpha H_t^{1-\alpha} - z_t\phi, \tag{2.12}$$

[9] For the households that are not allowed to reoptimize, the indexation scheme is $W_{j,t+1} = (\pi_t^c)^{\kappa_w} (\bar{\pi}_{t+1}^c)^{(1-\kappa_w)} \mu_{z,t+1} W_{jt}$, where κ_w is an indexation parameter.

$$S_t B_{t+1}^* = S_t P_t^x \left(C_t^x + I_t^x\right) - S_t P_t^* \left(C_t^m + I_t^m\right) + R_{t-1}^* \Phi(a_{t-1}, \widetilde{\phi}_{t-1}) S_t B_t^*, \tag{2.13}$$

$$\nu W_t H_t = \mu_t M_t - Q_t, \tag{2.14}$$

where G_t is government expenditures, C_t^x and I_t^x are the foreign demand for export goods which follow CES aggregates with elasticity η_f, and $\mu_t = M_{t+1}/M_t$ is the monetary injection by the central bank. When defining the demand for export goods, we introduce a stationary asymmetric (or foreign) technology shock $\tilde{z}_t^* = z_t^*/z_t$, where z_t^* is the permanent technology level abroad, to allow for temporary differences in permanent technological progress domestically and abroad.

2.1.4. Structural shocks, government, foreign economy

The structural shock processes in the model are given by the univariate representation

$$\hat{\varsigma}_t = \rho_\varsigma \hat{\varsigma}_{t-1} + \varepsilon_{\varsigma t}, \qquad \varepsilon_{\varsigma t} \overset{iid}{\sim} N\left(0, \sigma_\varsigma^2\right) \tag{2.15}$$

where $\varsigma_t = \{\ \mu_{zt},\ \epsilon_t, \lambda_t^j,\ \zeta_t^c,\ \zeta_t^h,\ \Upsilon_t,\ \tilde{\phi}_t,\ \varepsilon_{Rt}, \bar{\pi}_t^c,\ \tilde{z}_t^*\},\ j = \{d, mc, mi, x\}$, and a hat denotes the deviation of a log-linearized variable from a steady-state level ($\hat{v}_t \equiv dv_t/v$ for any variable v_t, where v is the steady-state level). λ_t^j and ε_{Rt} are assumed to be white noise (that is, $\rho_{\lambda^j} = 0$, $\rho_{\varepsilon_R} = 0$).

The government spends resources on consuming part of the domestic good, and collects taxes from the households. The resulting fiscal surplus/deficit plus the seigniorage are assumed to be transferred back to the households in a lump sum fashion. Consequently, there is no government debt. The fiscal policy variables – taxes on labor income ($\hat{\tau}_t^y$), consumption ($\hat{\tau}_t^c$), and the payroll ($\hat{\tau}_t^w$), together with (HP-detrended) government expenditures ($\hat{g}_t$) – are assumed to follow an identified VAR model with two lags,

$$\Theta_0 \tau_t = \Theta_1 \tau_{t-1} + \Theta_2 \tau_{t-2} + S_\tau \varepsilon_{\tau t}, \tag{2.16}$$

where $\tau_t \equiv (\hat{\tau}_t^y, \hat{\tau}_t^c, \hat{\tau}_t^w, \hat{g}_t)'$, $\varepsilon_{\tau t} \sim N\left(0, I_\tau\right)$, S_τ is a diagonal matrix with standard deviations and $\Theta_0^{-1} S_\tau \varepsilon_{\tau t} \sim N\left(0, \Sigma_\tau\right)$.

Since Sweden is a small open economy we assume that the foreign economy is exogenous. Foreign inflation, π_t^*, output (HP-detrended), $\hat{y}_t^*$, and interest rate, R_t^*, are exogenously given by an identified VAR model with four lags,

$$\Phi_0 X_t^* = \Phi_1 X_{t-1}^* + \Phi_2 X_{t-2}^* + \Phi_3 X_{t-3}^* + \Phi_4 X_{t-4}^* + S_{x^*} \varepsilon_{x^* t}, \tag{2.17}$$

where $X_t^* \equiv (\pi_t^*, \hat{y}_t^*, R_t^*)'$, $\varepsilon_{x^*t} \sim N\left(0, I_{x^*}\right)$, S_{X^*} is a diagonal matrix with standard deviations and $\Phi_0^{-1} S_{x^*} \varepsilon_{x^* t} \sim N\left(0, \Sigma_{x^*}\right)$. Given our assumption of equal substitution elasticities in foreign consumption and investment, these three variables suffice to describe the foreign economy in our model setup.

2.1.5. Monetary policy

Monetary policy is modeled in two different ways. First, we assume that the central bank minimizes an intertemporal loss function under commitment. Let the intertemporal loss function in period t be

$$\mathrm{E}_t \sum_{\tau=0}^{\infty} \delta^{\tau} L_{t+\tau}, \tag{2.18}$$

where $0 < \delta < 1$ is a discount factor, and L_t is the period loss given by

$$L_t = (p_t^c - p_{t-4}^c - \bar{\pi}^c)^2 + \lambda_y (y_t - \bar{y}_t)^2 + \lambda_{\Delta i}(i_t - i_{t-1})^2, \tag{2.19}$$

where the central bank's target variables are; the model-consistent year-over-year CPI inflation, $p_t^c - p_{t-4}^c$, where p_t^c denotes the log of the CPI and $\bar{\pi}^c$ is the 2% inflation target; a measure of the output gap, $y_t - \bar{y}_t$, where y_t denotes output and $\bar{y}_t$ denotes potential output; and the first difference of the instrument rate, $i_t - i_{t-1}$, where i_t denotes the Riksbank's instrument rate, the repo rate, and λ_y and $\lambda_{\Delta i}$ are nonnegative weights on output-gap stabilization and instrument-rate smoothing, respectively. [10,11]

We compare results from three different measures of the output gap $(y_t - \bar{y}_t)$ in the loss function. The first measure, the *trend output gap* uses the trend production level as potential output $(\bar{y}_t)$, which is growing stochastically due to the unit-root stochastic technology shock in the model. This definition of potential output will resemble a potential output computed with an HP filter.[12] The second measure, the *unconditional output gap*, specifies potential output as the hypothetical output level that would arise if prices and wages were completely flexible and had been so for a very long time. Unconditional potential output therefore presumes different levels of the predetermined

[10] We use year-over-year inflation as a target variable rather than quarterly inflation, since the Riksbank and other inflation-targeting central banks normally specify their inflation target as a 12-month rate.

[11] The inflation target variable is assumed to be model-consistent CPI inflation since this measure more accurately captures the true import content in the consumption basket. In the model, where total consumption is a CES function of imported and domestic goods, model-consistent CPI inflation is

$$p_t^{c,\mathrm{model}} - p_{t-4}^{c,\mathrm{model}} = (1-\omega_c)\left(p^c/p^d\right)^{-(1-\eta_c)}(p_t^d - p_{t-4}^d) + \omega_c (p^c/p^{m,c})^{-(1-\eta_c)}(p_t^{m,c} - p_{t-4}^{m,c}), \tag{2.20}$$

where ω_c is the share of expenditures in the CPI spent on imported goods, p_t^d the (log) domestic price level and $p_t^{m,c}$ the (log) price of imported goods that the consumer has to pay. The weights used to calculate the model-consistent inflation differ from those in the data by the steady-state relative prices (p^c/p^d and $p^c/p^{m,c}$), which lower the import share in consumption. This definition of CPI inflation is consequently consistent with the notion that due to distribution costs etc., the import share of consumption is somewhat exaggerated in the official statistics.

[12] The correlation between the trend output gap and an output gap computed with the HP-filter is about 0.65 using 5000 observations of simulated data from the model.

variables, including the capital stock, from those in the actual economy. The third measure, the *conditional output gap*, makes potential output contingent upon the existing current predetermined variables. Conditional potential output is thus defined as the hypothetical output level that would arise if prices and wages suddenly become flexible in the current period and are expected to remain flexible in the future.[13] In precise form the three different concepts of *potential* output are

$$\bar{y}_t^{\text{trend}} = z_t,$$

$$\bar{y}_t^{\text{cond}} = F_{y\cdot}^f X_t,$$

$$\bar{y}_t^{\text{uncond}} = F_{y\cdot}^f X_t^f,$$

where z_t is the unit-root technology shock, the row vector $F_{y\cdot}^f$ expresses output as a function of the predetermined state variables in the flex-price economy, X_t is the vector of predetermined state variables in Ramses, and X_t^f is the state vector in the economy with flexible prices and wages (see Appendix A for a description of the model solution and these matrices).

We define the *flexprice equilibrium* under the assumption that prices and wages are completely flexible in the domestic economy (thus keeping the foreign economy distorted), and determine the nominal variables by assuming that CPI inflation is kept constant at its steady-state level ($\hat{\pi}_t^c = 0$). When computing the two cases of flexprice potential output we also disregard markup shocks and fiscal shocks, and set these to zero in the flexprice economy.

Second, we assume monetary policy obeys an instrument rule, following Smets and Wouters [23], where the central bank adjusts the short term interest rate in response to deviations of CPI inflation from the perceived inflation target, the trend output gap (measured as actual minus trend output)[14], the real exchange rate ($\widehat{\tilde{x}}_t \equiv \hat{S}_t + \hat{P}_t^* - \hat{P}_t^c$) and the interest rate set in the previous period. The instrument rule (expressed in log-linearized terms) follows:

$$\begin{aligned} i_t &= \rho_{Rt} i_{t-1} + (1-\rho_{Rt}) \left[\bar{\pi}_t^c + r_{\pi t} \left(p_t^c - p_{t-1}^c - \bar{\pi}_t^c \right) + r_{yt}(y_{t-1} - \bar{y}_{t-1}) + r_{xt} \widehat{\tilde{x}}_{t-1} \right] \\ &\quad + r_{\Delta\pi,t} \Delta(p_t^c - p_{t-1}^c) + r_{\Delta y,t} \Delta(y_t - \bar{y}_t) + \varepsilon_{Rt}, \end{aligned} \tag{2.21}$$

where Δ denotes the first-difference operator, $\bar{\pi}_t^c$ is a time-varying inflation target, a hat denotes log-deviations from steady-state, and ε_{Rt} is an uncorrelated monetary-policy shock.

[13] For a detailed description on how to calculate the unconditional and conditional potential output, see appendix C in ALLS [2].

[14] The trend output gap, rather than the unconditional output gap, seems to more closely correspond to the measure of resource utilization that the Riksbank has been responding to historically, see ALLV [5]. Del Negro, Schorfheide, Smets, and Wouters [10] report similar results for the US.

2.2. Parameterization

The model's parameters are estimated using Bayesian techniques on 15 Swedish macroeconomic variables during the period 1980Q1–2007Q3. We refer the reader to ALLS [2] for a detailed description of the estimation. To make the paper self-contained we report in appendix B the values for the calibrated parameters (table B.1), the prior distributions we use in the estimation and the obtained posterior results (table B.2). In the subsequent analysis the estimated posterior mode values under the estimated instrument rule are used for all the non-policy parameters. The estimates of the model parameters suggest that they are invariant with respect to our alternative assumptions about monetary policy during the inflation targeting period (1993-), so we treat them as structural and independent of the monetary policy (see table B.2). Clearly, this assumption is more of a stretch in the subsequent analysis when the deviations from past policy behavior is particularly large, i.e. for very small or large values of λ_y (see section 4).

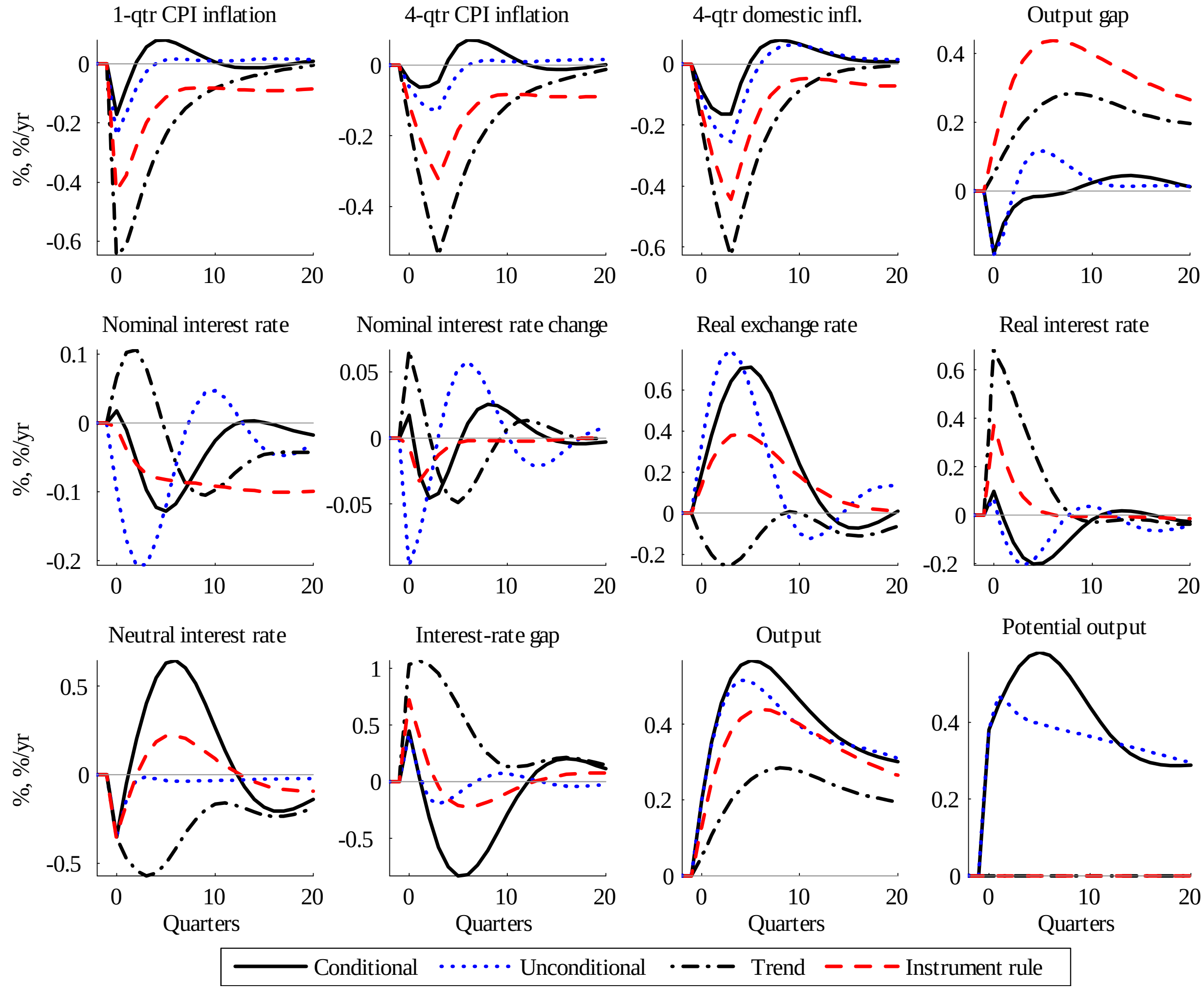
3. Monetary policy and the transmission of shocks

To gain intuition for the variance trade-off results in the next section, it is instructive to understand how the conduct of monetary policy affects the transmission of important shocks in the model. We do this by computing impulse response functions under optimal policy and under policy with the estimated instrument rule for two key shocks; technology and labor supply innovations. Impulse responses to stationary technology shocks are of key interest since movements in total factor productivity is a key driver of business cycles according to the estimated model. Labor supply shocks are important as this source of fluctuations creates an important trade-off between inflation and output-gap stabilization irrespective of which definition of output gap is used in the loss function.

Figure 3.1 depicts impulse responses to a positive stationary technology shock (one standard deviation) for optimal policy and for policy with the estimated instrument rule.[15] The impulse occurs in quarter 0. Before quarter 0, the economy is in the steady state with $X_t = 0$ and $\Xi_{t-1} = 0$ for $t \leq 0$ and $x_t = 0$ and $i_t = 0$ for $t \leq -1$. Under optimal policy, we use the estimated loss function ($\lambda_y = 1.10$, $\lambda_{\Delta i} = 0.37$) with the trend output gap (where the output gap between output and trend production is used), the unconditional output gap (the gap between output and unconditional flexprice potential output), and the conditional output gap (the gap between output and conditional

[15] The transmission of stationary technology shocks are also discussed in Adolfson et al. [2] and a subset of the impulses plotted in Figure 3.1 are provided there as well. However, since a thorough understanding of this shock is of particular importance for the results in this paper, we include an analysis of this shock here to make the paper self-contained.

Figure 3.1: Impulse response functions to a (one-standard deviation) stationary technology shock under optimal policy for different output gaps and under the estimated instrument rule.



flexprice potential output) and plot the corresponding impulse responses. It should be noted that this technology shock does not affect trend output in the model (which is influenced only by the unit-root technology shock). The output level under flexible prices and wages, flexprice potential output, is of course affected by the shock.

We start by comparing the impulse responses under policies using the trend output gap either as a response variable (instrument rule) or as a target variable in the loss function (optimal policy). Even if the instrument-rule parameters and the loss-function parameters are both estimated to capture the historical behavior of the central bank, the responses to a stationary technology shock are quite different when following the instrument rule (dashed curves) or using the quadratic loss function (dashed-dotted curves). The figure shows that optimal policy stabilizes inflation and the output gap more effectively over time than the instrument rule, although optimal policy initially allows a larger fall of both CPI and domestic inflation when using the trend output gap in its loss function. The nominal interest-rate response under the instrument rule is much more persistent than under optimal policy (which is even of the opposite sign), but inflation can still not be brought back to target as quickly. The real interest-rate response (level as well as the gap between the real interest rate and the state-dependent neutral real interest rate) is almost twice as high under optimal policy compared with the instrument rule and therefore reduces the increase in the trend output gap relative to the instrument rule. The stationary technology shock creates a trade-off for the central bank between balancing the induced decline in inflation and the higher (trend) output gap, and since the shock process is very persistent ($\rho_\varepsilon = 0.966$) this trade-off will last for many quarters. Under optimal policy such a trade-off is very costly in terms of the loss function, so the central bank invokes a forceful response to the technology shock. In contrast, the instrument rule cannot respond in an optimal fashion for each shock separately, but captures a realistic response based on inflation and the trend output gap derived from the historical behavior of the central bank. Had the central bank used larger (lower) coefficients on the inflation (trend output gap) variables in the instrument rule relative to the empirical estimates, inflation would approach target much faster after a shock to technology also under the instrument rule.

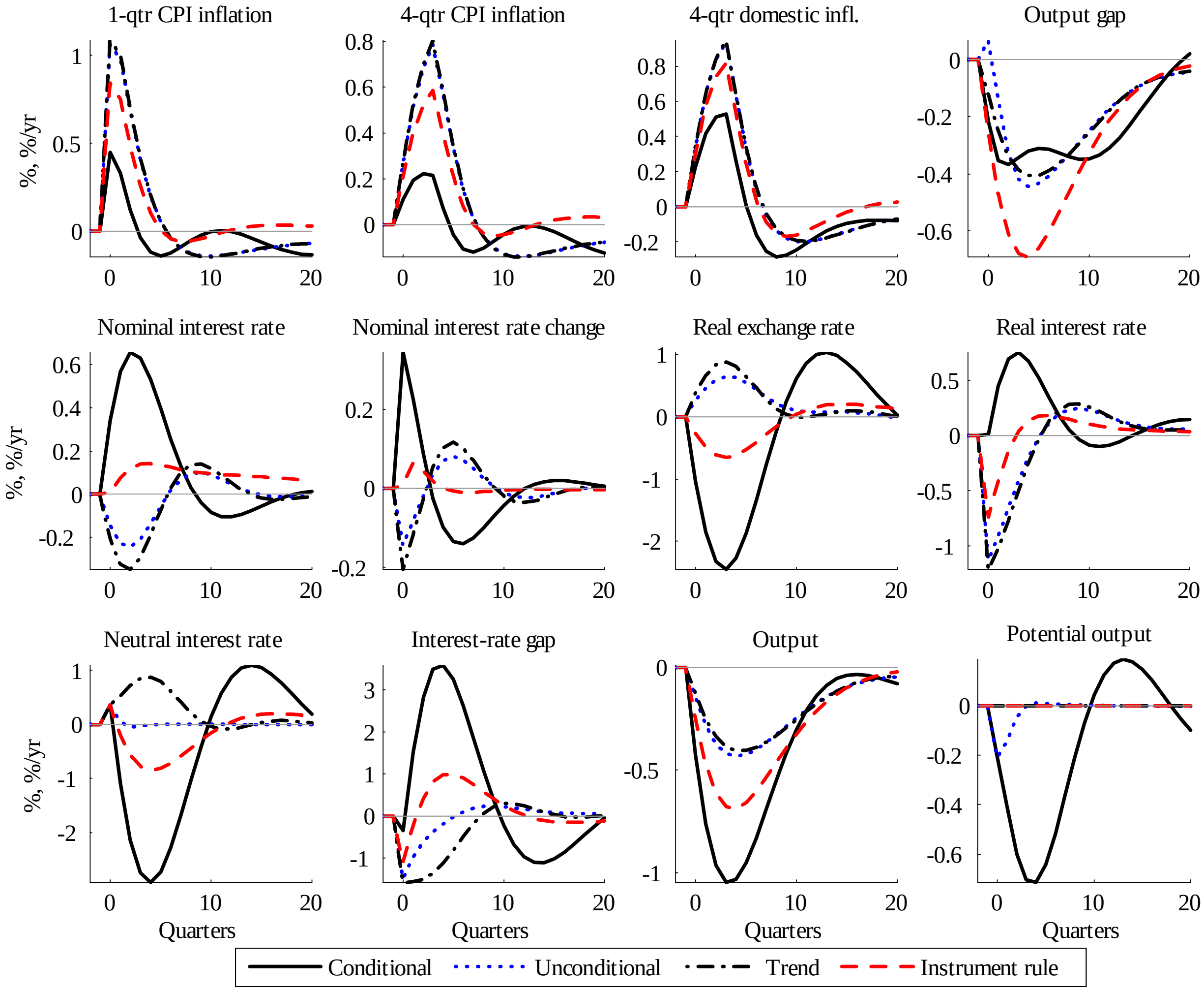
Figure 3.1 also illustrates the differences because of alternative output-gap measures in the central bank's loss function. The solid and dotted curves show impulse responses under optimal policy when potential output is specified as the output level prevailing under completely flexible prices and wages, where the flexprice equilibrium has lasted forever (unconditional flexprice potential output, dotted) or where prices and wages become flexible in the current period (conditional flexprice potential output, solid). The dashed-dotted curves, on the other hand, show impulses when the central bank stabilizes deviations of output from trend potential output. Due to sticky prices and wages, the stationary technology shock affects (unconditional/conditional) potential output quicker than actual output, and the flexprice output gap is therefore initially negative, whereas the trend output gap is positive (since trend output is by definition not affected by the shock). This important difference between the two output-gap definitions implies that the interest rate responses differ. The real interest rate response is negative when the central bank stabilizes the flexprice output gap and positive when it stabilizes the trend output gap. When the central bank stabilizes the flexprice output gap, it does not face the unfavorable trade-off between stabilizing inflation and the trend output gap. Therefore, the policy response can almost solely be directed at keeping inflation at target. This in effect implies that inflation can be stabilized much quicker than for the trend output gap, even though the weights in the loss function are the same in the two cases.

Another result from figure 3.1 is that the impulse responses of conditional and unconditional potential output differ. This is so because conditional potential output depends on the existing level of the predetermined variables in the actual economy with sticky prices and wages whereas unconditional potential output depends on the hypothetical level of the predetermined variables of the hypothetical economy with flexible prices and wages. When the shock hits the economy in quarter 0, the two output-gap definitions will be equal (since the economy by assumption starts out in steady state in quarter -1, which is the same for both the actual economy and the hypothetical flexprice economy), but in quarter 1 they will diverge. This is because conditional potential output in period 1 depends on the *actual* level of the predetermined variables in quarter 1 in the economy with sticky prices and wages, whereas unconditional potential output in quarter 1 depends on the *hypothetical* level of the predetermined variables in quarter 1 if prices and wages had been flexible (see appendix C in ALLS [2] for further details). The predetermined variables in quarter 1 in the sticky-price economy will differ from those in the flexprice economy because the forward-looking variables and the instrument rate in quarter 0 will differ between the sticky-price and the flexprice economy. Even if no new innovations have occurred between quarter 0 and quarter t, the levels of the predetermined variables used for computing the two potential output levels will thus differ. Since actual output and conditional potential output share the same predetermined variables in each period, the conditional output gap will normally be smaller than the the unconditional output gap. Moreover, with different output gaps in the loss function, the optimal policy responses will normally be different.

Figure 3.2 shows the impulse responses to a negative (one standard deviation) labor supply shock. For the set of observables we use to estimate our model, this shock is up to a scaling factor observationally equivalent to a (positive) wage markup shock. But consistent with the specification of the utility function 2.7 and the results in Galí, Smets and Wouters [15], we treat this shock as a genuine labor supply shock. Hence, we assume that it affects flexprice potential output.[16] This shock induces a negative output gap both measured as deviation from trend potential output as well as from (conditional and unconditional) flexprice potential output. Trend potential output is not at

[16] By using data on the real wage, employment and the unemployment rate, Galí, Smets and Wouters can distinguish between labor supply and wage-markup shocks as exogenous sources of labor market fluctuations. They find that labor supply shocks dominate, using data for the US.

Figure 3.2: Impulse response functions to a (one-standard deviation) labor supply shock under optimal policy for different output gaps and under the instrument rule.



all affected by the stationary wage markup shock, whereas (conditional and unconditional) flexprice potential output is. Because of wage and price stickiness, actual output is not directly adjusted to the disturbance. The higher real wage (not shown) pushes down hours worked (not shown) and thereby both potential and actual output. However, under flexible wages, the real wage and hours worked adjust very quickly to the new state, which feed into unconditional flexprice potential output and generates a negative output gap. The effects on the real wage, flexprice potential output and actual output are more short-lived compared with the technology shock, since the persistence of the labor supply shock is much lower ($\rho_{\varsigma_h} = 0.38$). Comparing figures 3.1 and 3.2 we see larger discrepancies between the different output gap measures after a technology shock relative to the labor supply shock. We also see that after a labor supply shock, the instrument rule is about as good as optimal policy, either with trend or unconditional output gap in the loss function, in bringing inflation back to target. With the conditional output gap in the loss function, it appears that inflation is stabilized more for the given weights in the loss function. Since conditional flexprice potential output is contingent upon the current state variables it resembles actual output more than unconditional flexprice potential output (cf. the solid curves), implying a somewhat smaller output gap and thereby more scope for inflation stabilization when the weights in the loss function are identical.

4. Variance trade-offs for the central bank

With a good understanding of the propagation of these key shocks, we now turn to an examination of the trade-offs the central bank is facing under optimal policy and under a simple instrument rule. As shown in Rudebusch and Svensson [21], when the intertemporal loss function (2.18) is scaled by $1-\delta$, the expected (conditional) intertemporal loss becomes equal to the unconditional mean of the period loss function when the discount factor approaches unity ($\lim_{\delta\to 1} \mathrm{E}_t \sum_{\tau=0}^{\infty}(1-\delta)\delta^{\tau} L_{t+\tau} = \mathrm{E}[L_t]$). The unconditional mean of the period loss function satisfies

$$\mathrm{E}[L_t] = \mathrm{Var}\left[p_t^c - p_{t-4}^c\right] + \lambda_y \mathrm{Var}[y_t - \bar{y}_t] + \lambda_{\Delta i}\mathrm{Var}[i_t - i_{t-1}] \tag{4.1}$$

under the assumption that the unconditional mean of 4-quarter CPI inflation equals the inflation target ($\mathrm{E}[p_t^c - p_{t-4}^c] = \bar{\pi}^c$) and the unconditional mean of the output gap equals zero ($\mathrm{E}[y_t - \bar{y}_t] = 0$). Under these assumptions, optimal policy for different loss-function weights λ_y and $\lambda_{\Delta i}$ results in efficient combinations of (unconditional) variances of inflation, the output gap, and the first-difference of the nominal interest rate. These variances for different loss-function weights provide the efficient relevant policy trade-offs between stabilization of inflation and the output gap and interest-rate smoothing. Appendix C shows how the unconditional variances are computed. To investigate the role of alternative measures of the output gap in the loss function, we show the variance trade-offs for either the trend output gap or the unconditional output gap in the loss function.[17] We first study the variance trade-offs when all shocks are active (figure 4.1), and then move on to an analysis of which type of shock influences the trade-offs most (figures 4.2 and 4.3).[18]

[17] In order not to lengthen the paper we have in this section chosen to only look at the unconditional output gap and not the conditional output gap.

[18] Since we want to explore what would happen if the central bank either follows an optimized simple instrument rule or commits to a loss function, we set the policy and inflation target shocks to zero in this section of the paper (i.e., $\varepsilon_t^R = 0$, and $\bar{\pi}_t^c = 0$).

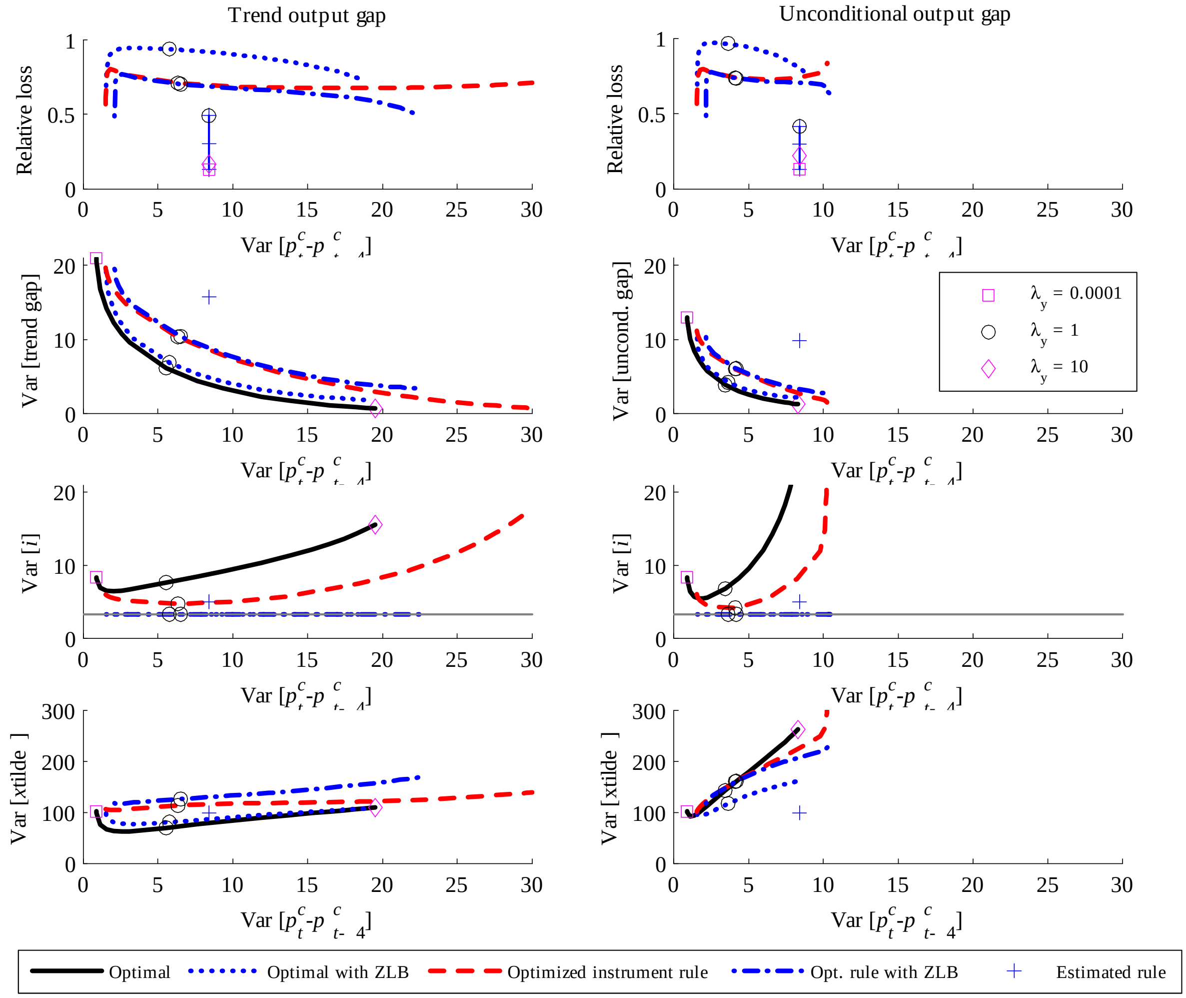
The curves referring to ZLB concern the case when the zero-lower-bound restriction on the nominal interest rate is imposed. They will be discussed in section 4.1.

In figure 4.1, the second row of the left column shows the variance of the trend output gap plotted against the variance of inflation, where inflation is 4-quarter CPI-inflation. The curve is obtained when varying the weight on output stabilization (λ_y) in the loss function with the trend output gap, given a fixed weight on interest-rate smoothing ($\lambda_{\Delta i} = 0.37$). The third row of the left column shows the corresponding variance of the nominal interest rate plotted against the variance of inflation, and the fourth row of the same column shows the variance of the real exchange rate plotted against the variance of inflation. Each λ_y results in a particular variance of inflation, and the figure should thus be read as if a vertical line through that level of inflation variance connects the three subplots. The curves are plotted for λ_y between 0.0001 and 10. A circle denotes the combination of variances resulting from $\lambda_y = 1$. On the solid curve only, the extreme low and high values for λ_y are marked by a square and diamond, respectively. The right column shows the variances when the unconditional output gap is used in the loss function instead of the trend output gap.

The top row of figure 4.1 shows the relative loss for the alternative policies we consider, expressed as the ratio between the unconditional mean loss under the optimal policy and the unconditional mean loss under the non-optimal (alternative) policies, plotted for each λ_y against the inflation variance of the non-optimal policy. Thus, the relative loss is bounded between zero and unity and shows what fraction of the loss for the non-optimal policy the loss for the optimal policy is. The vertical line marked with + shows the relative loss for the estimated instrument rule plotted against the (in this case constant) inflation variance for each λ_y of the loss function. Since the loss for the estimated rule is calculated according to equation (4.1) in this case the total loss will depend on the degree of output stabilization.

The figure shows that the gains from adhering to optimal policy instead of following the estimated instrument rule are substantial, especially for very small or large values of λ_y. For values of λ_y between 0.5 and 1.5, which seems most empirically relevant given the estimation results in Table B.2 in Appendix B, the estimated rule performs best relative to optimal policy for the trend output gap, but the loss is still about 50% higher relative to optimal policy. For the unconditonal output gap, the estimated rule performs somewhat worse. Given that the rule is estimated on the trend output gap, this finding is not surprising. As noted previously, we assume that the model parameters are invariant to the way we model monetary policy. Hence, the results far away from the past

Figure 4.1: Variance trade-offs when using either the trend or unconditional output gap in loss function and optimized simple instrument rule.



policy behavior should be interpreted more cautiously. Nevertheless, according to our estimated model, there are thus considerable gains in conducting optimal policy instead of adhering to the estimated rule for both measures of the output gap. To examine to which extent the gap between the estimated simple rule in (2.21) and optimal policy can be closed upon by simply optimizing the response coefficients in the rule, we consider a slightly simplified version of the instrument rule,

$$i_t = \rho_R i_{t-1} + (1-\rho_R)[\rho_\pi \left(p_t^c - p_{t-1}^c - \bar{\pi}^c\right) + \rho_y(y_t - \overline{y}_t)], \tag{4.2}$$

where the response coefficients ρ_R, ρ_π, and ρ_y are chosen to minimize the unconditional mean of the central bank loss function (4.1) for each given λ_y.[19] We use the same output gap (trend or

[19] We use Matlab's optimizers 'fminunc' and 'fminsearch' repeatedly to find the global optimum for the different response coefficients.

unconditional) in the simple instrument rule (4.2) and the unconditional mean of the loss function (4.1). The resulting optimized response coefficients in the simple instrument rule (4.2) are reported in table 4.1. We include forward-looking variables dated in period t in the simple instrument rule above, which means that the instrument rule not only depends on predetermined variables and is hence an implicit rather than explicit instrument rule. Consequently, since the interest rate then depends on forward-looking variables which in turn depend on the interest rate, the instrument rule is an equilibrium relation rather than an operational realistic instrument rule. We include forward-looking variables in the simple instrument rule as a way to allow the interest rate also in this case to respond to some current shocks and hence to be less at a disadvantage compared with the optimal policy.

Table 4.1: Optimized simple instrument rule
$i_t = \rho_R i_{t-1} + \rho_\pi(\pi_t - \bar{\pi}^c) + \rho_y(y_t - \bar{y}_t)$

	Trend output gap					Unconditional output gap				
λ_y	ρ_R	ρ_π	ρ_y	Loss	Optimal loss	ρ_R	ρ_π	ρ_y	Loss	Optimal loss
0.0001	1.03	0.51	0.003	1.97	1.11	1.01	0.51	-0.004	1.98	1.11
0.01	1.03	0.51	0.004	2.16	1.32	1.01	0.50	-0.003	2.09	1.24
0.11	1.02	0.47	0.01	4.07	3.17	1.01	0.47	0.004	3.17	2.37
0.21	1.01	0.44	0.02	5.88	4.71	1.01	0.44	0.01	4.19	3.29
0.31	1.01	0.42	0.02	7.59	6.03	1.02	0.41	0.02	5.15	4.08
0.41	1.01	0.39	0.02	9.21	7.18	1.02	0.39	0.02	6.05	4.76
0.51	1.02	0.36	0.03	10.73	8.20	1.02	0.37	0.03	6.90	5.36
1	1.05	0.25	0.05	16.84	11.94	1.03	0.32	0.06	10.38	7.65
1.5	1.07	0.19	0.07	21.19	14.53	1.00	0.32	0.13	12.90	9.35
2	1.09	0.17	0.09	24.31	16.46	0.96	0.32	0.21	14.60	10.73
3	1.10	0.16	0.14	28.51	19.23	0.90	0.25	0.33	16.91	12.99
4	1.12	0.16	0.20	31.24	21.20	0.90	0.24	0.40	18.74	14.89
5	1.13	0.17	0.27	33.20	22.73	0.91	0.25	0.45	20.46	16.58
6	1.13	0.17	0.34	34.72	23.97	0.92	0.26	0.49	22.10	18.13
7	1.13	0.17	0.40	35.95	25.03	0.93	0.27	0.54	23.68	19.59
8	1.13	0.17	0.47	36.98	25.95	0.93	0.29	0.58	25.20	20.97
9	1.12	0.17	0.53	37.87	26.76	0.93	0.30	0.62	26.69	22.29
10	1.12	0.17	0.59	38.66	27.49	0.93	0.31	0.65	28.14	23.57

Note: "Loss" is the loss from equation (4.1) under the instrument rule with optimized coefficients, and "Optimal loss" is the loss under optimal policy.

Table 4.1 shows that the optimized coefficients of the instrument rule are such that the optimized simple instrument rules are generally "super-inertial," that is, ρ_R is above unity. The exception is that ρ_R is lower than unity for the unconditional output gap when λ_y is sufficiently high (roughly above 1). Another property is that the response coefficients on the inflation (output gap) decreases (increases) as the weight on output-gap stabilization is increased. This property is not obvious and not general, since the mapping from loss function weights to optimal instrument-rule response coefficients is complicated and model-dependent.

From the top row in figure 4.1 we see that optimizing the response coefficients of the simple instrument rule closes a substantial part of the gap between the estimated instrument rule and the optimal policy.

For $\lambda_y = 1$, we find that the optimized instrument rule closes 20 percentage points of the gap to optimal policy relative to the estimated rule for the trend output gap, but the loss fraction between optimal policy and the optimized rule is despite this only about 0.7, implying that inflation and output-gap variances are still inefficient and can be further reduced by optimal policy. By and large, similar findings apply for the unconditional output gap. Interestingly, the nominal interest rate variance is larger under optimal policy than under the estimated instrument rule, which contributes to the more favorable inflation output trade-off relative to the estimated instrument rule. Figure 4.1 also shows that the central bank appears to face a relatively sharp trade-off between stabilizing inflation and the output gap. If the central bank wants to decrease the variance of inflation from 20 to 1, then it has to accept an increase in the variance of output of about 20, that is, the variance frontier has an "average" slope of about -1.[20] As can be seen from the right column of the second row in figure 4.1, the slope of the trade-off is about the same if the unconditional output gap is used in the loss function instead of the trend output gap. However, we see that the variance trade-off is more favorable and the variance curve is closer to origin for the unconditional output gap compared with that for the trend output gap. Thus, it is easier to stabilize the unconditional output gap than the trend output gap. Finally, the bottom row shows that lower output gap variance and higher inflation variance go with higher variance of the real exchange rate, so the real exchange rate is apparently implicitly adjusted to stabilize the output gap.

To examine which different shocks create the trade-offs in figure 4.1, figures 4.2 and 4.3 plot the variance trade-offs between inflation and the output gap for different subsets of active shocks, as well as the corresponding variances of the nominal interest rate and the real exchange rate plotted against each λ_y. We have divided the shocks into four different categories: domestic technology shocks (that is, stationary technology and investment specific technology shocks), markup shocks (that is, domestic, imported consumption, imported investment and export markup shocks), preference

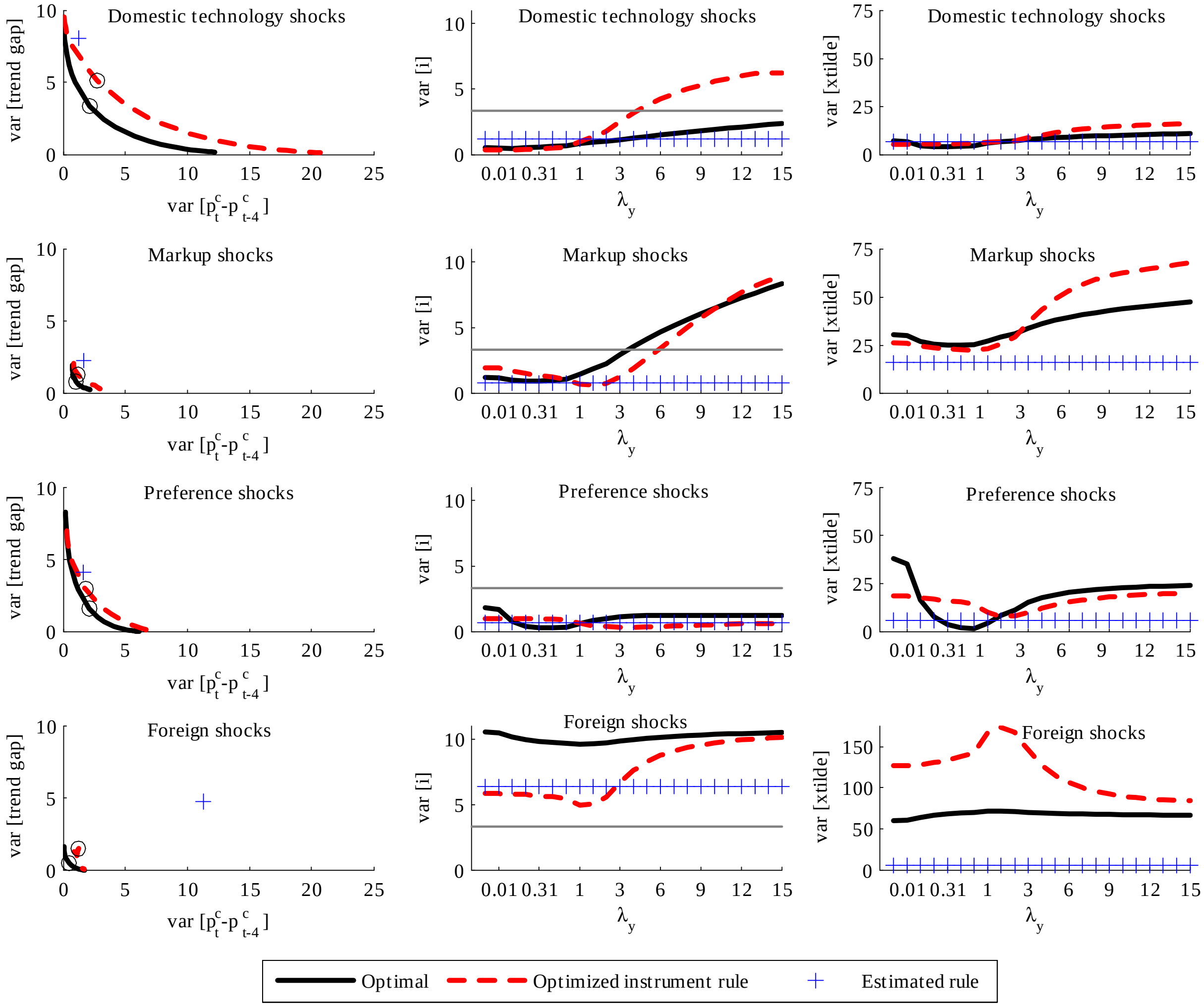
[20] We measure both inflation and the output gap in per cent, which implies that the variance is defined in terms of squared %.

shocks (that is, consumption preference and labor supply shocks), and foreign shocks (that is, unit-root technology, asymmetric technology, risk premium and foreign VAR shocks, which are foreign inflation, output and interest rate shocks). It is important to notice that the parameters in the optimized simple instrument rule are optimized on the full set of shocks, not on each subset separately. Therefore, it is possible that the variance trade-offs between inflation and the output gap are not always downward sloping for a particular subset of shocks for the optimized simple instrument rule (but they are always downward sloping for the optimal policy, which responds optimally to each shock separately (see (A.3) and (A.4)).

Figure 4.2 refers to the case with the trend output gap in the loss function and the variance of the trend output gap. It shows that, in that case, the variance trade-offs between inflation and the trend output gap is predominantly driven by the domestic technology shocks. The reason is that the stationary technology shock affects actual output but not trend output. Even if the shock is efficient in the sense that it lowers inflation pressure and increases actual output, trend productivity is not affected and the output gap therefore increases and thus creates a trade-off between stabilizing inflation and the trend output gap. We also see from the figure that the central bank needs to balance inflation stabilization against output-gap stabilization for most of the different sets of shocks, as the variance frontiers are downward sloping for all subsets of shocks. This is partly due to the interest-rate smoothing term in the loss function and partly due to the open-economy aspects of the model. Since all variations in the interest rate also lead to fluctuations in the exchange rate, it matters for policy how, for example, the consumers substitute between domestic and imported goods. The third column of the figure shows that the variance of the real exchange rate is high for all the different categories of shocks (cost-push shocks as well as demand-oriented shocks). Finally, we see that the variances for the estimated instrument rule is rather close to the variance trade-off for the optimal policy for all the categories of shocks except the foreign shocks, for which the estimated instrument rule is found to be very inefficient.

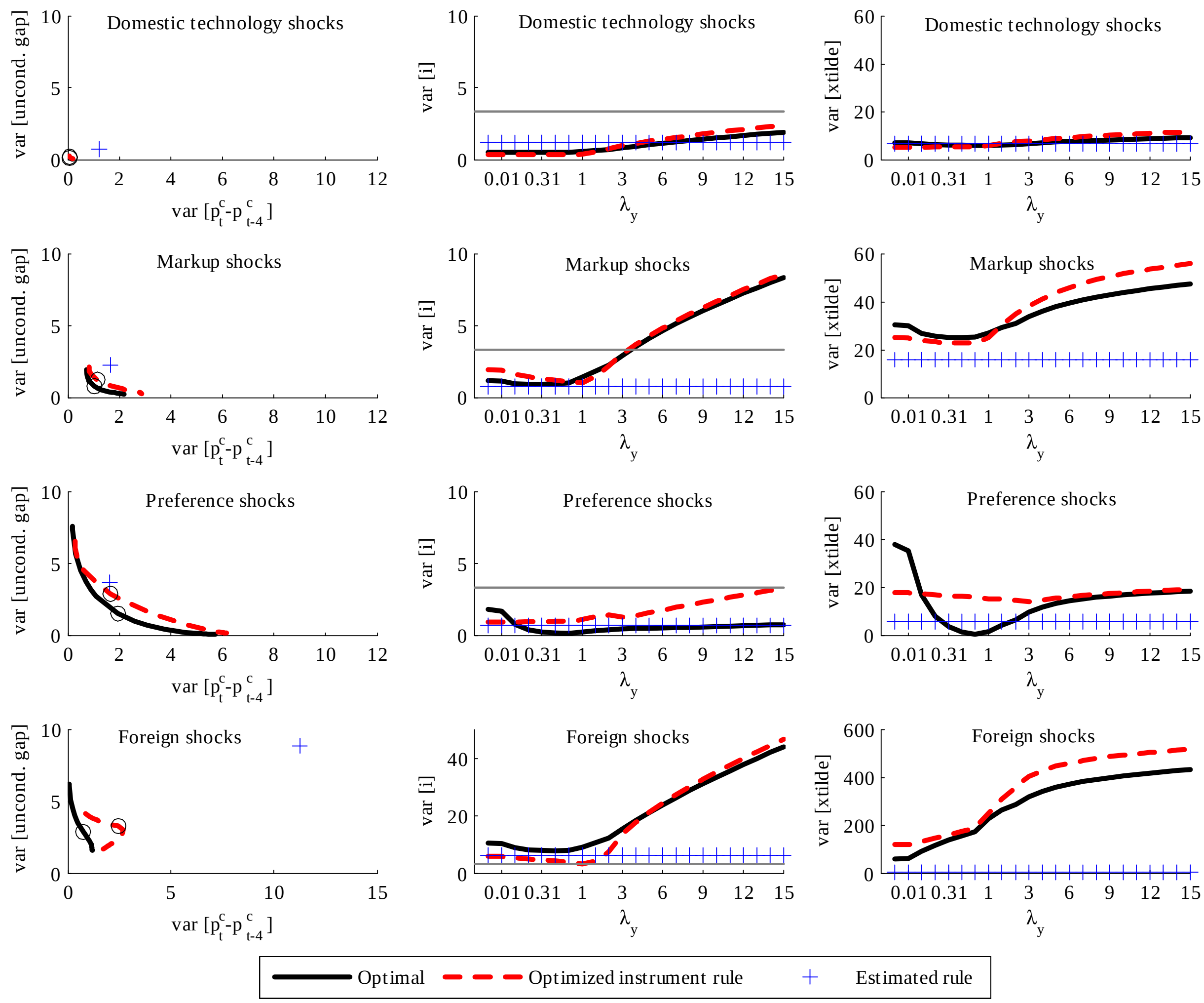
Figure 4.3 refers to the case with the unconditional output gap in the loss function and the variance of the unconditional output gap. In this case, the variance trade-off is mainly caused by preference shocks rather than domestic technology shocks. Since productivity shocks influence both unconditional potential output and actual output, the unconditional output gap will be less affected by this type of shock compared with the trend output gap. This means that the central bank does not have to trade off inflation stability for output-gap stability to the same extent when using the unconditional output gap in the loss function. It should also be noted that the unconditional

Figure 4.2: Variance trade-offs between when different shocks are active. Trend output gap in the loss function.



output gap is negatively affected whereas the trend output gap is positively affected by an increase in productivity (see figure 3.1), which implies that a lower interest rate stabilizes inflation and the flexprice output gap simultaneously. The labor supply shock and the consumption preference shock, on the other hand, affect inflation and the output gap with opposite signs irrespective of which output gap definition is used in the loss function. In figure 3.2 we saw that the central bank cannot simultaneously stabilize both the increase in inflation and the negative (unconditional, conditional, and trend) output gap in response to labor supply (wage markup) shocks. So this shock, along with the price markup shocks are the main sources of the trade-off between inflation and the unconditional output gap.

Figure 4.3: Variance trade-offs when different subsets of shocks are active. Unconditional output gap in loss function.



4.1. The zero lower bound on nominal interest rates

As is evident in figure 4.1, the interest-rate variance is relatively high under both the optimized instrument rule and the optimal policy. This means that the zero lower bound (ZLB) for the nominal interest rate may occasionally bind when shocks hit the economy. An approximation to the (non-linear) constraint that the nominal interest rate must be non-negative is to limit the variance of the nominal interest rate and thereby reduce the probability that the interest rate violates the ZLB. This approximation allows us to keep the linear-quadratic approach and focus on the second moments, but a potential drawback is of course that the approximation also limits upward movements in the nominal interest rate. We nevertheless adopt this approximation (see

Woodford [25] for a discussion).[21]

When optimizing the response coefficients of the simple instrument rule we therefore add the restriction there is only a 1% probability of hitting the ZLB. With an assumed steady state value for the nominal interest rate of 4.25% this implies that the variance of the nominal interest rate is not allowed to be larger than 3.34%. The dashed-dotted curves in figure 4.1 show the outcome of this procedure. Limiting the variance in the nominal interest rate implies the central bank can not stabilize output and inflation to the same extent, and the variance frontier moves slightly further out compared with when the ZLB is not imposed. For small λ_y the difference is not particularly pronounced, but for large λ_y the ZLB restriction results in a much larger output-gap variance that is not compensated by the decrease in inflation variance and hence the loss increases substantially. A larger output variability also feeds into a somewhat higher variance of the real exchange rate when the trend output gap is considered. It should, however, also be noted that the restriction on the variance of the nominal interest rate is strongly binding. For large weights on output-gap stabilization, the interest rate variance is almost six (nine) times as high when the ZLB is not imposed in the trend (unconditional) output-gap case. From this perspective, the impact on the inflation-output variance trade-off seems rather modest.

We also impose the zero lower bound on the nominal interest rate on the optimal policy, in this case by adding an extra interest-rate variance argument, $\lambda_i {i_t}^2$, to the loss function in (2.19) and gradually increasing λ_i until the variance of the nominal interest rate is not larger than 3.34%. The resulting variance trade-offs are depicted as dotted curves in figure 4.1. We see that the trade-off between inflation and output-gap variance shifts out a bit but not much. So even if the interest-rate variance in the unrestricted case is larger with optimal policy than with the optimized simple instrument rule, the zero lower bound appears to have about the same impact on the trade-off between inflation and output–gap variance.[22]

5. Conclusions

Within a small open economy framework, this paper has examined how the trade-offs between stabilizing CPI inflation and alternative measures of the output gap depend on the conduct of monetary policy. We have shown that it matters substantially which output-gap definition the

[21] In a smaller model it would be possible to deal with the consequences of the zero lower bound in a more rigorous fashion, for example along the lines of Adam and Billi [1] and Hebden, Lindé and Svensson [16].

[22] The interest-rate variance can also be reduced by increasing the weight on interest-rate smoothing, $\lambda_{\Delta i}$. However, this deteriorates the trade-off between inflation and output-gap variance quite a bit (not shown) and is hence an inefficient way of reducing the inteterest-rate variance compared to increasing the weight on interest-rate variance, λ_i.

central bank uses in its loss function. Depending on whether it is the trend output gap (between output and trend output) or the unconditional output gap (between output and unconditional flexprice potential output, the hypothetical output level that would prevail if prices and wages were entirely flexible and had been so forever) that is included in the loss function, the central bank faces different trade-offs between stabilizing inflation and the output gap. According to our analysis, the trade-off between stabilizing inflation and the output gap is more favorable for the unconditional output gap than for the commonly used trend output gap. However, abandoning the trend output gap in favor of the unconditional or conditional output gap would also be associated with an increase in the variance of output since conditional/unconditional potential output fluctuates more than trend output due to the fact that stationary but persistent technology shocks are important to explain business cycle fluctuations in the Swedish economy. On the other hand, because the trade-off between output-gap stabilization and inflation stabilization is more favorable for unconditional and conditional output gaps than for trend output gaps, abandoning the trend output gap for one of the other output gap measures should be associated with lower inflation variability.

The sensitivity of the results when imposing the zero lower bound for the nominal interest rate was also examined. While we acknowlegde that our approach to address the effects of imposing the zero lower bound is a crude approximation and should therefore be treated with grain of salt, our results do suggest that this assumption has about the same implications for the optimized simple instrument rule and the optimal policy.

Appendix

A. Model solution

After log-linearization, Ramses is a log-linear model with forward-looking variables. It can be written in the following state-space form,

$$\begin{bmatrix} X_{t+1} \\ Hx_{t+1|t} \end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + Bi_t + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_{t+1}. \tag{A.1}$$

Here, X_t is an n_X-vector of *predetermined* variables in period t (where the period is a quarter); x_t is an n_x-vector of *forward-looking* variables; i_t is an n_i-vector of *instruments* (the forward-looking variables and the instruments are the *nonpredetermined* variables);[23] ε_t is an n_ε-vector of i.i.d. shocks with mean zero and covariance matrix I_{n_ε}; A, B, and C, and H are matrices of the appropriate dimension; and $y_{t+\tau|t}$ denotes $\mathrm{E}_t y_{t+\tau}$ for any variable y_t, the rational expectation of $y_{t+\tau}$ conditional on information available in period t. The variables are measured as differences from steady-state values, so their unconditional means are zero. The elements of the matrices A, B, C, and H are estimated with Bayesian methods and considered fixed and known for the policy simulations. Hence the conditions for certainty equivalence are satisfied. The appendix of ALLS [2] provides details on Ramses, including the elements of the vectors X_t, x_t, i_t, and ε_t.

First we assume monetary policy can be described as minimizing an intertemporal loss function under commitment. Let Y_t be an n_Y-vector of *target* variables, measured as the difference from an n_Y-vector Y^* of time invariant *target levels*. We assume that the target variables can be written as a linear function of the predetermined, forward-looking, and instrument variables,

$$Y_t = D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} \equiv [D_X \quad D_x \quad D_i] \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}, \tag{A.2}$$

where D is an $n_Y \times (n_X + n_x + n_i)$ matrix and partitioned conformably with X_t, x_t, and i_t.

Under the assumption of optimization under commitment in a timeless perspective, the optimal policy and resulting equilibrium can be described by the following difference equations,

$$\begin{bmatrix} x_t \\ i_t \end{bmatrix} = F \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix}, \tag{A.3}$$

$$\begin{bmatrix} X_{t+1} \\ \Xi_t \end{bmatrix} = M \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix} + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_{t+1}, \tag{A.4}$$

[23] A variable is predetermined if its one-period-ahead prediction error is an exogenous stochastic process (Klein [18]). For (A.1), the one-period-ahead prediction error of the predetermined variables is the stochastic vector $C\varepsilon_{t+1}$.

for $t \geq 0$, where X_0 and Ξ_{-1} are given. The Klein algorithm returns the matrices F and M. These matrices depend on A, B, H, D, W, and δ, but they are independent of C. The independence of C demonstrates the certainty equivalence of the optimal policy and equilibrium. The n_X-vector Ξ_t consists of the Lagrange multipliers of the lower block of (A.1), the block determining the forward-looking variables. The initial value Ξ_{-1} for $t = 0$ is given by the optimization for $t = -1$ (or equal to zero in the case of commitment from scratch in $t = 0$). The choice and calculation of the initial Ξ_{-1} is further discussed in ALLS [2].

B. Parameters

In table B.1 we report the parameters we have chosen to calibrate. These parameters are mostly related to the steady-state values of the observed variables (that is, the great ratios: C/Y, I/Y and G/Y). Table A.2 shows the prior and posterior distributions obtained in ALLS [2].

Table B.1: Calibrated parameters

Parameter	Description	Calibrated value
β	Households' discount factor	0.999999
α	Capital share in production	0.25
η_c	Substitution elasticity between C_t^d and C_t^m	5
σ_a	Capital utilization cost parameter	$1,000,000$
μ	Money growth rate (quarterly rate)	1.010445
μ_z	Technology growth rate (quarterly rate)	1.005455
σ_L	Labor supply elasticity	1
δ	Depreciation rate	0.025
λ_w	Wage markup	1.30
ω_i	Share of imported investment goods	0.50
ω_c	Share of imported consumption goods	0.35
ν	Share of wage bill financed by loans	1
τ^y	Labor income tax rate	0.30
τ^c	Consumption tax rate	0.24
τ^k	Capital income tax rate	0.00
$\rho_{\bar{\pi}}$	Inflation target persistence	0.975
g_r	Government expenditures-output ratio	0.30

Table B.2: Prior and posterior distributions

Parameter		Prior distribution			Posterior distribution					
					Policy rule		Commitment		Loss params.	
		type	mean	std.d. /df	mode	std.d. Hess.	mode	std.d. Hess.	mode	std.d. Hess.
Calvo wages	ξ_w	beta	0.750	0.050	0.719	0.045	0.719	0.042		
Calvo domestic prices	ξ_d	beta	0.750	0.050	0.712	0.039	0.737	0.043		
Calvo import cons. prices	$\xi_{m,c}$	beta	0.750	0.050	0.868	0.018	0.859	0.016		
Calvo import inv. prices	$\xi_{m,i}$	beta	0.750	0.050	0.933	0.010	0.929	0.011		
Calvo export prices	ξ_x	beta	0.750	0.050	0.898	0.019	0.889	0.025		
Indexation wages	κ_w	beta	0.500	0.150	0.445	0.124	0.422	0.115		
Indexation prices	κ_d	beta	0.500	0.150	0.180	0.051	0.173	0.050		
Markup domestic	λ_d	truncnormal	1.200	0.050	1.192	0.049	1.176	0.050		
Markup imported cons.	$\lambda_{m,c}$	truncnormal	1.200	0.050	1.020	0.028	1.021	0.029		
Markup.imported invest.	$\lambda_{m,i}$	truncnormal	1.200	0.050	1.137	0.051	1.154	0.049		
Investment adj. cost	$\tilde{S}''$	normal	7.694	1.500	7.951	1.295	7.684	1.261		
Habit formation	b	beta	0.650	0.100	0.626	0.044	0.728	0.035		
Subst. elasticity invest.	η_i	invgamma	1.500	4.0	1.239	0.031	1.238	0.030		
Subst. elasticity foreign	η_f	invgamma	1.500	4.0	1.577	0.204	1.794	0.318		
Risk premium	$\tilde{\phi}$	invgamma	0.010	2.0	0.038	0.026	0.144	0.068		
UIP modification	$\tilde{\phi}_s$	beta	0.500	0.15	0.493	0.067	0.488	0.029		
Unit root tech. shock	ρ_{μ_z}	beta	0.750	0.100	0.790	0.065	0.765	0.072		
Stationary tech. shock	ρ_{ε}	beta	0.750	0.100	0.966	0.006	0.968	0.005		
Invest. spec. tech shock	ρ_{Υ}	beta	0.750	0.100	0.750	0.077	0.719	0.067		
Asymmetric tech. shock	$\rho_{\tilde{z}^*}$	beta	0.750	0.100	0.722	0.052	0.736	0.058		
Consumption pref. shock	ρ_{ζ_c}	beta	0.750	0.100	0.919	0.034	0.881	0.038		
Labour supply shock	ρ_{ζ_h}	beta	0.750	0.100	0.382	0.082	0.282	0.064		
Risk premium shock	$\rho_{\tilde{\phi}}$	beta	0.750	0.100	0.852	0.059	0.885	0.041		
Unit root tech. shock	σ_{μ_z}	invgamma	0.200	2.0	0.127	0.025	0.201	0.039		
Stationary tech. shock	σ_{ε}	invgamma	0.700	2.0	0.457	0.051	0.516	0.054		
Invest. spec. tech. shock	σ_{Υ}	invgamma	0.200	2.0	0.441	0.069	0.470	0.065		
Asymmetric tech. shock	$\sigma_{\tilde{z}^*}$	invgamma	0.400	2.0	0.199	0.030	0.203	0.031		
Consumption pref. shock	σ_{ζ_c}	invgamma	0.200	2.0	0.177	0.035	0.192	0.031		
Labour supply shock	σ_{ζ_h}	invgamma	1.000	2.0	0.470	0.051	0.511	0.053		
Risk premium shock	$\sigma_{\tilde{\phi}}$	invgamma	0.050	2.0	0.454	0.157	0.519	0.067		
Domestic markup shock	σ_{λ_d}	invgamma	1.000	2.0	0.656	0.064	0.667	0.068		
Imp. cons. markup shock	$\sigma_{\lambda_{m,c}}$	invgamma	1.000	2.0	0.838	0.081	0.841	0.084		
Imp. invest. markup shock	$\sigma_{\lambda_{m,i}}$	invgamma	1.000	2.0	1.604	0.159	1.661	0.169		
Export markup shock	σ_{λ_x}	invgamma	1.000	2.0	0.753	0.115	0.695	0.122		
Interest rate smoothing	$\rho_{R,1}$	beta	0.800	0.050	0.912	0.019	0.900	0.023		
Inflation response	$r_{\pi,1}$	truncnormal	1.700	0.100	1.676	0.100	1.687	0.100		
Diff. infl response	$r_{\Delta\pi,1}$	normal	0.300	0.100	0.210	0.052	0.208	0.053		
Real exch. rate response	$r_{x,1}$	normal	0.000	0.050	−0.042	0.032	−0.053	0.036		
Output response	$r_{y,1}$	normal	0.125	0.050	0.100	0.042	0.082	0.043		
Diff. output response	$r_{\Delta y,1}$	normal	0.063	0.050	0.125	0.043	0.133	0.042		
Monetary policy shock	$\sigma_{R,1}$	invgamma	0.150	2.0	0.372	0.061	0.360	0.059		
Inflation target shock	$\sigma_{\bar{\pi}^c,1}$	invgamma	0.050	2.0	0.465	0.108	0.647	0.198		
Interest rate smoothing 2	$\rho_{R,2}$	beta	0.800	0.050	0.882	0.019				
Inflation response 2	$r_{\pi,2}$	truncnormal	1.700	0.100	1.697	0.097				
Diff. infl response 2	$r_{\Delta\pi,2}$	normal	0.300	0.100	0.132	0.024				
Real exch. rate response 2	$r_{x,2}$	normal	0.000	0.050	−0.058	0.029				
Output response 2	$r_{y,2}$	normal	0.125	0.050	0.081	0.040				
Diff. output response 2	$r_{\Delta y,2}$	normal	0.063	0.050	0.135	0.029				
Monetary policy shock 2	$\sigma_{R,2}$	invgamma	0.150	2.0	0.100	0.012				
Inflation target shock 2	$\sigma_{\bar{\pi}^c,2}$	invgamma	0.050	2.0	0.081	0.037				
Output stabilization	λ_y	truncnormal	0.5	100.0			1.091	0.526	1.102	0.224
Interest rate smoothing	$\lambda_{\Delta i}$	truncnormal	0.2	100.0			0.476	0.191	0.369	0.061
Log marg likelihood laplace					−2631.56		−2654.45			

C. Unconditional variances

As shown in Svensson [24], the model solution satisfies

$$\tilde{X}_{t+1} = M\tilde{X}_t + \tilde{C}\varepsilon_{t+1}, \tag{C.1}$$

$$\tilde{x}_t = F\tilde{X}_t, \tag{C.2}$$

where

$$\tilde{X}_t \equiv \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix}, \quad \tilde{x}_t \equiv \begin{bmatrix} x_t \\ i_t \end{bmatrix}, \quad \tilde{C} \equiv \begin{bmatrix} C \\ 0 \end{bmatrix}$$

(note that $\tilde{x}_t$ here does not denote the real exchange rate). The variance-covariance matrices of the predetermined variables, $\Sigma_{\tilde{X}\tilde{X}}$, and the forward-looking variables, $\Sigma_{\tilde{x}\tilde{x}}$, therefore satisfy the equations

$$\Sigma_{\tilde{X}\tilde{X}} = M\Sigma_{\tilde{X}\tilde{X}}M' + \tilde{C}\Sigma_{\varepsilon\varepsilon}\tilde{C}', \tag{C.3}$$

$$\Sigma_{\tilde{x}\tilde{x}} = F\Sigma_{\tilde{X}\tilde{X}}F', \tag{C.4}$$

where $\Sigma_{\varepsilon\varepsilon}$ is the variance-covariance matrix of the i.i.d. shocks ε_t.

The solution for the target variables and the observed variables are also functions of the predetermined variables,

$$Y_t = D\begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} = D\begin{bmatrix} I_{n_X} & 0 \\ & F \end{bmatrix}\tilde{X}_t \equiv \tilde{D}\tilde{X}_t,$$

$$Z_t = \bar{D}\begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} + \eta_t = \bar{D}\begin{bmatrix} I_{n_X} & 0 \\ & F \end{bmatrix}\tilde{X}_t + \eta_t \equiv \bar{\bar{D}}\tilde{X}_t + \eta_t.$$

Then their variance-covariance matrices, Σ_{YY} and Σ_{ZZ}, can be determined from the variance-covariance matrix of the predetermined variables,

$$\Sigma_{YY} = \tilde{D}\Sigma_{\tilde{X}\tilde{X}}\tilde{D}', \tag{C.5}$$

$$\Sigma_{ZZ} = \bar{\bar{D}}\Sigma_{\tilde{X}\tilde{X}}\bar{\bar{D}}' + \Sigma_{\eta\eta}, \tag{C.6}$$

where $\Sigma_{\eta\eta}$ is the variance-covariance matrix of the measurement errors η_t.

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