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The Magic of the Exchange Rate: Optimal Escape from a Liquidity Trap in Small and Large Open Economies*

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Abstract

The optimal escape from a liquidity trap involves generating private-sector expectations of a higher future price level and higher future inflation. This lowers the real interest rate and reduces the recession during the liquidity trap. The problem, emphasized by Krugman, is that central-bank promises of a higher future price level may not be credible.

The current exchange rate will be a good indicator of private-sector expectations of the future price level. An intentional currency depreciation (which is technically feasible) will create private-sector expectations of a future weaker currency and a higher future price level. An *intentional* currency depreciation and a crawling peg (as in the Foolproof Way) can implement the optimal escape from a liquidity trap and make this credible.

Optimal escape from a liquidity trap in a large economy does not prevent the rest of the world from achieving its monetary-policy objectives, if the rest of the world is not in a liquidity trap. For *negative* international output externalities (which may not be very realistic, since they rely on optimal international risk sharing), the rest of the world may fall into a liquidity trap. This nevertheless moves the world equilibrium towards the equilibrium corresponding to optimal international cooperation. For *positive* international output externalities, any initial liquidity trap in the rest of the world is alleviated or eliminated.

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1. Introduction

The optimal escape from a liquidity trap, with a binding zero lower bound for interest rates and a higher-than-optimal real interest rate, involves generating private-sector expectations of a higher future price level and higher future inflation. This implies a lower real interest rate and a milder recession during the liquidity trap, as demonstrated by Krugman [20] and, more recently, Jung, Teranishi and Watanabe [19] and Eggertsson and Woodford [14]. The problem, emphasized by Krugman [20], is that the private sector may not believe central-bank promises of a higher future price level, especially if the central bank has a reputation for achieving low inflation. This is the well-known credibility problem of escape from a liquidity trap. For instance, a current expansion of the monetary base need not imply a permanent expansion.

In this context, this paper shows, in a reasonably rigorous model of a two-country world, that the exchange rate has two important roles. *First*, under reasonable assumptions, the current exchange rate will vary approximately one-to-one with private-sector expectations of the future price level and hence be a good indicator of whether policy aimed at creating expectations of a higher future price level has succeeded. Success is indicated by a substantial current currency depreciation. Exchange-rate movements hence immediately reveal the success of failure of any policy attempting to influence such expectations. For instance, the dramatic expansion of the monetary base in Japan from March 2001—an increase to date of more than 60%—has apparently failed in having any impact on expectations of Japan's future price level. *Second*, an intentional currency depreciation (which can be shown to be technically feasible) will induce private-sector expectations of a future weaker currency. Under the reasonable assumption of unaffected *future* terms of trade, this implies expectations of higher future price level. As shown by Svensson [30], an intentional currency depreciation and a crawling peg (as in the can induce private-sector expectations of a higher future price level and escape from the liquidity trap—the Foolproof Way. This paper shows that such policy with an appropriately calibrated crawling peg can indeed implement the *optimal* policy for escape from a liquidity trap. This provides a solution to the credibility problem of the optimal escape. This is the magic of the exchange rate in the context of liquidity trap for an open economy.

A large economy implementing the optimal escape from a liquidity trap may have an impact on the rest of the world. This paper shows that, with *negative* international output externalities, the reduced recession following the optimal escape in a large economy will reduce the real and nominal interest rates in the rest of the world somewhat and possibly increase the risk that the rest of the world falls into a liquidity trap. This may seem to be a problem for the rest of the world. However, it is shown that, from the point of view of optimal international monetary-policy coordination, this is good, and it moves the rest of the world towards the world equilibrium corresponding to optimal international cooperation.

Negative international output externalities rely on complete international risk sharing, which is not very realistic. With incomplete international risk sharing, *positive* international output externalities are more realistic. With *positive* international output externalities, implementing the optimal escape in a large economy increases the natural interest rate in the rest of the world and alleviates or eliminates any liquidity trap in the rest of the world.

Section 2 lays out a model of a two-country world and derives the basic relations to be used between interest rates, inflation expectations, price levels, money supplies, exchange rates, potential outputs, natural interest rates and output gaps. Section 3 examines the nature of a liquidity trap in the special case of a small open economy, derives the optimal escape from a liquidity trap under credible commitment, and states the credibility problem of the optimal escape. Section 4 shows that the current exchange rate serves as an indicator of private-sector expectations of the future price level. It also demonstrates that an intentional currency depreciation and a crawling peg can implement the optimal escape from a liquidity trap and indeed solve the credibility problem.[1] Section 5 examines the impact on the rest of the world, the foreign country, of a large economy undertaking the optimal escape from a liquidity trap in a situation of noncooperation between the countries. This is compared to a situation of optimal monetary-policy cooperation between the countries. Section 6 provides some conclusions and further discussion. An appendix provides some technical details.

2. A world of two large countries

Consider a model of a world consisting of two large countries, home and foreign, a variant of the models of, for instance, Benigno and Benigno [5], Clarida, Galí and Gertler [10], Corsetti and Pesenti [13] and Obstfeld and Rogoff [25]. Let the home country have a continuum of identical home households h ($0 \leq h \leq 1-\alpha$), where $0 < \alpha < 1$, so $1-\alpha$ can be interpreted as the relative size (population) of the home country. Similarly, let the foreign country have a continuum of identical foreign households h^* ($1-\alpha \leq h^* \leq 1$), so α can be interpreted as the relative size

[1] Jeanne and Svensson [18] examine the credibility problem in further detail in a slightly different model of a small open economy.

(population) of the foreign country. Let all quantities in a country be measured per capita, that is, per household in that country. Consider a representative home household, h. It has the intertemporal utility function

$$\mathrm{E}_t \sum_{j=0}^{\infty} \delta^j \left[\frac{C_t^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} + V(\frac{M_t}{P_t^c}) - \frac{N_t^{1+\varphi}}{1+\varphi} \right]. \tag{2.1}$$

Here, E_t denotes expectations conditional on information available in period t; δ $(0 < \delta < 1)$ denotes the constant subjective discount factor; $\rho \equiv -\ln \delta > 0$ is the corresponding (continuously compounded) constant rate of time preference; C_t is the household's (aggregate) consumption in period t; $\sigma > 0$ is the intertemporal elasticity of substitution in consumption; $V(M_t/P_t^c)$ is the utility of the transactions services of the household's real money measured in consumption, M_t/P_t^c; M_t denotes the household's holdings of home nominal money; P_t^c is the consumer price index (CPI); N_t denotes the household's supply of labor; and $\varphi > 0$ is the elasticity of the marginal disutility of labor with respect to labor supply. Since C_t, M_t and N_t will be the same for all home households h, $0 \leq h \leq 1-\alpha$, the index h on these variables is suppressed. Money is base money; the household's share of the sector of financial intermediaries is for simplicity incorporated in the representative household.

I assume that the utility of transactions services is continuously differentiable and has the properties

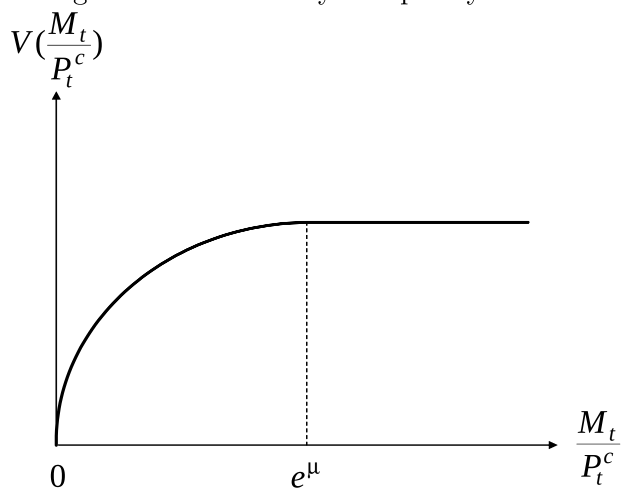
$$V'(\frac{M_t}{P_t^c}) > 0,\ V''(\frac{M_t}{P_t^c}) < 0,\ V(\frac{M_t}{P_t^c}) < V(e^{\mu}) \ \text{ for } 0 < \frac{M_t}{P_t} < e^{\mu};$$
$$V(\frac{M_t}{P_t^c}) = V(e^{\mu}) \ \text{ for } \frac{M_t}{P_t^c} \geq e^{\mu};$$
$$V'(\frac{M_t}{P_t^c}) \to \infty \ \text{ for } \frac{M_t}{P_t^c} \to 0.$$

That is, the utility of liquidity services is increasing in real money measured in consumption at a decreasing rate, up to a "satiation level," $M_t/P_t^c = e^{\mu}$, the log of which is given by a constant $\mu > 0$, as illustrated in figure 2.1. Beyond this satiation level, the utility of liquidity services is constant. Regardless of how high the nominal interest rate is, there is always a positive demand for real money.

The home household's consumption is an aggregate of the household's consumption of final home goods (produced in the home country), C_{ht}, and imported final foreign goods (produced in the foreign country), C_{ft}, according to the CES function

$$C_t \equiv [(1-\alpha)^{\frac{1}{\eta}} C_{ht}^{1-\frac{1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{ft}^{1-\frac{1}{\eta}}]^{\frac{1}{1-1/\eta}}, \tag{2.2}$$

Figure 2.1: The utility of liquidity services



where η is the intratemporal elasticity of substitution between home and foreign goods. Since C_t, C_{ht} and C_{ft} are measured per household, and the measure of home households is $1-\alpha$, it follows that total consumption, consumption of home goods and consumption of foreign goods in the home country are given by $(1-\alpha)C_t$, $(1-\alpha)C_{ht}$ and $(1-\alpha)C_{ft}$, respectively.

The home CPI is given by

$$P_t^c = \left[(1-\alpha)P_t^{1-\eta} + \alpha P_t^{f\ 1-\eta}\right]^{\frac{1}{1-\eta}} \tag{2.3}$$

$$\equiv P_t\left[(1-\alpha) + \alpha T_t^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{2.4}$$

where P_t and P_t^f are the home-currency prices of home and foreign goods, respectively, and $T_t \equiv P_t^f/P_t$ is the terms of trade, the price of foreign goods in terms of home goods, that is, the price of imported goods in terms of exported goods (an increase in T_t corresponds to a deterioration of the home country's terms of trade). The log-linear approximation around a steady state (to be determined) is

$$p_t^c = (1-\alpha)p_t + \alpha p_t^f = p_t + \alpha\tau_t, \tag{2.5}$$

where p_t^c, p_t and p_t^f denote the logs of the corresponding prices and

$$\tau_t \equiv p_t^f - p_t \tag{2.6}$$

denotes the log of the terms of trade (the steady-state level of the terms of trade will be normalized below to fulfill $\tau = 0$).

Home per household demand for home and foreign final goods will be

$$C_{ht} = (1-\alpha)C_t \left(\frac{P_t}{P_t^c}\right)^{-\eta}, \tag{2.7}$$

$$C_{ft} = \alpha C_t \left(\frac{P_t^f}{P_t^c}\right)^{-\eta}.$$

Prices are set in the currency of the producer and perfect exchange-rate pass-through is assumed, so the Law of One Price holds. Hence,

$$p_t^f = p_t^* + s_t, \tag{2.8}$$

where p_t^* is the (log) foreign-currency price of foreign goods and s_t is the (log) exchange rate (measured in units of home currency per unit of foreign currency).

Foreign quantities and foreign prices are denoted by $*$. A representative household h^* $(1-\alpha \leq h^* \leq 1)$ in the foreign country has the same intertemporal utility function as the home representative household (with the same δ, σ and φ), and with the arguments C_t^*, the foreign household's consumption, M_t^*/P_t^{c*}, the foreign household's holdings of foreign real base money (where P_t^{c*} is the foreign CPI expressed in foreign currency), and N_t^*, the foreign household's labor supply (the index h^* on these quantities are dropped, since they will be the same for each foreign household). The foreign household's consumption is the same aggregate of consumption of home and foreign final goods. The (loglinearized) foreign CPI will fulfill

$$p_t^{c*} = \alpha p_t^* + (1-\alpha)(p_t - s_t) = p_t^* - (1-\alpha)\tau_t \tag{2.9}$$

($p_t - s_t$ is the (log) foreign-currency price of home goods). It follows that purchasing-power parity (PPP) holds,

$$p_t^c = p_t^{c*} + s_t. \tag{2.10}$$

Home and foreign final goods are produced in two stages. In the second stage, production of home and foreign final goods, Y_t and Y_t^* (measured per household), occurs in each country under perfect competition with a continuum of nontraded intermediate inputs $Y_t(\iota)$ $(0 \leq \iota \leq 1)$ and $Y_t^*(\iota^*)$ $(0 \leq \iota^* \leq 1)$ (measured per household) of local differentiated intermediate goods, according to,

$$Y_t \equiv [\int_0^1 Y_t(\iota)^{1-\frac{1}{\xi}} d\iota]^{\frac{1}{1-1/\xi}}, \tag{2.11}$$

$$Y_t^* \equiv [\int_0^1 Y_t^*(\iota^*)^{1-\frac{1}{\xi}} d\iota^*]^{\frac{1}{1-1/\xi}}, \tag{2.12}$$

where $\xi > 1$ denotes the elasticity of substitution between differentiated goods. The corresponding price indices fulfill

$$P_t = [\int_0^1 P_t(\iota)^{1-\xi} d\iota]^{\frac{1}{1-\xi}}, \qquad (2.13)$$
$$P_t^f = [\int_0^1 P_t^f(\iota^*)^{1-\xi} d\iota^*]^{\frac{1}{1-\xi}},$$

where $P_t(\iota)$ and $P_t^f(\iota^*)$ denote the home-currency prices of home and foreign intermediate goods ι and ι^*, respectively. It follows that (per household) demand for differentiated good ι and ι^* is given by

$$Y_t(\iota) = Y_t \left(\frac{P_t(\iota)}{P_t}\right)^{-\xi}, \qquad (2.14)$$
$$Y_t^*(\iota^*) = Y_t^* \left(\frac{P_t(\iota^*)}{P_t}\right)^{-\xi}.$$

In the first stage, a continuum of home and foreign firms, denoted $0 \leq \iota \leq 1$ and $0 \leq \iota^* \leq 1$, produce home and foreign differentiated goods with a technology that is linear in labor input with country-wide exogenous stochastic productivity parameters, A_t and A_t^*,

$$Y_t(\iota) = A_t N_t(\iota), \qquad (2.15)$$
$$Y_t^*(\iota^*) = A_t^* N_t^*(\iota^*),$$

where $N_t(\iota)$ and $N_t^*(\iota)$ denote home and foreign input of labor (measured per household) in the production of good ι and ι^*, respectively. The producer of home (foreign) good ι (ι^*) maximizes profits subject to perfect competition in the home (foreign) labor market and monopolistic competition in the market for differentiated intermediate inputs (with the gross markup $\xi/(\xi-1)$ over marginal cost) and distributes the profits to home (foreign) households. Aggregate per household labor supply and demand in the home and foreign country will be given by

$$N_t \equiv \int_0^1 N_t(\iota) d\iota,$$
$$N_t^* \equiv \int_{1-\alpha}^1 N_t^*(\iota^*) d\iota^*.$$

Under the assumption of complete international risk-sharing and suitable initial conditions (see the appendix for details), both the marginal utility of consumption and, thereby, the quantity consumed are equalized between the countries,

$$c_t = c_t^*; \qquad (2.16)$$

the trade balance is zero in the steady state,

$$p^c + c = p + y, \tag{2.17}$$

$$p^{c*} + c^* = p^* + y^*, \tag{2.18}$$

where variables without subindex denote steady-state levels; home and foreign consumption fulfills

$$c_t = y_t - \alpha\eta\tau_t, \tag{2.19}$$

$$c_t^* = y_t^* + (1-\alpha)\eta\tau_t; \tag{2.20}$$

and the terms of trade fulfill

$$\tau_t = \frac{1}{\eta}(y_t - y_t^*). \tag{2.21}$$

The (log) terms of trade are proportional to the difference between (log) home and foreign output. Combination of (2.16) and (2.19)–(2.21) gives

$$c_t = c_t^* = (1-\alpha)y_t + \alpha y_t^*. \tag{2.22}$$

(Log) home and foreign consumption is an average of (log) home and foreign output. Furthermore, the units of home and foreign goods and labor can be normalized so the steady state is characterized by

$$c = c^* = y = y^* = \tau = 0.$$

As we shall see, this boils down to normalizing the steady state home and foreign log productivity levels accordingly.

2.1. Price setting

The firms producing differentiated goods are assumed to set prices for period $t+1$ in period t so as to maximize expected profits. Consider the price-setting problem in period t of a particular home firm ι ($0 \leq \iota \leq 1$). It sets its price for period $t+1$, $P_{t+1}(\iota)$, in monopolistic competition with a constant elasticity of demand $\xi > 1$ by (2.14). To a first-order approximation, expected profits are maximized if the price is set as a gross markup, $\xi/(\xi-1)$, of the expected marginal cost, $\mathrm{E}_t W_{t+1}/A_{t+1}$, where W_{t+1} is the nominal wage in period $t+1$. In a log-linear approximation,

$$p_{t+1}(\iota) = \ln\frac{\xi}{\xi-1} + w_{t+1|t} - \dot{a}_{t+1|t}, \tag{2.23}$$

where $p_t(\iota) \equiv \ln P_t(\iota)$, $w_t \equiv \ln W_t$, $\dot{a}_t \equiv \ln A_t$, and $z_{t+j|t} \equiv \mathrm{E}_t z_{t+j}$ denotes the expectation conditional on information in period t of the realization of any variable z_{t+j} in period $t+j$. It follows that all firms ι set the same price, so by (2.13),

$$p_t(\iota) = p_t \qquad (0 \leq \iota \leq 1). \tag{2.24}$$

It then follows from (2.14) and (2.15) that

$$\begin{aligned} y_t(\iota) &= y_t, \\ n_t(\iota) &= n_t \qquad (0 \leq \iota \leq 1), \\ y_t &= \dot{a}_t + n_t, \end{aligned} \tag{2.25}$$

where lowercase symbols denote the logs. Furthermore, perfect competition in the labor market implies, with obvious notation,

$$w_t = p_t^c + (w_t - p_t^c) = p_t^c + \varphi n_t + \frac{1}{\sigma} c_t, \tag{2.26}$$

where I use that the log real wage, W_t/P_t^c, in equilibrium will equal the marginal rate of substitution of consumption for consumption, $N_t^{\varphi}/C_t^{-1/\sigma}$, the log of which is $\varphi n_t + c_t/\sigma$. Using (2.5), (2.21), (2.19), (2.24) and (2.25) in (2.23), we get

$$p_{t+1} = \ln \frac{\xi}{\xi - 1} + p_{t+1|t} + \frac{1 + \sigma\varphi}{\sigma} y_{t+1|t} + \alpha(1 - \frac{\eta}{\sigma})\tau_{t+1|t} - (1 + \varphi)\dot{a}_{t+1|t}.$$

Normalizing the steady-state level of the log productivity level, $\dot{a}$,—that is, choosing units—such that

$$\dot{a} \equiv \frac{1}{1+\varphi} \ln \frac{\xi}{\xi - 1},$$

and letting $a_t \equiv \dot{a}_t - \dot{a}$ denote the *deviation* of the log home productivity level from that steady state level, we can write the price-setting equation, the aggregate-supply relation or Phillips curve,

$$p_{t+1} = p_{t+1|t} + \frac{1 + \sigma\varphi}{\sigma} y_{t+1|t} + \alpha(1 - \frac{\eta}{\sigma})\tau_{t+1|t} - (1 + \varphi)a_{t+1|t}. \tag{2.27}$$

Thus, the home price level in period $t+1$, p_{t+1}, is set in advance and hence predetermined, and it depends on private-sector expectations in period t of the price level, the output, the terms of trade and the productivity level in period $t+1$. Firms producing differentiated goods set next period's price proportional to the marginal cost, and the marginal cost is increasing in the price level and the output and decreasing in productivity. The dependence on the terms of trade is negative if $\sigma < \eta$. A unit increase in τ_{t+1} will increase p^c_{t+1} by α by (2.5), which, for a given real wage, $w_t - p^c_t$, by (2.26) will increase the log nominal wage and log marginal cost by α. But a unit increase in τ_{t+1} will also, for a given output level, by (2.19) reduce log consumption by $\alpha\eta$, which will increase the log marginal utility of consumption and reduce the log real wage and log marginal cost by $\alpha\eta/\sigma$. The net effect on log marginal cost is $\alpha(1 - \eta/\sigma)$, which term appears in (2.27).

However, by taking expectations in period t of (2.27) and eliminating the term $p_{t+1|t}$, we realize that, in equilibrium, the last three terms must sum to zero, so the pricing equation is simply

$$p_{t+1} = p_{t+1|t}. \tag{2.28}$$

In equilibrium, home firms simply set the price of home (intermediate) goods equal to the expected future home (final goods) price level. Similarly, foreign firms will set the foreign-currency price of foreign (intermediate) goods equal to the expected future foreign (final goods) price level,

$$p^*_{t+1} = p^*_{t+1|t}.$$

2.2. Potential output

Under the assumption of flexible prices in the home country, we can derive the corresponding flexprice equilibrium home output level, home potential output, for a given level of foreign output. More precisely, under flexible prices, we can write the profit-maximizing condition as unity equal to the product of the gross markup and the "product marginal cost", marginal cost deflated by the home-goods price,

$$1 = \frac{\xi}{\xi - 1} \frac{1}{A_t} \frac{W_t}{P_t}, \tag{2.29}$$

Taking logs, we get

$$\begin{aligned} 0 &= \ln \frac{\xi}{\xi - 1} - \dot{a}_t + (p^c_t - p_t) + (w_t - p^c_t) \\ &= \ln \frac{\xi}{\xi - 1} - \dot{a}_t + \alpha \frac{1}{\eta}(\bar{y}_t - y^*_t) + \varphi(\bar{y}_t - \dot{a}_t) + \frac{1}{\sigma} c_t \\ &= \alpha \frac{1}{\eta}(\bar{y}_t - y^*_t) + \varphi \bar{y}_t + \frac{1}{\sigma}[(1 - \alpha)\bar{y}_t + \alpha y^*_t] - (1 + \varphi) a_t, \end{aligned}$$

where $\bar{y}_t$ denotes the log potential output and I have used (2.5), (2.21) (2.22) and (2.26). Solving for $\bar{y}_t$, we then have

$$\bar{y}_t \equiv b_1 a_t - b_2 y^*_t, \tag{2.30}$$

where

$$b_1 \equiv \frac{\tilde{\sigma}(1+\varphi)}{1+\tilde{\sigma}\varphi} > 0, \tag{2.31}$$

$$b_2 \equiv \frac{\tilde{\sigma}}{1+\tilde{\sigma}\varphi}\alpha(\frac{1}{\sigma}-\frac{1}{\eta}) > 0, \tag{2.32}$$

$$\tilde{\sigma} \equiv \frac{\sigma}{1-\alpha+\alpha\frac{\sigma}{\eta}}, \tag{2.33}$$

where the inequality for b_2 holds if $\sigma < \eta$.

Thus, potential output depends not only on the productivity shock but also on the foreign output level. Furthermore, the sign of this latter effect depends on the relative size of σ and η, the intertemporal elasticity of substitution in consumption and the intratemporal elasticity of substitution between home and foreign goods. The reason is that foreign output affects the product marginal cost in (2.29) via two channels, a terms-of-trade channel and a consumption channel. In the terms-of-trade channel, a unit increase in log foreign output will lead to a fall in the terms of trade and a fall in $p_t^c - p_t$ equal to α/η, by (2.5) and (2.21). For a given CPI real wage, this reduces the product real wage and thereby the product marginal cost. This leads to a *rise* in potential output proportional to α/η. However, in the consumption channel, by (2.22), the same increase in foreign output increases log consumption by α, which reduces the log marginal utility of consumption by α/σ. For given terms of trade, this will increase the real CPI wage and thereby increases the product marginal cost. This leads to a *fall* in potential output proportional to α/σ. The net fall in potential output is proportional to $\alpha(1/\sigma - 1/\eta)$, which term enters into b_2 in (2.32).

Thus, if $\sigma = \eta$, the two effects cancel, $b_2 = 0$, and home potential output is independent of foreign output. Most estimates indicate that the intertemporal elasticity of substitution is lower than the intratemporal elasticity of substitution between home and foreign goods, so $\sigma < \eta$ is considered the realistic case.[2] I take this to be the base case, for which case $b_2 > 0$ and home potential output is decreasing in foreign output. We also note that, for $\sigma < \eta$, we have $\sigma < \tilde{\sigma} < \eta$.

Thus, this base case implies a *negative* international output externality: an increase in foreign output reduces home potential output. As explained, the source of this negative output

[2] Laxton et al. [21] use 0.41 for the intertemporal elasticity of substitution (denoted σ^{-1} in table 10, p. 47) and 0.99 for the elasticity of substitution between home and foreign goods (denoted $-\gamma_{m3}$ in table 11, p. 59). Bayoumi, Laxton and Pesenti [3], Chari, Kehoe and McGrattan [9] and Smets and Wouters [28] use 1.5 for the elasticity of substitution between home and foreign goods, whereas Hunt and Rebucci [17] use 3. As for φ, the elasticity of the marginal disutility of labor, the inverse of the elasticity of labor supply, Bayoumi, Laxton and Pesenti [3] and Hunt and Rebucci [17] use 3 as the main case, whereas Galí, Gertler and López-Salido [15] use 5.

externality is the assumption of complete risk-sharing, which implies that an increase in foreign output increases home consumption, reduces the marginal utility of home consumption, and increases the marginal cost of home production. With the intertemporal elasticity of substitution less than the intratemporal elasticity of substitution between home and foreign goods, this effect dominates over the effect of the home terms-of-trade improvement from the increase in foreign output, which in isolation reduces the marginal cost of home production. If we believe that the assumption of complete risk-sharing is unrealistic, we might doubt that the home consumption effect from an increase in foreign output dominates over the terms-of-trade effect. Then we might believe that there is a *positive* output externality rather than a negative, corresponding to $b_2 < 0$. Although I will maintain the negative output externality as the base case, I will also report the results under positive output externality, and in the concluding section 6 further discuss the two output-externality cases.

In deriving (2.28), we have already observed that the last three terms on the right side of (2.27) sum to zero. Using that to solve for $y_{t+1|t}$ and comparing with (2.30)–(2.33) gives $y_{t+1|t} = \bar{y}_{t+1|t}$, so

$$x_{t+1|t} = 0, \tag{2.34}$$

where

$$x_t \equiv y_t - \bar{y}_t, \tag{2.35}$$

denotes the home output gap. With the price equation (2.28), the expected future output gap is equal to zero.

Similarly, foreign potential output, $\bar{y}_t^*$, is given by

$$\bar{y}_t^* = b_1^* a_t^* - b_2^* y_t, \tag{2.36}$$

where $a_t^* \equiv \dot{a}_t^* - \dot{a}$ denotes the deviation of the log foreign productivity level $\dot{a}_t^* \equiv \ln A_t^*$ from the steady state $\dot{a}$,

$$\begin{aligned} b_1^* &\equiv \frac{\tilde{\sigma}^*(1+\varphi)}{1+\tilde{\sigma}^*\varphi} > 0, \\ b_2^* &\equiv \frac{\tilde{\sigma}^*}{1+\tilde{\sigma}^*\varphi}(1-\alpha)(\frac{1}{\sigma} - \frac{1}{\eta}) > 0, \\ \tilde{\sigma}^* &\equiv \frac{\sigma}{\alpha + (1-\alpha)\frac{\sigma}{\eta}}, \end{aligned}$$

where the inequality for b_2^* holds if $\sigma < \eta$. In analogy with (2.34), the expected future foreign output gap will equal zero,

$$x_{t+1|t}^* = 0, \tag{2.37}$$

where

$$x_t^* \equiv y_t^* - \bar{y}_t^* \tag{2.38}$$

denotes the foreign output gap.

2.3. Real interest rates, natural interest rates, output gaps and the trade balance

The first-order condition for optimal intertemporal consumption is

$$c_t = c_{t+1|t} - \sigma(r_t^c - \rho), \tag{2.39}$$

where r_t^c denotes the (continuously compounded) CPI real interest rate, defined by

$$r_t^c \equiv i_t - \pi_{t+1|t}^c,$$

the home nominal interest rate, i_t, less expected CPI inflation, $\pi_{t+1|t}^c$, where $\pi_t^c \equiv p_t^c - p_{t-1}^c$ is CPI inflation in period t. The home(-good) real interest rate, r_t, is defined by

$$r_t \equiv i_t - \pi_{t+1|t},$$

the nominal interest rate less expected home inflation, where $\pi_t \equiv p_t - p_{t-1}$ is home(-good) inflation in period t. By (2.5), the following relation holds between the CPI and the home-good real interest rates,

$$r_t = r_t^c + \alpha(\tau_{t+1|t} - \tau_t). \tag{2.40}$$

In analogy with potential output, the home natural interest rate, $\bar{r}_t$, is defined as the real interest rate that results in a flexprice equilibrium in the home country for given foreign output. By (2.40) and (2.21), it will fulfill the identity

$$\bar{r}_t \equiv \bar{r}_t^c + \alpha(\bar{\tau}_{t+1|t} - \bar{\tau}_t) \equiv \bar{r}_t^c + \alpha\frac{1}{\eta}[(\bar{y}_{t+1|t} - \bar{y}_t) - (y_{t+1|t}^* - y_t^*)], \tag{2.41}$$

where $\bar{r}_t^c$ and $\bar{\tau}_t$ denote the home natural CPI real interest rate and the home natural terms of trade (where "home natural" refers to a home flexprice equilibrium for given foreign output) and where I have used that, by (2.21), the home natural terms of trade depends on home potential output and foreign output and is defined according to

$$\bar{\tau}_t \equiv \frac{1}{\eta}(\bar{y}_t - y_t^*).$$

Furthermore, by (2.39) and (2.22), the home natural CPI real interest rate fulfills

$$\bar{r}_t^c \equiv \rho + \frac{1}{\sigma}(\bar{c}_{t+1|t} - \bar{c}_t) \equiv \rho + \frac{1}{\sigma}[(1-\alpha)(\bar{y}_{t+1|t} - \bar{y}_t) + \alpha(y_{t+1|t}^* - y_t^*)]. \tag{2.42}$$

where I have used that, by (2.22), the home natural consumption level, $\bar{c}_t$, fulfills

$$\bar{c}_t \equiv (1-\alpha)\bar{y}_t + \alpha y_t^*.$$

Using (2.42) in (2.41) gives

$$\bar{r}_t = \rho + d_1(a_{t+1|t} - a_t) + d_2(y_{t+1|t}^* - y_t^*), \tag{2.43}$$

where the coefficients fulfill

$$\begin{aligned} d_1 &\equiv \frac{b_1}{\tilde{\sigma}} \equiv \frac{1+\varphi}{1+\tilde{\sigma}\varphi} > 0, \\ d_2 &\equiv b_2\varphi \equiv \frac{\tilde{\sigma}\varphi}{1+\tilde{\sigma}\varphi}\alpha(\frac{1}{\sigma} - \frac{1}{\eta}) > 0 \end{aligned}$$

(where the inequality for d_2 holds for $\sigma < \eta$).

Thus, the home natural interest rate depends positively on the expected home productivity growth and the expected foreign output growth. From (2.41), we can interpret the effect of foreign output growth on the home natural interest rate as going through two parallel channels, the expected home natural terms-of-trade change (the second term on the right side of (2.41)) and the home natural CPI interest rate (the first term on the right side of (2.41)). Regarding the first channel, from (2.41), we see that, for a given home natural CPI real interest rate and for given expected home potential output growth, a unit increase in expected foreign output growth, $y_{t+1|t}^* - y_t^*$, leads to a fall in the expected home natural terms-of-trade change by $1/\eta$ and *fall* in the home natural interest rate by α/η. Regarding the second channel, from (2.42), we see that the same unit increase in expected foreign output growth leads to an increase in the expected home natural consumption growth, $\bar{c}_{t+1|t} - \bar{c}_t$, by α and a *rise* in the home natural CPI interest rate, $\bar{r}_t^c$, by α/σ. Hence, for *given* home potential output growth, the net rise in the home natural interest rate is $\alpha(1/\sigma - 1/\eta)$, which term appears in the coefficient d_2. Furthermore, by (2.30), we have

$$\bar{y}_{t+1|t} - \bar{y}_t = b_1(a_{t+1|t} - a_t) - b_2(y_{t+1|t}^* - y_t^*),$$

so a unit increase in expected foreign output growth will actually lead to a *fall* in expected home potential output growth by b_2. This will also affect the home natural interest rate through the two channels mentioned and will reduce the home natural interest rate by $b_2/\tilde{\sigma}$, which equals the fraction $1/(1+\tilde{\sigma}\varphi)$ of $\alpha(1/\sigma - 1/\eta)$. As a result, the *total* effect on the home natural interest rate of a unit increase in expected foreign output growth is the fraction $\tilde{\sigma}\varphi/(1+\tilde{\sigma}\varphi)$ of the term $\alpha(1/\sigma - 1/\eta)$. This explains the coefficient d_2. Thus, the negative international output externality, $b_2 > 0$, corresponds to the home natural interest rate being a decreasing function of foreign output.

Above, we noted that foreign output affects home potential output through two channels, a terms-of-trade channel and a consumption channel. This is obviously what results in the two channels through which expected foreign output growth affects the home natural interest rate, the terms-of-trade-change channel and the CPI-real-interest-rate channel, since the latter can be seen as a consumption-growth channel.

Using (2.19 and (2.40) in (2.39) gives the aggregate-demand relation

$$y_t = y_{t+1|t} - \tilde{\sigma}[r_t - \rho - \alpha(\frac{1}{\sigma} - \frac{1}{\eta})(y^*_{t+1|t} - y^*_t)]. \tag{2.44}$$

Then potential output and the natural interest rate will fulfill the identify

$$\bar{y}_t \equiv \bar{y}_{t+1|t} - \tilde{\sigma}[\bar{r}_t - \rho - \alpha(\frac{1}{\sigma} - \frac{1}{\eta})(y^*_{t+1|t} - y^*_t)]. \tag{2.45}$$

By subtracting (2.45) from (2.44), we get a convenient form of the aggregate demand relation,

$$x_t = -\,\tilde{\sigma}(r_t - \bar{r}_t), \tag{2.46}$$

where I have used (2.34). Thus, home output-gap is decreasing in the home real interest-rate gap, the difference between the real interest rate and the natural interest rate.

We see that $\tilde{\sigma}$, the elasticity of output-gap growth with respect to the real interest rate, replaces the intertemporal elasticity of substitution in the standard aggregate demand relation for a closed economy. For $\alpha = 0$, which corresponds to a closed economy, $\tilde{\sigma} = \sigma$. For the realistic case of $\sigma < \eta$, as noted above, we have $\sigma < \tilde{\sigma} < \eta$.

Analogously, the foreign(-good) real interest rate is defined as

$$r^*_t \equiv i^*_t - \pi^*_{t+1|t},$$

where $\pi^*_t \equiv p^*_t - p^*_{t-1}$ is foreign(-good) inflation in period t, and the foreign natural rate fulfills

$$\bar{r}^*_t = \rho + d^*_1(a^*_{t+1|t} - a^*_t) + d^*_2(y_{t+1|t} - y_t), \tag{2.47}$$

where the coefficients are given by

$$\begin{aligned} d^*_1 &\equiv \frac{b^*_1}{\tilde{\sigma}^*} \equiv \frac{1+\varphi}{1+\tilde{\sigma}^*\varphi} > 0, \\ d^*_2 &\equiv b^*_2\varphi \equiv \frac{\tilde{\sigma}^*\varphi}{1+\tilde{\sigma}^*\varphi}(1-\alpha)(\frac{1}{\sigma} - \frac{1}{\eta}) > 0 \end{aligned}$$

(where the inequality for d_2^* holds for $\sigma < \eta$). The foreign aggregate demand relation can be written,

$$x_t^* = -\tilde{\sigma}^*(r_t^* - \bar{r}_t^*), \tag{2.48}$$

where I have used (2.37).

Since the real interest rates, r_t and r_t^* are "own-good" real interest rates, that is, the nominal interest rate less the expected inflation for the own-produced good. They are are related by real interest-rate parity,

$$\tau_t = \tau_{t+1|t} - (r_t - r_t^*), \tag{2.49}$$

and they are equal only if there is no expected change in the terms of trade. The nominal interest rates are related by nominal interest-rate parity,

$$s_t = s_{t+1|t} - (i_t - i_t^*). \tag{2.50}$$

Any foreign-exchange risk premium or any other risk premium are disregarded (cf. Svensson [29] for details on various risk premia).

The home and foreign CPI real interest rates are equal

$$r_t^c \equiv i_t - \pi_{t+1|t}^c = i_t^* - \pi_{t+1|t}^{c*} \equiv r_t^{c*}, \tag{2.51}$$

since PPP holds ($\pi_t^{c*} \equiv p_t^{c*} - p_{t-1}^{c*}$ is foreign CPI inflation in period t).

The home country's trade balance and net export, as a share of steady-state output, is defined as

$$\mathrm{nx}_t \equiv \frac{P_t Y_t - P_t^c C_t}{P_t Y},$$

where $Y \equiv e^y = 1$ denotes the steady-state output. A linear approximation is

$$\mathrm{nx}_t = y_t - c_t - (p_t^c - p_t) = \alpha(y_t - y_t^*) - \alpha\tau_t = (\eta - 1)\alpha\tau_t = \alpha(1 - \frac{1}{\eta})(y_t - y_t^*). \tag{2.52}$$

We see that the Marshall-Lerner condition, that a deterioration of the terms of trade increases net export, holds if and only if $\eta > 1$, which I take to be the normal case (see footnote 2).

2.4. Money demand and supply and the zero lower bound for interest rates

The nominal interest rate fulfills the zero lower bound,

$$i_t \geq 0. \tag{2.53}$$

A negative nominal interest rate is not compatible with an equilibrium. A negative nominal interest rate would result in an unbounded supply of nominal bonds, since borrowing at a negative interest rate and investing in money paying zero interest would be a riskless arbitrage.

The first-order conditions for money and consumption choices will result in (see the appendix)

$$V'(\frac{M_t}{P_t^c}) = C_t^{\frac{1}{\sigma}}(1 - e^{-i_t}). \tag{2.54}$$

Solving for the real money demand measured in consumption results in the money demand function, $F(C_t, i_t)$,

$$\begin{aligned} \frac{M_t}{P_t^c} &= F(C_t, i_t) \quad (i_t > 0), \\ \frac{M_t}{P_t^c} &\geq e^{\mu} \quad\quad\quad (i_t = 0), \end{aligned}$$

where the function $F(C_t, i_t)$, by the assumptions on the utility from liquidity services, fulfills

$$\begin{aligned} F(C_t, i_t) &< e^{\mu}, \quad \frac{\partial F}{\partial C_t} > 0, \quad \frac{\partial F}{\partial i_t} < 0 \qquad (i_t > 0), \\ F(C_t, 0) &= e^{\mu}. \end{aligned}$$

Taking logs, we have

$$\begin{aligned} m_t - p_t^c &= \ln F(e^{c_t}, i_t) \quad (i_t > 0), \\ m_t - p_t^c &\geq \mu \quad\quad\quad\quad\quad (i_t = 0), \end{aligned} \tag{2.55}$$

where m_t denotes the (log nominal) money demand.

In equilibrium, money supply equals money demand, and we can interpret (2.55) as an equilibrium relation between m_t, interpreted as the supply of monetary base, consumption c_t, the CPI p_t^c and the interest rate i_t. Furthermore, using (2.5), (2.21) and (2.22), we can rewrite this as a relation between the supply of base money, the home price level, home output, the home interest rate and foreign output,

$$\begin{aligned} m_t - p_t &= l(y_t, y_t^*, i_t) \quad (i_t > 0), \\ m_t - p_t &\geq l(y_t, y_t^*, 0) \quad (i_t = 0), \end{aligned} \tag{2.56}$$

where the function $l(y_t, y_t^*, i_t)$ is defined by

$$l(y_t, y_t^*, i_t) \equiv \frac{\alpha}{\eta}(y_t - y_t^*) + \ln F(e^{(1-\alpha)y_t + \alpha y_t^*}, i_t)$$

and fulfills

$$\begin{aligned} l(y_t, y_t^*, i_t) &< l(y_t, y_t^*, 0), \quad \frac{\partial l}{\partial y_t} > 0, \quad \frac{\partial l}{\partial i_t} < 0 \qquad (i_t > 0) \\ l(y_t, y_t^*, 0) &\equiv \frac{\alpha}{\eta}(y_t - y_t^*) + \mu. \end{aligned}$$

When the nominal interest rate is zero, real money demand (measured in the home good) is greater than or equal to the satiation level of money demand, $l(y_t, y_t^*, 0) \equiv \frac{\alpha}{\eta}(y_t - y_t^*) + \mu$, the minimum real money demand for a zero nominal interest rate.[3]

In the foreign country, the corresponding relation is

$$\begin{aligned} m_t^* - p_t^* &= l^*(y_t^*, y_t, i_t^*) \quad (i_t^* > 0), \\ m_t^* - p_t^* &\geq l^*(y_t^*, y_t, 0) \quad (i_t^* = 0), \end{aligned}$$

where the function $l^*(y_t^*, y_t, i_t^*)$ is defined by

$$l^*(y_t, y_t^*, i_t^*) \equiv -\frac{1-\alpha}{\eta}(y_t - y_t^*) + \ln F(e^{(1-\alpha)y_t + \alpha y_t^*}, i_t^*)$$

and fulfills

$$\begin{aligned} l^*(y_t^*, y_t, i_t^*) &< l^*(y_t^*, y_t, 0), \quad \frac{\partial l^*}{\partial y_t^*} > 0, \quad \frac{\partial l^*}{\partial i_t^*} < 0 \qquad (i_t^* > 0), \\ l^*(y_t^*, y_t, 0) &\equiv -\frac{1-\alpha}{\eta}(y_t - y_t^*) + \mu. \end{aligned}$$

We can interpret the home central bank as controlling the domestic interest rate by controlling the supply of the monetary base and exploiting (2.56), and vice versa for the foreign central bank.

2.5. Monetary-policy objectives

The home central bank has an intertemporal loss function in period t corresponding to "flexible own-inflation targeting" with the constant discount factor δ, an inflation target for home(-good) inflation equal to $\pi \geq 0$ and a relative weight on output-gap variability equal to $\lambda > 0$,

$$\mathrm{E}_t \sum_{j=0}^{\infty} (1-\delta)\delta^j \frac{1}{2}[(\pi_{t+j} - \pi)^2 + \lambda x_{t+j}^2]. \tag{2.57}$$

The foreign central bank has an analogous loss function, also corresponding to flexible own-inflation targeting, with the same discount factor δ, an inflation target for foreign(-good) inflation equal to $\pi^* \geq 0$, and a relative weight on output-gap variability equal to $\lambda^* > 0$,

$$\mathrm{E}_t \sum_{j=0}^{\infty} (1-\delta)\delta^j \frac{1}{2}[(\pi_{t+j}^* - \pi^*)^2 + \lambda^* x_{t+j}^{*2}]. \tag{2.58}$$

[3] Equation (2.54) is not suitable for loglinearization, since the right side of it is independent of C_t for $i_t = 0$. Therefore, I prefere to use the exact function $l(y_t, y_t^*, i_t)$ to represent the equilibrium money demand.

3. A liquidity trap in a simple case of a small open economy

In order to illustrate the central problem of a liquidity trap in the simplest possible way, consider a particularly simple case of the above economy. First, assume that the foreign country can be treated as exogenous for the home country and in particular fulfills

$$\begin{aligned} y_t^* &= y^* = 0, \\ r_t^* &= \bar{r}_t^* = \rho > 0, \\ \pi_t^* &= \pi^*, \\ i_t^* &= i^* \equiv \rho + \pi^* > 0, \end{aligned}$$

for all periods t. This is effectively assuming the case of a small open economy (although with some market power in the market for its export). Section 5 will deal with the large-economy case.

Second, assume that the productivity a_t is iid. Then the expected future productivity fulfills

$$a_{t+1|t} = 0.$$

It follows from (2.30) and (2.43) that potential output and the natural interest rate are given by

$$\begin{aligned} \bar{y}_t &= b_1 a_t, \\ \bar{r}_t &= \rho - d_1 a_t, \end{aligned} \tag{3.1}$$

with $\bar{y}_{t+1|t} = 0$ and $\bar{r}_{t+1|t} = \rho$. Hence, the natural interest rate depends on the rate of time preference and the productivity parameter only. The natural interest rate depends negatively on a_t, the deviation of the productivity from the steady-state level. The natural interest rate depends positively on the expected growth of productivity, and a higher current productivity implies less growth back to steady state productivity in the future.

Third, suppose that the variance of the natural interest rate is sufficiently small and the inflation target π is sufficiently large so that only with a small probability will the natural interest rate fulfill

$$\bar{r}_t + \pi < 0. \tag{3.2}$$

This requires $\bar{r}_t < -\pi \leq 0$, that is, the natural interest rate has to be sufficiently negative, which requires that the productivity shock is sufficiently high relative. Inequality (3.2) will be the condition for a binding the zero lower bound for the nominal interest rate and a liquidity trap, as we shall see. For a given probability distribution of $\bar{r}_t$, the higher the inflation target, the lower the probability that the zero lower bound will bind. With a high probability, the natural interest rate will fulfill

$$\bar{r}_t + \pi \geq 0. \tag{3.3}$$

This inequality will be the condition for no liquidity trap. It requires that the productivity shock is not too high.

We shall think of (3.3) as the normal case for the economy. It allows an equilibrium where,

$$\pi_{t+1} = \pi_{t+1|t} = \pi, \tag{3.4}$$

$$x_t = 0, \tag{3.5}$$

$$i_t = \bar{r}_t + \pi \geq 0, \tag{3.6}$$

$$r_t = \bar{r}_t, \tag{3.7}$$

$$m_t = p_t + l(\bar{y}_t, 0, \bar{r}_t + \pi). \tag{3.8}$$

That is, expected and actual future inflation equals the inflation target, the output gap equals zero, the nominal interest rate equals the natural interest rate plus the inflation target, the real interest rate equals the natural interest rate, and the central bank sets the money supply to achieve the corresponding nominal interest rate. Because potential output and the natural interest rate are stochastic, money growth will in this equilibrium be stochastic but with a mean equal to the inflation target. This is the *ideal equilibrium*, when the central bank achieves its target for inflation, π, and target for the output gap, 0.

I now consider the home economy in period 1 (the "present") and the consequences of a possible liquidity trap in the present. I assume that the economy has been in the ideal equilibrium for a long time before period 1, so the realizations of the natural interest rate has fulfilled (3.3), expected and actual inflation has been equal to the inflation target, and the output gap has been equal to zero. Furthermore, for any given price level in period 2, the economy is expected to continue in the ideal equilibrium from period 2 on ("the future"), so private-sector expectations in period 1 are assumed to fulfill

$$\pi_{3|1} \equiv p_{3|1} - p_{2|1} = \pi, \tag{3.9}$$

$$\begin{aligned} x_{2|1} &= 0, \\ y_{2|1} &= \bar{y}_{2|1} = y = 0, \\ r_{2|1} &= \bar{r}_{2|1} = \rho > 0, \\ i_{2|1} &= \rho + \pi > 0. \end{aligned} \tag{3.10}$$

That is, inflation after period 2 is expected to equal the inflation target, the expected future output gap is zero, the expected output and potential output is zero, the expected real interest rate equals the average natural interest rate, and the expected nominal interest rate equals the sum of the average real interest rate and the inflation target.

By (2.56), the expected future price level in (3.9) will be directly related to the expected future money supply according to

$$p_{2|1} = m_{2|1} - l(0, 0, \rho + \pi) \tag{3.11}$$

(where, in a first-order approximation, the nonlinearity of $l(y_t, y_t^*, i_t)$ is disregarded). Private-sector expectations of the future price level are directly related to the expectations of the future money supply. It also follows from (2.28) and the above assumptions that the price level in period 2 is determined by period-1 private-sector expectations of the price level,

$$p_2 = p_{2|1}. \tag{3.12}$$

The period-1 price level, p_1, is by (2.27) determined by period-0 expectations and given in period 1, $p_1 = p_{1|0}$. By (2.46), we have the aggregate-demand relation in period 1,

$$x_1 = -\tilde{\sigma}(i_1 - \pi_{2|1} - \bar{r}_1), \tag{3.13}$$

$$i_1 \geq 0, \tag{3.14}$$

where I restate the zero lower bound for the nominal interest rate. By (3.1), $\bar{r}_1$ fulfills

$$\bar{r}_1 = \rho - d_1 a_1.$$

This model can now be seen as a more formal version of that in Krugman [20] and a simplified version of that in Jung, Teranishi and Watanabe [19] and Eggertsson and Woodford [14].

Given the above assumptions, the home central bank's intertemporal loss function (2.57) in period 1 can be simplified to

$$L_1 = \frac{1}{2}[\lambda x_1^2 + \delta(\pi_{2|1} - \pi)^2]. \tag{3.15}$$

3.1. The optimal escape from a liquidity trap

The zero lower bound on nominal interest rates is a constraint on policy that binds and increases the central-bank loss in some states of the world. By the optimal escape from a liquidity trap, I mean the optimal policy under the assumption of commitment, taking the zero lower bound into account. Commitment here means that the central bank in period 1 can commit to any money-supply function in period 2, so as to via (3.11) generate any private-sector expectations of the period-2 price level, $p_{2|1}$, and thereby, any private-sector inflation expectations, $\pi_{2|1} = p_{2|1} - p_1$. More precisely, by committing to a money-supply function such that

$$m_2 = \tilde{p}_2 + l(\bar{y}_2, 0, \bar{r}_2 + \pi) \tag{3.16}$$

for a given $\tilde{p}_2$ and any period-2 realizations of potential output and the natural interest rate, $\bar{y}_2$ and $\bar{r}_2$, the central bank will generate private-sector expectations (to a first-order approximation)

$$m_{2|1} = \tilde{p}_2 + l(0, 0, \rho + \pi) \tag{3.17}$$

and, by (3.11), $p_{2|1} = \tilde{p}_2$, which in turn by (3.12) will result in the actual prices $p_2 = \tilde{p}_2$.

Accordingly, choosing $\pi_{2|1}$ and x_1 so as to minimize (3.15) subject to (3.13) and (3.14) for given $\bar{r}_1$ gives the optimal policy under commitment. The two constraints (3.13) and (3.14) can be rewritten as the single aggregate-demand constraint

$$x_1 \leq \tilde{\sigma}(\bar{r}_1 + \pi_{2|1}), \tag{3.18}$$

and the corresponding interest rate can then be inferred from (3.13).

The corresponding Lagrangian is

$$\mathcal{L}_1 = \frac{1}{2}[\lambda x_1^2 + \delta(\pi_{2|1} - \pi)^2] - \phi_1[\tilde{\sigma}(\bar{r}_1 + \pi_{2|1}) - x_1],$$

where the Lagrange multiplier, $\phi_1 \geq 0$ (not to be confused with φ, the elasticity of the marginal disutility of labor with respect to labor supply), fulfills the complementarity slackness condition

$$\phi_1[\tilde{\sigma}(\bar{r}_1 + \pi_{2|1}) - x_1] = 0.$$

The first-order condition with respect to $\pi_{2|1}$ is

$$\delta(\pi_{2|1} - \pi) - \phi_1 \tilde{\sigma} = 0. \tag{3.19}$$

The first-order condition with respect to x_1 is

$$\lambda x_1 + \phi_1 = 0.$$

The first-order conditions and the complementary slackness conditions can be consolidated into the following *optimal targeting rule* (see Svensson [32] on targeting rules):

(N) No liquidity trap: If possible, set $\pi_{2|1} = \pi$ and choose $i_1 \geq 0$ so as to fulfill the target criterion

$$x_1 = 0.$$

(L) Liquidity trap: If this is not possible, set $i_1 = 0$ and choose $\pi_{2|1} > \pi$ so as to fulfill the target criterion

$$\pi_{2|1} - \pi = -\frac{\lambda\tilde{\sigma}}{\delta} x_1 > 0. \tag{3.20}$$

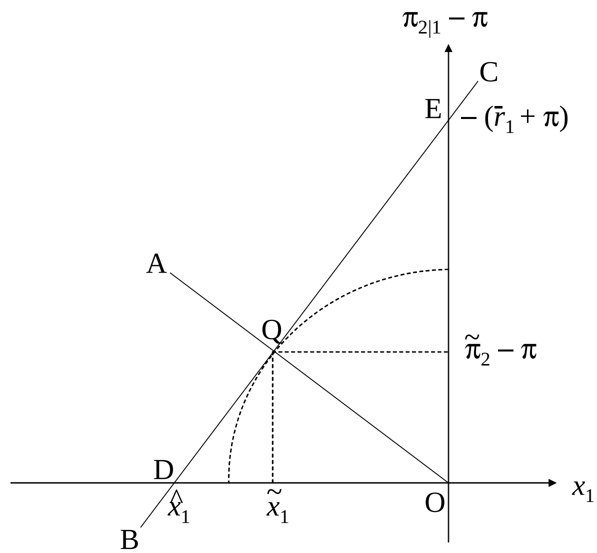
Thus, two cases, (N) and (L), are possible. First, if and only if (3.3) is fulfilled, we have $\phi_1 = 0$, and the zero lower bound is not binding. Then (N) is the relevant case, and the ideal equilibrium results,

$$\begin{aligned} \pi_{2|1} &= \pi, \\ x_1 &= 0, \\ r_1 &= \bar{r}_1, \\ i_1 &= \bar{r}_1 + \pi \geq 0, \\ m_1 &= p_1 + l(\bar{y}_1, 0, \bar{r}_1 + \pi). \end{aligned} \tag{3.21}$$

Expected future inflation equals the inflation target. The output gap is zero, and the nominal supply is set such that the resulting nominal interest rate makes the real interest rate equal to the natural interest rate. The loss is at a minimum, with $L_1 = 0$.

Second, if and only if (3.2) holds, we have $\phi_1 > 0$, and the zero lower bound is binding. Then, (L) is the relevant case. The economy is in a liquidity trap, and the central-bank loss will be higher than for (N).

Figure 3.1: The optimal escape from a liquidity trap



In the liquidity trap, the equilibrium under the optimal policy, denoted by $\tilde{}$, is

$$i_1 = 0, \tag{3.22}$$

$$m_1 \geq p_1 + l(\bar{y}_1 + \tilde{x}_1, 0, 0), \tag{3.23}$$

$$\pi_{2|1} = \pi - \frac{\lambda\tilde{\sigma}^2}{\delta + \lambda\tilde{\sigma}^2}(\bar{r}_1 + \pi) \equiv \tilde{\pi}_2 > \pi, \tag{3.24}$$

$$r_1 = -\pi_{2|1} = -\tilde{\pi}_2 \equiv \tilde{r}_1 > \bar{r}_1 \tag{3.25}$$

$$x_1 = \tilde{\sigma}(\bar{r}_1 + \tilde{\pi}_2) = \frac{\delta\tilde{\sigma}}{\delta + \lambda\tilde{\sigma}^2}(\bar{r}_1 + \pi) \equiv \tilde{x}_1 < 0. \tag{3.26}$$

The period-1 money supply is set so the nominal interest rate is zero. The central bank commits to a period-2 money-supply function (3.16) that results in the expected period-2 inflation overshooting the inflation target. The real interest rate is higher than the natural interest rate, and the output gap is negative. The expected overshooting of the inflation target implies that the real interest rate and the magnitude of the negative output gap is reduced somewhat, compared to if inflation expectations were equal to the inflation target. The minimum loss is positive because of the binding zero lower bound, $\tilde{L}_1 > 0$.

We can illustrate this in figure 3.1. The figure shows the period-1 output gap, x_1, along the horizontal axis and the expected period-2 inflation overshoot, $\pi_{2|1} - \pi$, along the vertical axes. The dashed curve shows the part of an iso-loss curves for the home central bank that falls in the northwest quadrant. A complete iso-loss curve is an ellipse around the origin O ($x_1 = 0$ and $\pi_{2|1} - \pi = 0$), where the loss is minimized and equal to zero. Iso-loss curves further out from the origin correspond to higher losses.

The positively sloped line BC shows the aggregate-demand constraint (3.18) with equality. Its slope is $1/\tilde{\sigma}$. Points on and to the left of the line fulfill the inequality (3.18). The line hits the vertical axis at point E, for $x_1 = 0$ and $\pi_{2|1} - \pi = -(\bar{r}_1 + \pi)$. If $\bar{r}_1 + \pi \geq 0$, point E lies below the origin O, the line BC is to the right of the origin, the constraint is not binding, and the central bank can reach the origin. This is the case (N), no liquidity trap, resulting in the ideal equilibrium, $L_1 = 0$.

When $\bar{r}_1 + \pi < 0$, point E lies above the origin (as drawn in figure 3.1), the line BC is to the left of the origin, the origin is no longer attainable, and the constraint is binding. This is the case (L), a liquidity trap. The line BC hits the horizontal axis at point D. This is the large negative output gap,

$$x_1 = \tilde{\sigma}(\bar{r}_1 + \pi) \equiv \hat{x}_1 < 0,$$

that results when $\pi_{2|1} - \pi = 0$, the expected period-2 inflation equals the inflation target. This large negative output gap, denoted by $\hat{x}_1$, will be prominent in this paper. The minimum loss occurs at point Q, where an iso-loss curve is tangent to the constraint. The ray OA corresponds to the target criterion (3.20), the locus of tangency points between iso-loss curves and the binding aggregate-demand constraint when the constraint shifts because of changes in the natural interest rate. Point Q gives the optimal output gap, $\tilde{x}_1$, and the optimal expected inflation overshoot, $\tilde{\pi}_2 - \pi$, given the liquidity trap.

The optimal policy under commitment hence trades off the right amount of expected overshooting of the future inflation target for the appropriate reduction in the magnitude of the output gap from point D to point Q. The nature of this optimal policy was clarified in Krugman [20]. A precise derivation of the optimal policy in some specific circumstances was provided, more recently, in Jung, Teranishi and Watanabe [19] and Eggertsson and Woodford [14].

The optimal tradeoff obviously depends on λ. I take the normal case to be $\lambda > 0$ (flexible inflation targeting) with a target criterion (3.20) corresponding to the negatively sloped ray OA in figure 3.1. If $\lambda = 0$ (strict inflation targeting), the target criterion (3.20) corresponds to a horizontal ray OA, we have $\pi_{2|1} = \pi$ regardless of the period-1 output gap, the minimum loss occurs at point D, and the magnitude of the negative output gap is larger, $x_1 = \hat{x}_1$. If $\lambda = \infty$ (strict output-gap targeting), the target criterion corresponds to a vertical ray OA, we have $x_1 = 0$ regardless of the inflation target, the minimum loss occurs at point E, and expected future inflation is higher, $\pi_{2|1} - \pi = -(\bar{r}_1 + \pi)$. This is the expected period-2 inflation required to make the real interest rate equal to the natural rate, $r_1 = -\pi_{2|1} = \bar{r}_1$.

A *liquidity trap* is hence a situation when the zero lower bound is binding, in the sense that optimal policy in the absence of the zero lower bound would imply a negative nominal interest rate. Furthermore, an expansion of the monetary base in the period has no effect on prices or quantities (other than the monetary base).

3.2. The credibility problem of the optimal escape from a liquidity trap

However, as Krugman [20] emphasized, the problem is that this optimal policy may not be credible. Absent any mechanism by which the central bank can commit in period 1 to a period-2 money-supply function (3.16), the central bank may not be able to generate the required private-sector expectations of higher future inflation. The private sector may simply believe that future inflation will equal past inflation and the central bank's inflation target. If so, the economy ends up in a bad equilibrium with a more negative output gap, the one corresponding to point D in figure 3.1 and denoted by ˆ above,

$$\begin{aligned} i_1 &= 0, \\ m_1 &\geq p_1 + l(\bar{y}_1 + \hat{x}_1, 0, 0), \\ \pi_{2|1} &= \pi \; < \; \tilde{\pi}_2, \\ r_1 &= -\pi \; \equiv \; \hat{r}_1 \; > \; \tilde{r}_1, \\ x_1 &= \tilde{\sigma}(\bar{r}_1 + \pi) \; \equiv \; \hat{x}_1 \; < \tilde{x}_1. \end{aligned} \tag{3.27}$$

This equilibrium has a higher loss, $\hat{L}_1 > \tilde{L}_1$. I will refer to it as the *bad equilibrium.* I will refer to the equilibrium corresponding to the optimal escape, the equilibrium at point Q in figure 3.1, as the *good equilibrium.*

In order to avoid the bad equilibrium and instead get to the good equilibrium, the central bank would need to commit itself to the period-2 money-supply function (3.16), and also communicate this commitment to the private sector. But with the interest rate already constant at zero, it is difficult to demonstrate any commitment. There is simply no obvious commitment mechanism, at least not in a closed economy.

Many authors have discussed whether or not a current expansion of the monetary base will get the economy out of the liquidity trap and, in particular, induce private-sector expectations of a higher future price level (see, for instance, Benhabib, Schmitt-Grohé and Uribe [4], Bernanke [6], Clouse et al. [11], Goodfriend [16], Meltzer [23], and Orphanides and Wieland [26]). However, the precise mechanism through which an expansion of the monetary base will alter expectations about the future price level is not clear. The problem is why an expansion of the monetary base in period 1 should be viewed as a commitment to a higher money supply in period 2. While the liquidity trap lasts and the interest rate is zero, the demand for monetary base is perfectly elastic, and excess liquidity is easily absorbed by the private sector. However, once the liquidity trap is over and the nominal interest rate is positive, demand for money may shrink drastically, in most cases requiring a drastic reduction of the monetary base. Bank of Japan has expanded the monetary base by more than 60% since the spring of 2001 (Bank of Japan [2]); given this step, it will definitely have to contract the monetary base once the liquidity trap is over (unless nominal income is at least some 50% higher in the future, which seems unlikely). Thus, a commitment not to reduce the monetary base at all in the future is not credible, but a commitment to reduce it by less than otherwise is a more complex matter. The situation is hence more complex than just making a permanent expansion of the monetary base, proposed by Auerbach and Obstfeld [1]. In terms of the simple model above, the optimal policy calls for an expected future money supply equal to (3.17), but this may very well be less than the period-1 money supply in (3.27).

In practice, the central bank will end up supplying whatever future quantity of base money that is demanded at the future desired interest rate-and output levels, for a given future price level. There is simply no mechanism, at least in a closed economy, by which a credible commitment to a particular future money supply can be made.

So, the big problem with the optimal escape from a liquidity trap is how it can be made credible, so the private sector believes in a higher future inflation. We have a situation with multiple equilibria. Without credibility for the optimal escape from the liquidity trap, the private sector believes the future inflation will be π and firms will set prices to make this a self-fulfilling equilibrium. Then the economy is stuck in the bad equilibrium at point D in figure 3.1. If instead the private sector believes in the future inflation $\tilde{\pi}_2$, firms will set prices to make this a self-fulfilling equilibrium, and the economy will be in the good equilibrium at point Q. How can the central bank, absent any direct commitment mechanism for the future money supply, make the economy reach the good equilibrium in period 1 rather than the bad equilibrium?[4]

[4] The current model, with the assumption of flexible own-inflation targeting, has multiple equilibria under discretion. Regardless of the price level in period t and the previous price-level expectations, $p_t = p_{t|t-1}$, we have $x_t = 0$ if there is no liquidity trap and $x_t = \hat{x}_t = \tilde{\sigma}(\bar{r}_t + \pi) < 0$, if there is a liquidity trap. I focus on the equilibria where price-level expectations either fulfills $p_{t+1|t} = p_t + \pi$ (corresponding to the ideal equilibrium without any liquidity trap and the bad equilibrium with a liquidity trap) or $p_{t+1|t} = \tilde{p}_{t+1}$ (which is the case for the good equilibrium under the liquidity trap). In Jeanne and Svensson [18], with the assumption of flexible CPI inflation

4. The magic of the exchange rate

Enter the exchange rate. There is no zero lower bound for the exchange rate. Even if the nominal interest rate is zero, a depreciation of the currency provides a potentially powerful way to stimulate the economy out of a liquidity trap, as noted by, for instance, Bernanke [6], McCallum [22], Meltzer [23], and Orphanides and Wieland [26]. A currency depreciation will stimulate an economy directly by giving a boost to exporting and import-competing sectors.

More importantly, as noted by Svensson [30], a currency depreciation and a *peg* of the currency at a depreciated rate can serve as a conspicuous commitment to a higher future price level and higher future inflation, consistent with the optimal way to escape from a liquidity trap discussed above. Indeed, as noted in Svensson [31] (although without a rigorous model), an exchange-rate peg can induce private-sector expectations of a higher future price level and indeed implement the optimal escape from the liquidity trap. Thus, the appropriate exchange-rate management can solve the credibility problem of the optimal escape from a liquidity trap.

In order to show this, I first determine the exchange rate paths consistent with the bad and the good equilibria. By (2.6) and (2.8), we have

$$p_{2|1} = p^*_{2|1} + s_{2|1} + \tau_{2|1}.$$

By (2.21) and the above assumptions, we have

$$\tau_{2|1} = \tau = 0.$$

It follows that

$$s_{2|1} = p_{2|1} - p^*_{2|1}; \tag{4.1}$$

for given expectations about the future foreign price level, private-sector expectations in period 1 of the exchange rate and price level in period 2 are directly related. Furthermore, the present exchange rate and private-sector expectations of the future exchange rate are related by

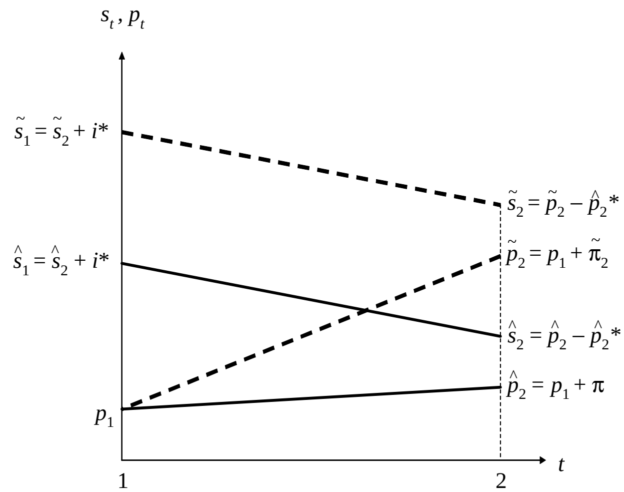
$$s_1 = s_{2|1} - (i_1 - i^*). \tag{4.2}$$

In the bad equilibrium, we then have

$$\begin{aligned} s_{2|1} &= p_{2|1} - p^*_{2|1} = \hat{p}_2 - \hat{p}^*_2 \equiv \hat{s}_2, \\ s_1 &= s_{2|1} + i^* = \hat{s}_2 + i^* \equiv \hat{s}_1 > \hat{s}_2, \end{aligned}$$

targeting, the equilibrium under discretion is unique.

Figure 4.1: Price levels and exchange rates



where $\hat{p}_2 \equiv p_1 + \pi$ is the expected future price level in the bad equilibrium, and where I have used that by the above assumptions the foreign price level is expected to grow at the steady rate π^*, so

$$p^*_{2|1} = p^*_1 + \pi^* \equiv \hat{p}^*_2.$$

In the good equilibrium, we instead have, from (4.1) and (4.2),

$$\begin{aligned} s_{2|1} &= \tilde{p}_2 - \hat{p}^*_2 \equiv \tilde{s}_2 > \hat{s}_2, \\ s_1 &= \tilde{s}_2 + i^* \equiv \tilde{s}_1 > \tilde{s}_2, \end{aligned} \tag{4.3}$$

where $\tilde{p}_2 \equiv p_1 + \tilde{\pi}_2$ is the expected future price level in the good equilibrium.

This is illustrated in figure 4.1, with period 1 and 2 along the horizontal axis and the log price level and exchange rate along the vertical axis. In the bad equilibrium, the private sector expects the price level to rise from p_1 to $\hat{p}_2 = p_1 + \pi$, where the slope of the solid line $p_1\hat{p}_2$ equals $\pi \geq 0$. Furthermore, the private sector expects the exchange rate to fall (the currency to appreciate) from $\hat{s}_1$ to $\hat{s}_2$, where the negative slope of the solid line $\hat{s}_1\hat{s}_2$ equals the foreign interest rate. In the good equilibrium, the private sector expects the price level to rise more, from p_1 to $\tilde{p}_2 = p_1 + \tilde{\pi}_2$, corresponding to the steeper sloped dashed line $p_1\tilde{p}_2$. Furthermore, the private sector expects the exchange rate to fall from $\tilde{s}_1$ to $\tilde{s}_2$, corresponding to the dashed line $\tilde{s}_1\tilde{s}_2$, still with the same negative slope. Thus, the expected exchange-rate path shifts up by the excess of $\tilde{\pi}_2$ over π (the term $-[\lambda\tilde{\sigma}^2/(\delta + \lambda\tilde{\sigma}^2)](\bar{r}_1 + \pi)$ in (3.24).

Two important results follow from this: From (4.1), we see that private-sector expectations of the future exchange rate are directly related to private-sector expectations of the future price level. From (4.3), we see that the present exchange rate is directly related to the expected future exchange rate. It follows that the present exchange rate varies one-to-one with expectations of the future price level. Thus, the first important result is that *the present exchange rate serves as an indicator of private-sector expectations of the future price level.*

As already noted above, the optimal policy under a liquidity trap, clarified by Krugman [20], precisely derived by Jung, Teranishi and Watanabe [19] and Eggertsson and Woodford [14], and restated for the simple case above, involves generating private-sector expectations of a higher future price level and moving the economy from the bad equilibrium corresponding to point D in figure 3.1 to the good equilibrium corresponding to point Q. Furthermore, the crucial problem for the central bank is how to induce such expectations and make a credible commitment to a higher future price level. It follows that any success or failure in inducing such expectations will immediately be revealed by the exchange rate. If the private sector expects a higher price level in the future, the present exchange rate will rise and hence the currency depreciate. If no such depreciation occurs, the central bank has not succeeded in inducing such expectations.

This allows simple empirical tests of whether policy measures to escape from a liquidity trap have any effect on expectations. Regarding Japan, from 1999 to the summer of 2003, the yen has fluctuated in the interval 105–130 yen per dollar with an average of about 117. In the year to the summer of 2003, the average rate has been about 120. Hence, there has not been any substantial depreciation. Consequently, any policy in Japan, including the "quantitative easing" with the 50% expansion of the monetary base in the two years to the summer 2003 (Bank of Japan [2]), has apparently not succeeded in any substantial increase in the expected future price level.

The second important result is that, in a small open economy, an *intentional* currency depreciation gives the central bank a way to *induce* the desired private-sector expectations of a higher future price level and higher future inflation. Thus, *an intentional currency depreciation is a potential solution to the crucial problem of making a higher future price level credible.*

In the simple case above, the central bank can indeed implement the optimal policy to escape from a liquidity trap by announcing an initial depreciation of the currency and a crawling peg that starts at the initial rate $\tilde{s}_1$ with a steady appreciation of the currency at the rate of the foreign interest rate to the level $\tilde{s}_2$ in the future. The central bank can achieve the initial depreciation by committing itself to buying and selling unlimited amounts of foreign exchange at the rate $\tilde{s}_1$. If the peg would fail, the domestic currency would appreciate back to the vicinity of the exchange rate before the announcement, making the currency a good investment. Thus, initially, before the peg's credibility has been established, there will be excess demand for the currency. This is easily fulfilled, though, since the central bank can print unlimited amounts of its currency and trade it for foreign exchange.[5] Indeed, there is a big difference between defending a fixed exchange rate for a strong currency under appreciation pressure (when foreign-exchange reserves rise) and for a weak currency under depreciation pressure (when foreign-exchange reserves fall and eventually run out). Thus, the peg can be defended, and after a short time, perhaps a few days, the crawling peg's credibility will have been established. The rate of crawl at the foreign interest rate then corresponds to and results in a domestic nominal interest rate equal to zero, as required by the optimal policy.

Moreover, once the central bank has established the initial rate $\tilde{s}_1$, the private sector cannot believe in a future rate lower than $\tilde{s}_2$, since that would require a negative domestic nominal interest rate to be an equilibrium. Finally, since the private sector expects the terms of trade to revert to the steady-state level in the future, if it believes that the future exchange rate will be $\tilde{s}_2$, it must also believe that the future price level will be $\tilde{p}_2$. Thus, the initial depreciation and the crawling peg *must* induce the desired private-sector expectations of a higher future price level and hence implement the optimal policy to escape from a liquidity trap.

Thus, the crawling peg provides the central bank with a mechanism and an action by which it can directly affect private-sector expectations of the future exchange rate and price level. This appears to be feasible in the absence of a commitment mechanism by which the central bank can commit itself to a particular future money-supply function.

During the initial defense of the peg, the central bank may end up accumulating substantial foreign-exchange reserves. Once the peg's credibility has been established, it may be able to unload these and rebalance its balance sheet, while still maintaining a certain level of excess liquidity so as to make sure that the domestic interest rate stays at zero. Interestingly, sizeable foreign-exchange reserves provides the central bank with an internal balance-sheet incentive to maintain the peg, since a sudden appreciation of the currency would then result in a capital

[5] Furthermore, no currency trader can trade at a different exchange rate than $\tilde{s}_1$: Suppose a trader offered to buy and sell the domestic currency at an exchange rate $s < \tilde{s}_1$. Then other traders could make a profit by buying the domestic currency cheaply from the central bank at $\tilde{s}_1$ and selling it to this trader more expensively at s, instantaneously making a (log) profit of $\tilde{s}_1 - s > 0$ per unit of foreign currency traded (recall that the exchange rate is defined as units of domestic currency per unit of foreign currency). This trader would accumulate excess holdings of the domestic currency and would be unable to sell them without a loss, since any buyer can always buy domestic currency from the central bank at $\tilde{s}_1$. The trader would soon be out of a job.

loss on the foreign-exchange reserves (when these are evaluated in domestic currency, as is the practice).[6]

Furthermore, note that once the peg has become credible and private-sector expectations of the future exchange rate and price level have adjusted, the crawling peg is no longer necessary and binding. The currency could be floated, as long as the private-sector expectations are consistent with the optimal escape from the liquidity trap.

4.1. The original and the optimal Foolproof Way

Svensson [30] advocates the Foolproof Way (FPW) to escape from a liquidity trap. The FPW is to announce and implement (1) a price-level target path, starting above the current price level by a "price gap" to undo and increasing at the rate of the long-run inflation target, (2) a depreciation and a crawling peg of the currency, and (3) an exit strategy in the form the future abandonment of the peg in favor of inflation targeting when the price-level target path has been reached. The rate of crawl originally proposed in the FPW is the difference between a domestic long-run inflation target and average world inflation, $\pi - \pi^*$. Once credibility is established, the domestic interest rate would fulfill

$$i_1 = i^* + \pi - \pi^* \tag{4.4}$$

and normally be *positive* rather than zero. In practice, with a small difference between the domestic inflation target and average world inflation, the peg would be approximately fixed and the domestic interest rate would be approximately equal to the foreign interest rates and hence normally positive. Thus, the original FPW implies that the economy would normally immediately escape from the liquidity trap, in the sense that the nominal interest rate is positive and there is no excess liquidity.

In contrast, the *optimal* crawling peg outlined above has a *zero* domestic interest rate and a rate of appreciation equal to the foreign interest rate. Indeed, we can conceive of the Optimal Foolproof Way (OFPW), having (1) the upward-sloping price-level target path, $\{\hat{p}_t^{\mathrm{o}}\}_{t=1}^{\infty}$, with

$$\hat{p}_t^{\mathrm{o}} = \tilde{p}_2 + (t-2)\pi \qquad (t \geq 1),$$

[6] Jeanne and Svensson [18] show in detail, in a slightly different model of a small open economy, that a central bank's realistic concerns about its independence and thereby capital allows it to commit to a higher future price level through a currency depreciation and a crawling peg. The bank wishes to maintain its independence from the government. A negative capital would require a capital injection and put the bank at the government's mercy. In order to avoid this, the bank never voluntarily allows its capital to fall below a certain minimum level. Because a future currency appreciation would imply a capital loss on the bank's foreign-exchange reserves, a minimum capital level provides a lower bound on the future exchange rate (an upper bound on future currency appreciation). By managing its capital such that the minimum capital level is reached for the exchange rate consistent with the desired higher future price level, the bank can commit itself to that higher future price level. This provides a mechanism for a commitment to the optimal escape from a liquidity trap along the lines of the Foolproof Way.

which coincides with $\tilde{p}_2$ in period 2 and increases at the rate of the long-run inflation target, (2) the crawling peg

$$s_t = \tilde{s}_1 - (t-1)i^* \qquad (t \geq 1),$$

and (3) the abandonment of the peg in favor of flexible inflation targeting as in (2.57) once the price-level target path has been reached. In the above model, the price-level target path would be reached in period 2. The implicit optimal price-level gap to be undone would be, by (3.24),

$$\hat{p}_1^{\mathrm{o}} - p_1 = \tilde{\pi}_2 - \pi = -\frac{\lambda\tilde{\sigma}^2}{\delta + \lambda\tilde{\sigma}^2}(\bar{r}_1 + \pi) > 0.$$

Thus, although effective in escaping from a liquidity trap, because the original FPW has a positive domestic interest rate rather than zero, it is not quite optimal. For the same initial exchange rate as the OFPW, $\tilde{s}_1$, the original FPW would result in a higher-than-optimal expected future exchange rate,

$$s_{2|1} = \tilde{s}_1 + \pi - \pi^* > \tilde{s}_2 = \tilde{s}_1 - i^*$$

(where the inequality holds if $i^* + \pi - \pi^* > 0$) and a correspondingly higher-than-optimal expected future price level,

$$p_{2|1} = \tilde{p}_2 + i^* + \pi - \pi^* > \tilde{p}_2$$

(where the inequality holds if $i^* + \pi - \pi^* > 0$), whereas the resulting real interest rate and output gap in period 1 would still be equal to the optimum one. In figure 4.1, the original FPW would then correspond to an approximately horizontal line starting at $\tilde{s}_1$, implying a higher expected exchange rate and and a higher expected price level in period 2. Alternatively, the original FPW could achieve the optimal expected price level $\tilde{p}_2$ and the optimal expected inflation, $\tilde{\pi}_2$, by a lower initial depreciation in period 1 (lower by i^*), which then would correspond to a higher than optimal real interest rate and lower than optimal output gap in period 1. (It can be shown that the "*optimal* original FPW," characterized by optimization under the constraint that the domestic interest rate fulfills (4.4), would result in a (somewhat) lower initial depreciation, higher expected future price level, higher real interest rate and larger negative output gap than the above outcome of the original FPW.)

For Japan, assume that a domestic long-run inflation target would equal about 1% per year. Interpret the U.S. as having an inflation target of about 2% per year. The current (November 2003) U.S. short rate is about 1%. Thus, with such a low U.S. interest rate, the original Foolproof Way would, after the initial depreciation, imply a Japanese interest rate of approximately 0%, in this case equal to the optimal FPW. Thus, in some cases (more precisely, when $i^* + \pi - \pi^* \approx 0$), there is little difference between the original and the optimal FPW.

5. The international impact in a world of two large economies

Above, the foreign variables have been treated as exogenous and independent of the home country. In particular, foreign output, interest rates and price levels have been taken as exogenous. This is equivalent to assuming that the home country is small and does not affect the rest of the world (except by having some monopoly power for home goods). Now I will assume that the foreign country is no longer necessarily exogenous for the home country and instead examine the impact on the foreign country of policy in the home country. The channels of impact are each country's potential output and natural interest rate's dependence on the other country's output, (2.30), (2.36), (2.43) and (2.47).

The foreign country's productivity level, a_t^*, is assumed to be iid, in analogy with the home country. I continue to focus on period 1, the present, and the consequences of a possible liquidity trap in the present. I assume that the foreign economy has been in the ideal equilibrium for a long time before period 1, so the realizations of the natural interest rate has fulfilled

$$\bar{r}_t^* + \pi^* \geq 0$$

(the analog of (3.3) for the home economy), expected and actual inflation has been equal to the inflation target, and the output gap has been equal to zero. Furthermore, in period 1, for any given expected period-2 price level, $p_{2|1}^*$, the foreign country is expected to continue in the ideal equilibrium from period 2 on ("the future"), so private-sector expectations in period 1 are assumed to fulfill

$$\begin{aligned}
\pi_{3|1}^* &\equiv p_{3|1}^* - p_{2|1}^* = \pi^*, \\
x_{2|1}^* &= 0, \\
y_{2|1}^* &= \bar{y}_{2|1}^* = y = 0, \\
r_{2|1}^* &= \bar{r}_{2|1}^* = \rho > 0, \\
i_{2|1}^* &= \rho + \pi^* > 0.
\end{aligned} \tag{5.1}$$

That is, inflation after period 2 is expected to equal the inflation target, the expected future output gap is zero, the expected output and potential output are zero, the expected real interest rate equals the average natural interest rate, and the expected nominal interest rate equals the sum of the average real interest rate and the inflation target.

Furthermore, the period-1 foreign price level, $p_1^* = p_{1|0}^*$, is by the foreign analog of (2.27) determined by period-0 expectations and given in period 1. The period-1 foreign output gap is given by

$$x_1^* = -\tilde{\sigma}^*(i_1^* - \pi_{2|1}^* - \bar{r}_1^*), \tag{5.2}$$

$$i_1^* \geq 0. \tag{5.3}$$

Given the foreign central bank's intertemporal loss function, (2.58) and the above assumptions, the relevant loss function in period 1 can be simplified to

$$L_1^* = \frac{1}{2}[\lambda^* x_1^{*2} + \delta(\pi_{2|1}^* - \pi^*)^2]. \tag{5.4}$$

5.1. Noncooperation

First, I will examine the case of noncooperation, when the home and foreign central banks conduct independent monetary policy and have independent objectives.[7]

It is practical to express the equilibrium in the space of output gaps. Then, it is practical to express both outputs and the natural interest rates as functions of the output gaps. We can express outputs as a function of the output gaps by combining (2.30), (2.35), (2.36) and (2.38), which leads to

$$y_1 = \bar{\bar{y}}_1 + f_1 x_1 - f_2 x_1^*, \tag{5.5}$$

$$y_1^* = \bar{\bar{y}}_1^* + f_1 x_1^* - f_2^* x_1. \tag{5.6}$$

Here

$$\bar{\bar{y}}_1 \equiv \frac{b_1 a_1 - b_2 b_1^* a_1^*}{1 - b_2 b_2^*},$$

$$\bar{\bar{y}}_1^* \equiv \frac{b_1^* a_1^* - b_2^* b_1 a_1}{1 - b_2 b_2^*},$$

denote the *world* home and foreign potential output levels, the home and foreign output levels that would result in a simultaneous flexprice equilibrium in both the home and foreign country

[7] The model is set up such that, in the absence of a liquidity trap in both countries, policy under either noncooperation or cooperation and under either commitment or discretion results in an equilibrium corresponding to the origin in figure 3.1, the first-best outcome where $x_1 = x_1^* = 0$, $\pi_{2|1} = \pi$ and $\pi_{2|1}^* = \pi^*$ and $L_1 = L_1^* = 0$, so the issues discussed in Canzoneri and Henderson [8] and Persson and Tabellini [27] do not arise.

(for which case the output gaps would be zero in both countries). The coefficients are given by

$$\begin{aligned} f_1 &\equiv \frac{1}{1-b_2 b_2^*} > 0, \\ f_2 &\equiv \frac{b_2}{1-b_2 b_2^*} > 0, \\ f_2^* &\equiv \frac{b_2^*}{1-b_2 b_2^*} > 0, \end{aligned}$$

where the inequalities for f_2 and f_2^* hold if $\sigma < \eta$.[8]

Furthermore, using (2.43), (2.47), the above assumptions, (5.5) and (5.6), we can express the natural interest rates as functions of the output gaps,

$$\begin{aligned} \bar{r}_1 &= \rho - d_1 a_1 - d_2 y_1^* \\ &\equiv \bar{\bar{r}}_1 + g_1 x_1 - g_2 x_1^* \equiv \bar{r}_1(x_1, x_1^*), \qquad (5.7) \\ \bar{r}_1^* &\equiv \rho - d_1^* a_1^* - d_2^* y_1 \\ &\equiv \bar{\bar{r}}_1^* + g_1^* x_1^* - g_2^* x_1 \equiv \bar{r}_1^*(x_1^*, x_1). \qquad (5.8) \end{aligned}$$

Here

$$\begin{aligned} \bar{\bar{r}}_1 &\equiv \rho - d_1 a_1 - d_2 \bar{\bar{y}}_1^*, \\ \bar{\bar{r}}_1^* &\equiv \rho - d_1^* a_1^* - d_2^* \bar{\bar{y}}_1, \end{aligned}$$

denote the *world* home and foreign natural interest rates, the real interest rates that would arise in a simultaneous flexprice equilibrium in both countries. The coefficients fulfill

$$\begin{aligned} g_1 &\equiv d_2 f_2^* > 0, \\ g_2 &\equiv d_2 f_1 > 0, \\ g_1^* &\equiv d_2^* f_2 > 0, \\ g_2^* &\equiv d_2^* f_1 > 0, \end{aligned}$$

where the inequalities for g_2 and g_2^* hold if $\sigma < \eta$. In that case, $g_2 > 0$, and an increase in the foreign output gap reduces the home natural interest rate. The increase in the foreign output gap increases foreign output, which reduces expected foreign output growth, which in turn reduces the home natural interest rate.

[8] As shown above, for $\sigma < \eta$, $b_2 > 0$ and $b_2^* > 0$. Throughout, I assume that $|b_2| < 1$ and $|b_2^*| < 1$, which is the case for reasonable parameters.

The aggregate-demand constraints for both countries can now be written

$$x_1 \leq \tilde{\sigma}[\bar{r}_1(x_1, x_1^*) + \pi_{2|1}], \tag{5.9}$$

$$x_1^* \leq \tilde{\sigma}^*[\bar{r}_1^*(x_1^*, x_1) + \pi_{2|1}^*], \tag{5.10}$$

where the functions $\bar{r}_1(x_1, x_1^*)$ and $\bar{r}_1^*(x_1^*, x_1)$ are given by (5.7) and (5.8).

I first consider the situation under commitment, when both central banks can commit to a future money-supply function and thereby affect private-sector expectations of future inflation and reach the good equilibrium described in section 3. Thus, I assume that the home central bank minimizes (3.15) under commitment and subject to (5.9), taking y_1^* and thereby $\bar{r}_1$ as given. This will lead to the following targeting rule:

(N) No liquidity trap: If possible, set $\pi_{2|1} = \pi$ and choose $i_1 \geq 0$ so as to fulfill the target criterion

$$x_1 = 0.$$

(L) Liquidity trap: If this is not possible, set $i_1 = 0$ and choose $\pi_{2|1} > \pi$ so as to fulfill the target criterion

$$\pi_{2|1} - \pi = -\frac{\lambda\tilde{\sigma}}{\delta}x_1 > 0. \tag{5.11}$$

Analogously, I assume that the foreign central bank minimizes (5.4) under commitment and subject to (5.10), taking y_1 and thereby $\bar{r}_1^*$ as given. This will lead to the analogous targeting rule:

(N*) No liquidity trap: If possible, set $\pi_{2|1}^* = \pi^*$ and choose $i_1^* \geq 0$ so as to fulfill the target criterion

$$x_1^* = 0. \tag{5.12}$$

(L*) Liquidity trap: If this is not possible, set $i_1^* = 0$ and choose $\pi_{2|1}^* > \pi^*$ so as to fulfill the target criterion

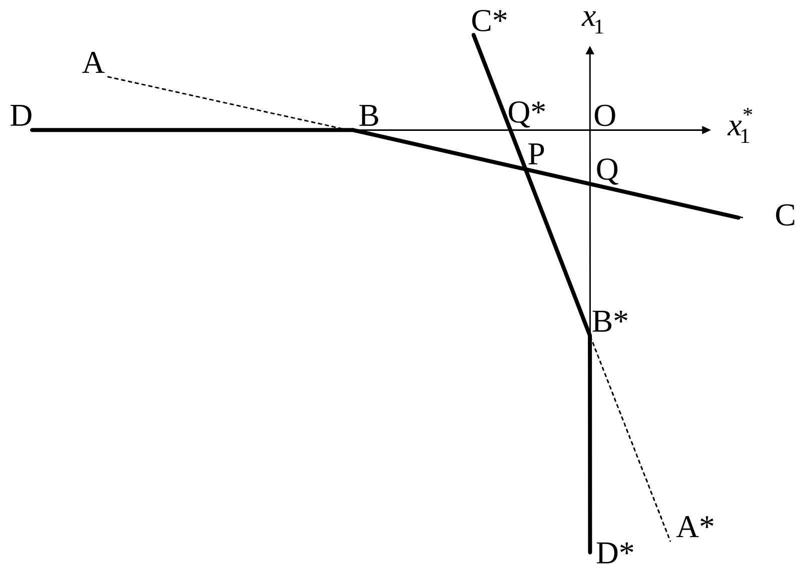
$$\pi_{2|1}^* - \pi^* = -\frac{\lambda^*\tilde{\sigma}^*}{\delta}x_1^* > 0. \tag{5.13}$$

Combining this with the constraints and expressions for the natural interest rates gives the following equation system for the equilibrium output gaps in the noncooperative equilibrium under commitment, the optimal escape from the liquidity traps:

$$x_1 = \min\{\tilde{\sigma}(\bar{\bar{r}}_1 + g_1 x_1 - g_2 x_1^* + \pi - \frac{\lambda\tilde{\sigma}}{\delta}x_1), 0\}, \tag{5.14}$$

$$x_1^* = \min\{\tilde{\sigma}^*(\bar{\bar{r}}_1^* + g_1^* x_1^* - g_2^* x_1 + \pi^* - \frac{\lambda^*\tilde{\sigma}^*}{\delta}x_1^*), 0\}. \tag{5.15}$$

Figure 5.1: The good equilibrium under noncooperation



Depending on the realizations of $\bar{\bar{r}}_1$ and $\bar{\bar{r}}_1^*$, four different equilibria are possible, denoted (N, N*), (L, N*), (N, L*) and (L, L*). The different cases are easy to illustrate graphically in (x_1, x_1^*)-space, the space of the home and foreign output gap. In figure 5.1, with x_1^* along the horizontal axis and x_1 along the vertical axis, the flat negatively sloped line AC represents the equation

$$x_1 = \frac{\delta\tilde{\sigma}(\bar{\bar{r}}_1 + \pi - g_2 x_1^*)}{\delta(1 - \tilde{\sigma}g_1) + \lambda\tilde{\sigma}^2}, \tag{5.16}$$

which is equation (5.14) when the minimum over zero is disregarded.[9] It hits the vertical axis, $x_1^* = 0$, at a point Q below the origin O, where

$$x_1 = \frac{\delta\tilde{\sigma}(\bar{\bar{r}}_1 + \pi)}{\delta(1 - \tilde{\sigma}g_1) + \lambda\tilde{\sigma}^2} < 0. \tag{5.17}$$

Thus, it is drawn for the case $\bar{\bar{r}}_1 + \pi < 0$. The line AC hits the horizontal axes at point B, where $x_1 = 0$ and

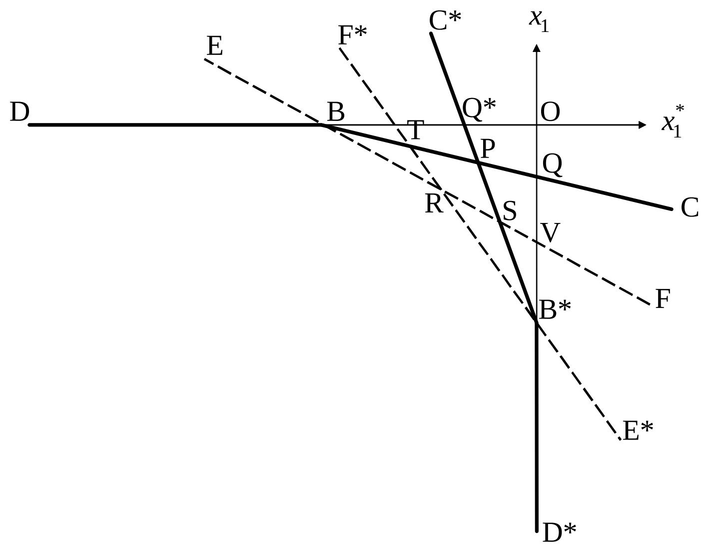
$$x_1^* = \frac{\bar{\bar{r}}_1 + \pi}{g_2} < 0. \tag{5.18}$$

When the minimum over zero in (5.14) is taken into account, equation (5.14), describing the home output gap for a given level of the foreign output gap, is represented by the kinked solid line DBC.

Similarly, the kinked solid line C*B*D* represents equation (5.15), the foreign output gap for a given level of the home output gap. It hits the horizontal axis at point Q* to the left of

[9] I assume $\tilde{\sigma}g_1 < 1$ and $\tilde{\sigma}^* g_1^* < 1$, which is the case for reasonable parameters.

Figure 5.2: The bad equilibrium under noncooperation



the origin, where

$$x_1^* = \frac{\delta\tilde{\sigma}^*(\bar{\bar{r}}_1^* + \pi^*)}{\delta(1 - \tilde{\sigma}^* g_1^*) + \lambda^* \tilde{\sigma}^{*2}} < 0. \tag{5.19}$$

Thus, it is drawn for the case $\bar{\bar{r}}_1^* + \pi^* < 0$. Point B* is given by $x_1^* = 0$ and

$$x_1 = \frac{\bar{\bar{r}}_1^* + \pi^*}{g_2^*} < 0. \tag{5.20}$$

The noncooperative equilibrium, $(\tilde{x}_1, \tilde{x}_1^*)$, is in this case at point P, where both countries are in a liquidity trap, (L, L*), and have negative output gaps. For given home and foreign equilibrium output gaps, the optimal home and foreign expected period-2 inflation overshoots, $\tilde{\pi}_2 - \pi$ and $\tilde{\pi}_2^* - \pi^*$, are given by (5.11) and (5.13).[10]

If $\bar{\bar{r}}_1 + \pi > 0$ and $\bar{\bar{r}}_1^* + \pi^* > 0$, point B in figure 5.1 is to the right of the origin, point O, and point B* is above point O. Then no country is in a liquidity trap, the equilibrium (N, N*), and the equilibrium is given by point O. If $\bar{\bar{r}}_1 + \pi < 0$ but $\bar{\bar{r}}_1^* + \pi^* > 0$, point B is to the left of point O but point B* is above point O. Then only the home country is in a liquidity trap, the equilibrium (L, N*), and the equilibrium is at point Q. This case will be further examined in section 5.1.1. If $\bar{\bar{r}}_1 + \pi > 0$ but $\bar{\bar{r}}_1^* + \pi^* < 0$, point B is to the right of point O but point B* is below point O. Then only the foreign country is in a liquidity trap, the equilibrium (N, L*), and the equilibrium is at point Q*.

The above discussion and figure 5.1 is under the assumption of commitment and hence good

[10] For reasonable parameters, the line AC has a slope less than one and the line C*A* a slope larger than one.

equilibria for both countries. If the home country is stuck in a bad equilibrium, $\pi_{2|1} = \pi$, and the relevant constraint for the home country is

$$x_1 \leq \tilde{\sigma}(\overline{\overline{r}}_1 + g_1 x_1 - g_2 x_1^* + \pi). \tag{5.21}$$

Since, under noncooperation, it is never optimal for the the home central bank to choose a positive output gap, the home equilibrium for a given foreign output gap is given by

$$x_1 = \min\{\tilde{\sigma}(\overline{\overline{r}}_1 + g_1 x_1 - g_2 x_1^* + \pi), 0\}. \tag{5.22}$$

This equilibrium is illustrated in figure 5.2. The dashed line EF represents the equation

$$x_1 = \frac{\tilde{\sigma}(\overline{\overline{r}}_1 + \pi - g_2 x_1^*)}{1 - \tilde{\sigma} g_1}, \tag{5.23}$$

the constraint (5.21) with equality. Points on and to the left of the line fulfills (5.21). The line intersects the horizontal axis in the same point B as line BC, (5.18), but it is steeper than line BC (for $\lambda > 0$), as a comparison of (5.16) and (5.23) shows. When the minimum over zero is taken into account, the equilibrium is given by the kinked line DBF.

Similarly, if the foreign country is stuck in a bad equilibrium, $\pi_{2|1}^* = \pi^*$, and the the foreign equilibrium for given home output gap is given by

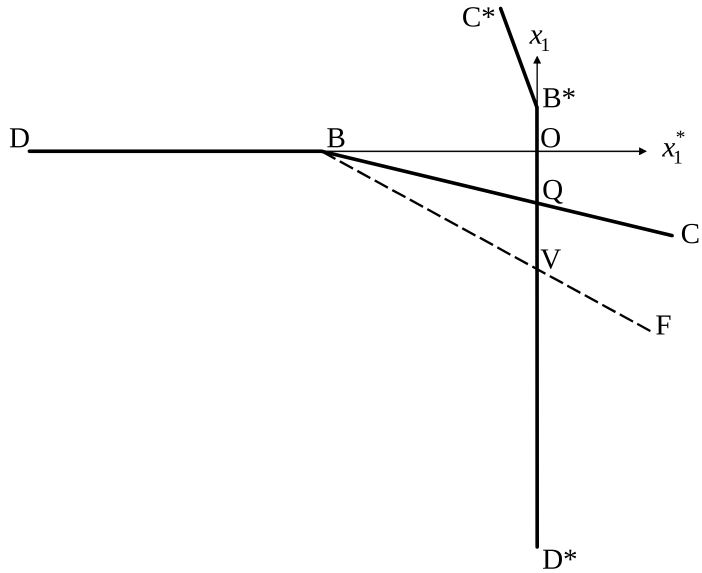
$$x_1^* = \min\{\tilde{\sigma}^*(\overline{\overline{r}}_1^* + g_1^* x_1^* - g_2^* x_1 + \pi^*), 0\}. \tag{5.24}$$

This corresponds to the kinked line F*B*D* in figure 5.2.

If both the home and the foreign country are stuck in a liquidity trap and in a bad equilibrium, the home and the foreign equilibria for given output in the other country are given by the kinked lines DBF and F*B*D*, respectively, and the world equilibrium will be at point R, where the two kinked lines intersect. If the home country is in a bad equilibrium but the foreign country is in a good equilibrium, the relevant kinked lines are DBF and C*B*D*, respectively, and the world equilibrium is at S. If the home country is in a good equilibrium and the foreign country is in a bad equilibrium, the relevant kinked lines are DBC and F*B*D*, and the world equilibrium is at point T. If both countries are in the good equilibrium, the equilibrium is at point P, as we have seen above.

Suppose that the home country is in a liquidity trap and in the bad equilibrium, but manages to move to the good equilibrium. Suppose that the foreign country is also in a liquidity trap. Then, if the foreign country is in a bad equilibrium, the world equilibrium shifts from point R to

Figure 5.3: The equilibrium (L, N*) under noncooperation



point T. If the foreign country is in a good equilibrium, the world equilibrium shifts from point S to point P. In both countries, while the negative output gap decreases in magnitude in the home country, it increases in magnitude in the foreign country. As noted above, the negative international output externality ($b_2 > 0$, $b_2^* > 0$) implies the natural interest rates are decreasing in the other country's output ($d_2 > 0$, $d_2^* > 0$) and output gap ($g_2 > 0$, $g_2^* > 0$). In a liquidity trap, this causes a negative output-gap externality, so a less negative output gap in one country causes a more negative output gap in the other country.

5.1.1. An interesting special case, (L, N*)

If the world foreign natural interest rate, $\bar{\bar{r}}_1^*$, rises, the line C*A* shifts to the right in figure 5.1 and point B* in figures 5.1 and 5.2 shifts up towards O. If $\bar{\bar{r}}_1^*$ rises sufficiently, point B* meets and passes point Q, and point P reaches Q. Then the foreign country is no longer in a liquidity trap, whereas the home country remains in a liquidity trap, (L, N*). The equilibrium is then at point Q. This equilibrium is illuminating.

Let me consider this equilibrium, illustrated in figure 5.3. I assume that the world home and foreign natural interest rates fulfill the conditions

$$\bar{\bar{r}}_1 + \pi < 0, \tag{5.25}$$

$$\bar{\bar{r}}_1^* + \pi^* > 0. \tag{5.26}$$

Condition (5.25) implies that point B lies on the horizontal axis to the left of the origin, and condition (5.26) implies that point B* lies on the vertical axis above the origin.

The foreign country will not be in a liquidity trap, so we have (N*) and

$$\begin{aligned}
\pi^*_{2|1} &= \pi^*, \\
x^*_1 &= 0, \\
i^*_1 &= \bar{r}^*(0, x_1) + \pi^* \;\equiv\; \bar{\bar{r}}^*_1 - g^*_2 x_1 + \pi^* \;\geq 0.
\end{aligned}$$

Thus, $\pi^*_{2|1} = \pi^*$ and x^*_1 are given and independent of the home country, whereas i^*_1 depends on x_1 because the foreign natural interest rate depends on x_1. If $\sigma < \eta$, we have $g^*_2 > 0$, and the foreign interest rate is a decreasing function of the home output gap.

The home country will be in a liquidity trap. I will consider the home country both in the bad equilibrium, when $\pi_{2|1} = \pi$ and $x_1 = \hat{x}_1$, and in the good equilibrium, the optimal escape, when $\pi_{2|1} = \tilde{\pi}_2 > \pi$ and $x_1 = \tilde{x}_1$. The bad equilibrium corresponds to point V in figure 5.3 and will fulfill

$$\begin{aligned}
\pi_{2|1} &= \pi, \\
r_1 &= -\pi \;\equiv\; \hat{r}_1 \;>\; \bar{r}_1(\hat{x}_1, 0) \;\equiv\; \bar{\bar{r}}_1 + g_1\hat{x}_1, \\
x_1 &= \tilde{\sigma}(\bar{\bar{r}}_1 + g_1 x_1 + \pi) \;=\; \frac{\tilde{\sigma}}{1 - \tilde{\sigma} g_1}(\bar{\bar{r}}_1 + \pi) \;\equiv\; \hat{x}_1 \;<\; 0 \\
i^*_1 &= \bar{\bar{r}}^*_1 - g^*_2\hat{x}_1 + \pi^* \;\equiv\; \hat{i}^*_1 \;\geq\; 0
\end{aligned}$$

The good equilibrium corresponds to point Q in figure 5.3 and will fulfill

$$\pi_{2|1} = \pi - \frac{\lambda\tilde{\sigma}}{\delta}\tilde{x}_1 \;\equiv\; \pi - \frac{\lambda\tilde{\sigma}^2}{\delta(1 - \tilde{\sigma} g_1) + \lambda\tilde{\sigma}^2}(\bar{\bar{r}}_1 + \pi) \;\equiv\; \tilde{\pi}_2 \;>\; \pi, \tag{5.27}$$

$$r_1 = -(\pi + \tilde{\pi}_2) \;\equiv\; \tilde{r}_1 \;<\; \hat{r}_1,$$

$$x_1 = \tilde{\sigma}(\bar{\bar{r}}_1 + g_1 x_1 + \tilde{\pi}_2) \;=\; \frac{\delta\tilde{\sigma}}{\delta(1 - \tilde{\sigma} g_1) + \lambda\tilde{\sigma}^2}(\bar{\bar{r}}_1 + \pi) \;\equiv\; \tilde{x}_1 \;>\; \hat{x}_1, \tag{5.28}$$

$$i^*_1 = \bar{\bar{r}}^*_1 - g^*_2\tilde{x}_1 + \pi^* \;\equiv\; \tilde{i}^*_1 \;<\; \hat{i}^*_1,$$

where the last inequality holds if $\sigma < \eta$.

We note that the home country going from the bad equilibrium V to the good equilibrium Q, from $\hat{x}_1$ to $\tilde{x}_1$, has no impact on the foreign output gap, $\tilde{x}^*_1 = 0$, or expected future inflation in the foreign country, $\pi^*_{2|1} = \pi^*$. When $\sigma < \eta$, the increase in the home output gap reduces the foreign natural, real and nominal interest rates by $g^*_2(\tilde{x}_1 - \hat{x}_1)$.

If point B* lies above point Q, as is the case in figure 5.3, the fall in the foreign natural real interest rate simply leads to a corresponding fall in the real and nominal interest rate and an unchanged foreign zero output gap. However, in the case where point B* lies between point V and point Q, a move of the home country from the bad equilibrium to the good equilibrium would lead to an equilibrium on the segment BQ to the left of point Q, with a liquidity trap and negative output gap in the foreign country. Then fall in the foreign natural interest rate causes the foreign country to hit the zero lower bound for the interest rate and throws it into a liquidity trap. I will return to that particular case in the discussion of cooperation in section 5.2.

5.1.2. Exchange-rate paths

Let me also examine the exchange-rate paths in the case (L, N*). The exchange-rate path in the bad equilibrium is given by

$$\begin{aligned} s_{2|1} &= \hat{p}_2 - \hat{p}_2^* \equiv \hat{s}_2, \\ s_1 &= \hat{s}_2 + \hat{\imath}_1^* \equiv \hat{s}_1. \end{aligned}$$

In the good equilibrium, if the foreign country is not in a liquidity trap, the expected foreign period-2 price level continues to equal to $\hat{p}_2^* \equiv p_1^* + \pi^*$. The exchange rate-path for the good equilibrium in the optimal escape is then given by

$$\begin{aligned} s_{2|1} &= \tilde{p}_2 - \hat{p}_2^* \equiv \tilde{s}_2 > \hat{s}_2, \\ s_1 &= \tilde{s}_2 + \tilde{\imath}_1^* \equiv \tilde{s}_1 > \hat{s}_1. \end{aligned} \tag{5.29}$$

If $\sigma < \eta$, the foreign nominal interest rate is lower in the optimal escape than in the bad equilibrium,

$$\tilde{\imath}_1^* - \hat{\imath}_1^* = -\, g_2^*(\tilde{x}_1 - \hat{x}_1) < 0.$$

The inequality $\tilde{s}_1 > \hat{s}_1$ holds under the reasonable assumption that the difference between $\tilde{p}_2 \equiv p_1 + \tilde{\pi}_2$ and $\hat{p}_2 \equiv p_1 + \pi$ dominates over the difference in the foreign interest rate (which is likely for reasonable parameters, since the former may involve some 20–30% whereas the latter probably involves less than half a percentage point).

Thus, if the home central bank implements the optimal escape by a depreciation of the currency and a crawling peg, the initial depreciation can be a little less than for the small open economy, and the rate of appreciation during the crawl can be a little less. This modification of the crawling peg is likely to be small and seems unlikely to have any practical consequences.

5.2. Optimal cooperation

Next, I will examine the case when monetary policy in the two countries are coordinated and have a common objective. I write the constraints with the natural interest rates explicitly depending on the output gaps,

$$x_1 \leq \tilde{\sigma}(\overline{\overline{r}}_1 + g_1 x_1 - g_2 x_1^* + \pi_{2|1}), \tag{5.30}$$

$$x_1^* \leq \tilde{\sigma}^*(\overline{\overline{r}}_1^* + g_1^* x_1^* - g_2^* x_1 + \pi_{2|1}^*). \tag{5.31}$$

The world loss is taken to be

$$(1-\alpha)L_1 + \alpha L_1^* = (1-\alpha)\frac{1}{2}[\lambda x_1^2 + \delta(\pi_{2|1} - \pi)^2] + \alpha\frac{1}{2}[\lambda^* x_1^{*2} + \delta(\pi_{2|1}^* - \pi^*)^2], \tag{5.32}$$

where the weights on the countries correspond to their relative size. Optimal cooperation under commitment involves choosing $\pi_{2|1}$, $\pi_{2|1}^*$, x_1 and x_1^* so as to minimize the world loss subject to (5.30) and (5.31), taking into account the natural interest rates' depending on the output gaps in both countries. The corresponding Lagrangian is

$$\begin{aligned}
\mathcal{L}_1 \;=\;& (1-\alpha)L_1 + \alpha L_1^* \\
& - (1-\alpha)\phi_1[\tilde{\sigma}(\overline{\overline{r}}_1 + g_1 x_1 - g_2 x_1^* + \pi_{2|1}) - x_1] \\
& - \alpha\phi_1^*[\tilde{\sigma}^*(\overline{\overline{r}}_1^* + g_1^* x_1^* - g_2^* x_1 + \pi_{2|1}^*) - x_1^*],
\end{aligned}$$

where the Lagrange multipliers $\phi_1 \geq 0$ and $\phi_1^* \geq 0$ fulfill the complementary slackness conditions

$$\begin{aligned}
\phi_1[\tilde{\sigma}(\overline{\overline{r}}_1 + g_1 x_1 - g_2 x_1^* + \pi_{2|1}) - x_1] &\geq 0, \\
\phi_1^*[\tilde{\sigma}^*(\overline{\overline{r}}_1^* + g_1^* x_1^* - g_2^* x_1 + \pi_{2|1}^*) - x_1^*] &\geq 0.
\end{aligned}$$

The first-order conditions with respect to $p_{2|1}$, x_1, $p_{2|1}^*$ and x_1^* are, respectively,

$$\begin{aligned}
\delta(\pi_{2|1} - \pi) - \phi_1\tilde{\sigma} &= 0, \\
\lambda x_1 - \phi_1\tilde{\sigma}g_1 + \phi_1 + \frac{\alpha}{1-\alpha}\phi_1^*\tilde{\sigma}^* g_2^* &= 0, \\
\delta(\pi_{2|1}^* - \pi^*) - \phi_1^*\tilde{\sigma}^* &= 0, \\
\lambda^* x_1^* - \phi_1^*\tilde{\sigma}^* g_1^* + \phi_1^* + \frac{1-\alpha}{\alpha}\phi_1\tilde{\sigma}g_2 &= 0.
\end{aligned}$$

This, together with the complementary slackness conditions, leads to the following targeting rule for the home central bank:

(N) No liquidity trap: If possible, set $\pi_{2|1} = \pi$ and choose $i_1 \geq 0$ so as to fulfill the target criterion

$$x_1 = -\frac{\alpha}{1-\alpha}\frac{\delta g_2^*}{\lambda}(\pi_{2|1}^* - \pi^*) \leq 0.$$

(L) Liquidity trap: If this is not possible, set $i_1 = 0$ and choose $\pi_{2|1} > \pi$ so as to fulfill the target criterion

$$\pi_{2|1} - \pi = -\frac{\lambda\tilde{\sigma}}{\delta(1-\tilde{\sigma}g_1)}x_1 - \frac{\alpha}{1-\alpha}\frac{\tilde{\sigma}g_2^*}{1-\tilde{\sigma}g_1}(\pi_{2|1}^* - \pi^*) > 0. \tag{5.33}$$

We can understand the targeting criterion (5.33) in the following way. Suppose that the home country is in a liquidity trap with a given negative period-1 output gap, $x_1 < 0$, and a positive expected period-2 inflation overshoot, $\pi_{2|1} - \pi > 0$, that fulfill the constraint (5.30) with equality. Suppose that the central bank considers a marginal increase $dx_1 > 0$ in the output gap, that is, a reduction in the magnitude of the negative output gap. This would bring a direct change in the home loss from the negative output gap by

$$\frac{\partial L_1}{\partial x_1}dx_1 = \lambda x_1 dx_1 < 0,$$

that is, a reduction of the home loss. The change in the world loss is $1-\alpha$ times the change in the home loss, taking into account the size of the home country. For a constant natural interest rate, by (5.30), the increase in the output gap dx_1 requires an increase in the expected period-2 inflation overshoot of $dx_1/\tilde{\sigma}$. However, the natural interest rate is not constant but increases by $d\bar{r}_1 = g_1 dx_1 > 0$, due to the effect of the home output gap on foreign potential and actual output and the effect of that on the home natural interest rate. The increase in the natural interest rate makes the constraint (5.30) less binding, which allows a reduction of the expected inflation overshoot by $-g_1 dx_1$. The total required change in the inflation overshoot is therefore $(1-\tilde{\sigma}g_1)dx_1/\tilde{\sigma}$. The change in the home loss due to this is

$$\frac{\partial L_1}{\partial(p_{2|1} - \hat{p}_2)}(1-\tilde{\sigma}g_1)\frac{1}{\tilde{\sigma}}dx_1 = \delta(\pi_{2|1} - \pi)(1-\tilde{\sigma}g_1)\frac{1}{\tilde{\sigma}}dx_1 > 0$$

(the discount factor enters because the overshoot occurs in period 2), a net loss. The change in the world loss is $1-\alpha$ times the change in the home loss. Suppose that the foreign country is also in a liquidity trap, with a negative output gap, $x_1^* < 0$, and a positive expected inflation overshoot, $\pi_{2|1}^* - \pi^* > 0$. The increase in the home output gap will change the foreign natural interest rate by $d\bar{r}_1^* = -g_2^* dx_1 < 0$, a reduction. This makes the foreign constraint (5.31) more binding. If the foreign output gap is held constant (which is convenient, since otherwise we need to keep track of its effect on the home natural interest rate), this requires an equal rise in the expected foreign inflation overshoot, $g_2^* dx_1 > 0$. The increase in the foreign loss from this is

$$\frac{\partial L_1^*}{\partial(\pi_{2|1}^* - \pi^*)} g_2^* dx_1 = \delta(\pi_{2|1}^* - \pi^*) g_2^* dx_1 > 0,$$

and the change in the world loss is α times the change in the foreign loss. In an optimum, all the changes in the world loss from a change in the home output gap for a given foreign output gap must sum to zero, which implies

$$(1-\alpha)[\lambda x_1 dx_1 + \delta(\pi_{2|1} - \pi)(1 - \tilde{\sigma} g_1)\frac{1}{\tilde{\sigma}} dx_1] + \alpha\delta(\pi_{2|1}^* - \pi^*) g_2^* dx_1 = 0.$$

Solving for $\pi_{2|1} - \pi$ results in the targeting criterion (5.33).

The analogous targeting rule for the foreign central bank is:

(N*) No liquidity trap: If possible, set $\pi_{2|1}^* = \pi^*$ and choose $i_1^* \geq 0$ so as to fulfill the target criterion

$$x_1^* = -\frac{1-\alpha}{\alpha}\frac{\delta g_2}{\lambda^*}(\pi_{2|1} - \pi) \leq 0.$$

(L*) Liquidity trap: If this is not possible, set $i_1^* = 0$ and choose $\pi_{2|1}^* > \pi^*$ so as to fulfill the target criterion

$$\pi_{2|1}^* - \pi^* = -\frac{\lambda^* \tilde{\sigma}^*}{\delta(1 - \tilde{\sigma}^* g_1^*)} x_1^* - \frac{1-\alpha}{\alpha}\frac{\tilde{\sigma}^* g_2}{1 - \tilde{\sigma}^* g_1^*}(\pi_{2|1} - \pi) > 0.$$

The combination of these targeting rules with the constraints (5.30) and (5.31) will then determine the four kinds of equilibria, (N, N*), (L, N*), (N, L*) and (L, L*).

5.2.1. The special case, (L, N*)

Let me look at the special case when the home country is in a liquidity trap but the foreign country is not, (L, N*). I assume that the natural interest rates fulfills the conditions (5.25) and (5.26), so the noncooperative equilibrium corresponds to figure 5.3, with points B and B* located as in the figure.

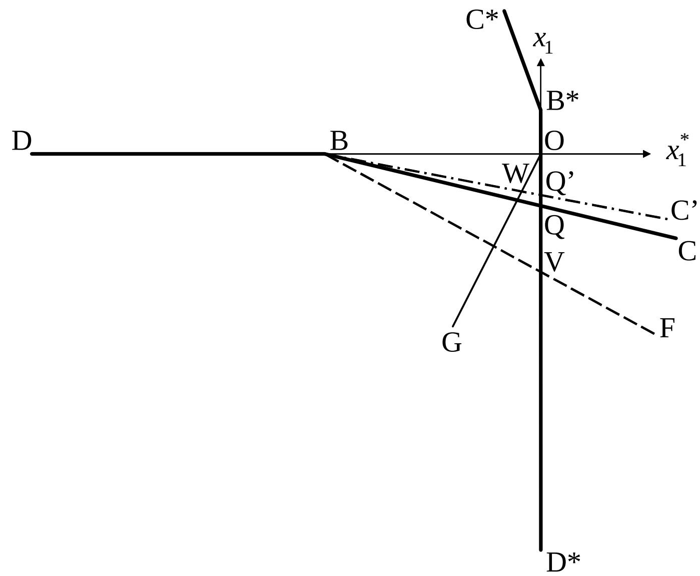
Since the foreign country is not in a liquidity trap, (N*), we have

$$\pi_{2|1}^* = \pi^*, \tag{5.34}$$

$$x_1^* = -\frac{1-\alpha}{\alpha}\frac{\delta g_2}{\lambda^*}(\pi_{2|1} - \pi) \leq 0, \tag{5.35}$$

$$i_1^* = \bar{\bar{r}}_1^* + g_1^* x_1^* - g_2^* x_1 + \pi^* - \frac{1}{\tilde{\sigma}^*} x_1^* \geq 0. \tag{5.36}$$

Figure 5.4: The equilibrium (L, N*) under cooperation



Thus, $\pi^*_{2|1}$ is given by the foreign inflation target and independent of the home country, whereas $x_1^* < 0$ if $\pi_{2|1} > \pi$, which is the case if the home country is in a liquidity trap.

When the foreign country is not in a liquidity trap, if the home country is not in a liquidity trap, the home central bank simply sets $\pi_{2|1} = \pi$ and chooses $i_1 \geq 0$ so as to achieve $x_1 = 0$. If the home country is in a liquidity trap, the home central bank sets $i_1 = 0$ and chooses $\pi_{2|1}$ so as to achieve the target criterion

$$\pi_{2|1} - \pi = -\frac{\lambda\tilde{\sigma}}{\delta(1-\tilde{\sigma}g_1)}x_1 \ > \ 0. \tag{5.37}$$

We can combine this and (5.30) to express the home equilibrium output for given foreign output as

$$x_1 = \min\{\tilde{\sigma}[\bar{\bar{r}}_1 + g_1 x_1 - g_2 x_1^* + \pi - \ \frac{\lambda\tilde{\sigma}}{\delta(1-\tilde{\sigma}g_1)}x_1], 0\}.$$

This is illustrated as the kinked line DBC' in figure 5.4, similar to the kinked line DBC for the noncooperative equilibrium. The dashed-dotted segment BC' corresponds to the line

$$x_1 = \frac{\delta\tilde{\sigma}(\bar{\bar{r}}_1 + \pi - g_2 x_1^*)}{\delta(1-\tilde{\sigma}g_1) + \frac{\lambda\tilde{\sigma}^2}{1-\tilde{\sigma}g_1}}, \tag{5.38}$$

which is flatter than the segment BC (which corresponds to (5.16)), since cooperation takes into account that, for a given foreign output gap, an increase in the home output gap increased the home natural interest rate and makes the constraint (5.30) less binding.

For a zero foreign output gap, the world equilibrium would be at point Q', where the segment BC' meets the vertical axis. This would correspond to a situation when the foreign central bank is not cooperating and taking home output and hence the foreign natural interest rate as given, whereas the home country incorporates the endogeneity of the foreign output and thereby the home natural interest rate and consequently behaves as a Stackelberg leader.

However, under cooperation, the foreign targeting criterion is (5.35) rather than (5.12). Combining this with the home target criterion (5.37) gives the condition

$$x_1^* = \frac{1-\alpha}{\alpha}\frac{\lambda}{\lambda^*}\frac{\tilde{\sigma} g_2}{1-\tilde{\sigma} g_1} x_1 < 0. \tag{5.39}$$

This equation corresponds to the ray OG in figure 5.4. Thus, the equilibrium under optimal cooperation is given by point W, where the ray OG intersects the kinked line DBC'. I let this equilibrium be denoted by $(\tilde{\tilde{x}}_1, \tilde{\tilde{x}}_1^*)$. Comparing with the noncooperative equilibrium at point Q, $(\tilde{x}_1, \tilde{x}_1^*)$, we see that

$$\tilde{\tilde{x}}_1^* \;<\; \tilde{x}_1^* \;=\; 0, \tag{5.40}$$

$$0 \;>\; \tilde{\tilde{x}}_1 \;>\; \tilde{x}_1. \tag{5.41}$$

In the equilibrium under optimal cooperation, the foreign country has a negative output gap in spite of not being in a liquidity trap. As a result, the magnitude of the home negative output gap is reduced compared to the noncooperative case.

The slope of the ray OG and the location of the equilibrium W depends on the relative home and foreign weights on output gap stabilization, λ/λ^*, as (5.39) shows. For a given home weight λ, we see that, for $\lambda^* = \infty$, the ray becomes vertical and corresponding to $x_1^* = 0$ and OD*, which results in an a world equilibrium at point Q'. When the weight on foreign output-gap stabilization becomes infinitely large, the foreign output gap is held at zero regardless of the outcome for the home output gap. For $\lambda^* = 0$, the ray becomes horizontal and corresponding $x_1 = 0$ and OD, which results in a world equilibrium at point B. When there is zero weight on stabilizing the foreign output gap, it optimal to let it grow so negative and large that the home output gap becomes zero. This is done by increasing the home natural interest rate so it fulfills the condition

$$\bar{r}_1(0, \tilde{\tilde{x}}_2) + \pi \equiv \bar{\bar{r}}_1 - g_2 \tilde{\tilde{x}}_1 + \pi = 0,$$

so the home country is no longer in a liquidity trap.

Above, when discussing noncooperation in the case (L, N*), we noticed that the home country moving from the bad to the good equilibrium results in the world equilibrium moving from point

V to point Q in figures 5.3 and 5.4, with the foreign country remaining in the ideal equilibrium. This brings the world equilibrium closer to the optimal equilibrium under cooperation, point W. It reduces the home loss, without any impact on the foreign loss. This is obviously the case as long as point B* does not fall below point Q, which is the case as long as the world foreign natural interest rate, $\bar{\bar{r}}_1^*$, is not too low but fulfills the condition

$$\bar{\bar{r}}_1^* + \pi^* \geq \frac{\delta\tilde{\sigma}g_2^*(\bar{\bar{r}}_1 + \pi)}{\delta(1 - \tilde{\sigma}g) + \lambda\tilde{\sigma}^2} \tag{5.42}$$

(this condition follows from the right side of (5.20) greater than or equal to the right side of (5.17)).

Suppose, however, that condition (5.42) is violated and that point B* falls between point V and Q. This means that, when the home country moves from the bad to the good equilibrium and the foreign natural interest rate falls, the foreign country falls into a liquidity trap and develops a negative output gap. Under noncooperation, the world equilibrium would move from point V to a point P on the segment BQ to the left of point Q, where the segment C*B* (with B* below Q) would intersect BQ. This would be in the vicinity of point W, the equilibrium under the assumption of no liquidity trap for the foreign country. Clearly, from a the point of view of the world loss, the move from point V to that point P would be a good one, since it would reduce world loss.

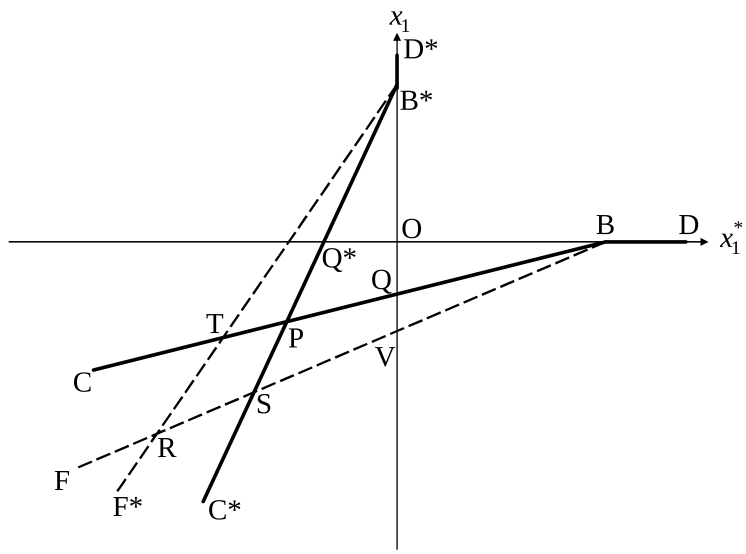
Intuitively, the equilibrium under optimal coordination involves some degree of equalization of the home and foreign output gap. Instead of a zero foreign output gap and a negative home output gap, it is better to have negative foreign output gap and this way reduce the magnitude of the negative home output gap. This means that the home country going from the bad to the good equilibrium reduces world loss, even if it would create a negative foreign output gap.

5.3. The trade balance

The international impact above has been discussed in terms of the effect on the natural interest rates and the output gaps. Some discussion of the consequences of a currency depreciation has emphasized the impact on the trade balance, perhaps because of its visibility. From (2.52), (5.5) and (5.6), we can write the share of home period-1 net export in steady state GDP, nx_1, as a function of the output gaps,

$$\mathrm{nx}_1 = \alpha(1 - \frac{1}{\eta})(y_1 - y_1^*) = \alpha(1 - \frac{1}{\eta})[\bar{\bar{y}}_1 - \bar{\bar{y}}_1^* + (f_1 + f_2^*)x_1 - (f_1 + f_2)x_1^*].$$

Figure 5.5: The good and bad equilibria under noncooperation with positive output externalites



Let us assume the Marshall-Lerner condition, so $\eta > 1$ and net export is positively related to the period-1 terms of trade τ_1.

Consider the case when the home country is in a liquidity trap and the foreign country is not, (L, N*). Under noncooperation, when $\tilde{x}_1^* = 0$, a move from the bad to the good equilibrium, with a closing of the negative output gap from $\hat{x}_1 < 0$ to $\hat{x}_1 < \tilde{x}_1 < 0$, results in a rise in τ_1 and a rise in home net export. For some readers, this might seem to be a problem for the foreign country. However, this is a terms-of-trade improvement for the foreign country, and as such beneficial for the foreign country, without any impact on the foreign output gap.

Furthermore, from the point of view of the equilibrium under international cooperation, the move of net export is in the right direction. A move from the good equilibrium under noncooperation to the good equilibrium under cooperation involves a further reduction of the magnitude of the negative home output gap and the creation of a negative foreign output gap, as we have seen in (5.40) and (5.41). This implies an even further increase in τ_1, that is, a further terms-of-trade improvement for the foreign country, and further rise in home net export.

5.4. The case with positive international output externalities

The case discussed above is the case of *negative* international output externalities, taken as the basic case in this paper. As mentioned above, the source of the negative output externality is the assumption of complete international risk-sharing. Then an increase in foreign output both reduces the terms of trade and increases home consumption. For given home consumption, the former leads to a fall in home marginal cost and a rise in home potential output. The latter leads to a rise in the home CPI wage, a rise in home marginal cost, and a fall in home potential output. Under the reasonable assumption of a lower intertemporal elasticity of substitution than the intratemporal elasticity of substitution between home and foreign goods, the latter effect dominates. If we believe that the assumption of complete risk-sharing is unrealistic, we might believe that the terms-of-trade effect dominates, resulting in *positive* international output externalities. This subsection summarizes the results in that case.

With positive international output externalities, we have $b_2 < 0$ and $b_2^* < 0$, that is, home potential output is *increasing* in foreign output, and vice versa. Then we also have $d_2 < 0$ and $d_2^* < 0$, so the natural interest rates are decreasing in the other country's expected output growth. In period 1, under the assumptions in section 2, the natural interest rates will be *increasing* in the other country's potential output, $g_2 < 0$ and $g_2^* < 0$. Figure 5.5 is drawn for this case, under the assumptions $\overline{\overline{r}}_1 + \pi < 0$ and $\overline{\overline{r}}_1^* + \pi^* < 0$, the same as for figures 5.1 and 5.2, so it can be compared to those figures. Points Q and Q* are the same (corresponding to (5.17) and (5.19), respectively). However, the lines BC and C*B* are now positively sloped (since $g_2 < 0$ and $g_2^* < 0$). Points B and B* are now to the right and above the origin, respectively, and given by (5.18) and (5.20), but with opposite signs.

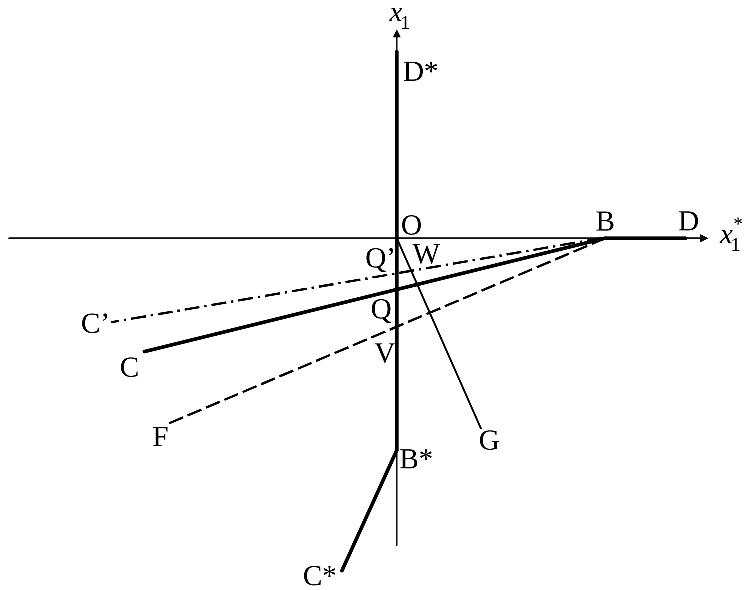
Thus, the kinked line CBD shows the good home equilibrium under noncooperation for given foreign output. The kinked line C*B*D* shows the good foreign equilibrium under noncooperation for given home output. The world equilibrium with both countries in the good equilibrium under noncooperation is at point P.

The kinked line FBD shows the bad home equilibrium for given foreign output. The kinked line F*B*D* shows the bad foreign equilibrium for given home output. The world equilibrium with both countries in the bad equilibrium is at point R.

Suppose that the home country implements the optimal escape and moves from the bad equilibrium to the good equilibrium. If the foreign country is in the bad equilibrium, the world equilibrium moves from point R to point T. If the foreign country is in the good equilibrium, the world equilibrium moves from point S to point T. In both cases, the magnitudes of the negative output gaps are reduced for both countries.

Figure 5.6 shows the case (L, N*), when the foreign country is not in a liquidity trap. It is drawn for the case (5.25) and (5.26), which now implies that point B* is below the origin.

Figure 5.6: The equilibrium (L, N*) under cooperation with positive output externalities



Assume that point B* is sufficiently below the origin to be below point V, as in figure 5.6.

Assume noncooperation. Then, if the home country is in a bad equilibrium, the world equilibrium is at point V. If the home country implements the optimal escape from the liquidity trap and moves to the good equilibrium, the world equilibrium moves to point Q. The foreign output gap is zero in both equilibria. The foreign potential output and natural interest rate both increase when the home country moves from the bad equilibrium to the good equilibrium.

Assume cooperation. The home equilibrium for given foreign output is now given by the kinked line C'BD, where the segment C'B is flatter than the segment CB because the home central bank takes into account that an increase in the home output gap increases the home natural interest rate and makes the constraint (5.30) less binding. The foreign targeting criterion corresponds to ray OG, (5.39) with the opposite sign. Thus, the world equilibrium under cooperation for the case (L, N*) is at point W, where the ray OG intersects the segment C'B, with a positive foreign output gap and reduced magnitude of the negative home output gap.

Suppose that the point B* is between points V and Q. Under noncooperation, if the home country is in a bad equilibrium, the foreign country is in a liquidity trap, with the world equilibrium on the segment FV, to the left of point V. If the home country moves to the good equilibrium, the equilibrium would move to point Q, and the foreign country would be out of its liquidity trap.

Thus, positive output externalities removes any of the conflicts arising with negative output externalities.

6. Conclusions

The optimal policy in a liquidity trap—the optimal escape from a liquidity trap—involves creating private-sector expectations of a higher future price level and higher future inflation, as noted by Krugman [20] and recently demonstrated in detail by Jung, Teranishi and Watanabe [19] and Eggertsson and Woodford [14]. This reduces the real interest rate and mitigates the recession associated with the liquidity trap. As emphasized by Krugman [20], there is a credibility problem with this optimal policy, in that it is difficult to make the private-sector believe in a higher future inflation, especially if the central bank has a reputation for achieving low inflation. Absent any mechanism for a commitment to a higher future price level or future money supply, private-sector expectations of the higher future price level are unlikely to arise.

In this context, this paper has emphasized two important roles for the exchange rate. First, as noted by Svensson [31] and demonstrated in detail in this paper, the current exchange rate serves as an indicator of private-sector expectations of the future price level. If any policy succeeds in substantially raising those expectations, this will be directly revealed by a substantial current currency depreciation. Correspondingly, if policy fails in substantially raising those expectations, this will be revealed by the absence of any substantial depreciation. An example of the latter is provided by the "quantitative easing" in Japan, where a more than 60% expansion of the monetary base since the spring of 2001 has not been accompanied by any substantial yen depreciation.

Second, as argued in Svensson [30], an intentional currency depreciation and a crawling peg—the Foolproof Way to escape from a liquidity trap—can induce private-sector expectations of a higher future price level, reduce the real interest rate and mitigate the recession in the liquidity trap. Furthermore, as argued in Svensson [31] and demonstrated in detail in this paper, a variant of the Foolproof Way, the Optimal Foolproof Way, can indeed implement the optimal escape from a liquidity trap.[11] The currency depreciation and the crawling peg are technically possible, since the main threats are appreciation pressure and excess demand for the currency,

[11] As noted above, the difference between the Optimal Foolproof Way presented in this paper and the original Foolproof Way presented in Svensson [30] is in practice small; the former has a zero domestic interest rate whereas the latter has a domestic interest rate that would in most cases be positive, being equal to the foreign interest rate plus the difference between the home inflation target and the average foreign inflation rate.

which can easily be countered by increased issue of the currency. The visibility, verifiability and technical feasibility of the exchange-rate peg makes a commitment to an exchange-rate peg and the credibility of such a commitment much more realistic than a commitment to a particular future money supply or a particular future price level. The central bank can quickly demonstrate that it is both able and willing to maintain the peg and this way make it credible. Once the peg is credible, the private-sector must expect a higher future exchange rate (a weaker currency in the future), higher by the same magnitude as the initial currency depreciation. Furthermore, for given expectations of the future terms of trade and the future foreign price level, the private-sector must expect a higher future domestic price level. Thus, the initial currency depreciation and the commitment to the crawling peg serves as a commitment to the higher future price level and therefore provides a solution to the credibility problem emphasized by Krugman.

This paper also examines the impact of the rest of the world of a country implementing the optimal escape from a liquidity trap. In a two-country world, the paper clarifies how the international impact can be expressed in terms of the effect of one country's output on the other country's potential output and natural interest rate. The international equilibria with and without liquidity traps in the countries are characterized under international noncooperation and cooperation. Under noncooperation, each country independently tries to achieve its inflation target and stabilize its output gap at zero, taking the situation in the other country as given. Under cooperation, both countries jointly minimize the deviation of their inflation rates from their targets and their output gaps.

For the case of *negative* international output externalities, which results under the assumption of complete international risk sharing and the intertemporal elasticity of substitution being less than the elasticity of substitution between home and foreign goods, implementing the optimal escape and mitigating the recession in the home country under noncooperation will lower the foreign natural interest rate somewhat. If the foreign country is not in a liquidity trap and its nominal interest rate is positive, under noncooperation, its optimal policy is to reduce its nominal and real interest rate, which allows it to achieve both its inflation target and maintain a zero output gap. However, if the foreign nominal interest rate is close to zero, the lower foreign natural interest rate could lead to a binding liquidity trap for the foreign country, which would imply a somewhat negative output gap. This may seem to be an undesirable negative impact on the foreign country of the optimal escape from the liquidity trap of the home country.

However, a relevant comparison is with the optimal policy under international cooperation.

This policy results in the smoothing of the output gaps between the counties, with a milder recession in the home country and larger recession in the foreign country than in the noncooperative equilibrium. Thus, if the home country under noncooperation implements the optimal escape from a liquidity trap and happens to cause a recession in the foreign country (which happens only if the foreign country thereby falls into a liquidity trap), this moves the world equilibrium towards the equilibrium under optimal international cooperation.

For the case of *positive* international output externalities, which may result under less than perfect international risk sharing and therefore be more realistic, implementing the optimal escape increases the foreign natural interest rate and reduces the severity or eliminates any foreign liquidity trap.

My conclusion is that the Foolproof Way is an effective policy to escape from a liquidity trap in small and large open economies and that the international impact of such a policy is not a problem. This conclusion is separately supported by several simulations of the outcome of the Foolproof Way for Japan, for instance, in Coenen and Wieland [12] and Meredith [24].

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A. Appendix

The home household has access to a complete world financial market and faces the period-t budget constraint,

$$P_t^c C_t + M_t + \mathrm{E}_t Q_{t+1,t} D_{t+1} = \Pi_t + W_t N_t - Z_t + M_{t-1} + D_t,$$

where $Q_{t+1,t}$ is the stochastic market home-currency discount factor, the home-currency value in period t of one state-contingent unit of home currency in period $t+1$; D_{t+1} is the state-contingent home-currency value of the household's financial assets in period $t+1$ including foreign assets but excluding money; $\mathrm{E}_t Q_{t+1,t} D_{t+1}$ is the home-currency value of these assets in period t; $M_t + D_{t+1}$ is the total state-contingent home-currency value of the home household's financial wealth in period $t+1$, including dividends and interest; $\Pi_t \equiv \int_{\iota=0}^{1} (P_t(\iota) - \frac{W_t}{A_t}) Y_t(\iota) d\iota$ is the home-currency value of profits of home firms; and Z_t is the home-currency value of lumpsum net taxes to the government.

The budget constraint can be rewritten in terms of the opportunity cost of holding money, $1 - \mathrm{E}_t Q_{t+1,t}$,

$$P_t^c C_t + (1 - \mathrm{E}_t Q_{t+1,t}) M_t + \mathrm{E}_t Q_{t+1,t} (M_t + D_{t+1}) = \Pi_t + W_t N_t - Z_t + M_{t-1} + D_t.$$

Furthermore, the continuously compounded nominal interest rate, i_t, fulfills

$$e^{-i_t} = \mathrm{E}_t Q_{t+1,t}. \tag{A.1}$$

The consolidated government consists of a central bank and a fiscal authority. The central bank changes the supply of base money by open-market operations and foreign-exchange interventions and delivers the surplus from these transactions, the seignorage, to the fiscal authority. The fiscal authority levies net lumpsum taxes on home households. There is no government consumption. The budget constraint of the consolidated government is

$$M_{t-1} + D_t^g = Z_t + M_t + \mathrm{E}_t Q_{t+1,t} D_{t+1}^g,$$

where D_{t+1}^g is the state-contingent home-currency value of the consolidated government liabilities in period $t+1$, including foreign-exchange reserves but excluding money.

World market equilibrium for financial assets gives

$$(1-\alpha) D_t + \alpha S_t D_t^* = (1-\alpha) D_t^g + \alpha S_t D_t^{g*},$$

where D_t^* and D_t^{g*}denote the period-t state-contingent foreign-currency value of the foreign household's financial assets and the foreign consolidated government's liabilities excluding money, respectively.

The first-order condition for the home households optimal intertemporal consumption is

$$Q_{t+1,t} = \frac{\delta C_{t+1}^{-1/\sigma}/P_{t+1}^c}{C_t^{-1/\sigma}/P_t^c}. \tag{A.2}$$

Loglinearization of (A.1), using (A.2), gives

$$c_t = c_{t+1|t} - \sigma[i_t - (p_{t+1|t}^c - p_t^c) - \rho].$$

The first-order condition for foreign household's optimal intertemporal consumption will be

$$Q_{t+1,t}^* = \frac{\delta C_{t+1}^{*-1/\sigma}/P_{t+1}^{*c}}{C_t^{*-1/\sigma}/P_t^{*c}},$$

where $Q_{t+1,t}^*$ is the stochastic foreign-currency market discount factor. The home- and foreign-currency market discount factors will fulfill

$$Q_{t+1,t} = Q_{t+1,t}^* S_t/S_{t+1}.$$

It the follows from (2.10) that home and foreign per-household consumption will be proportional,

$$\frac{C_{t+1}}{C_t} = \frac{C_{t+1}^*}{C_t^*}.$$

For suitable initial conditions, the home and foreign consumption will be equal,

$$C_t = C_t^*. \tag{A.3}$$

World market equilibrium for home final goods gives

$$(1-\alpha)Y_t = (1-\alpha)C_{ht} + \alpha C_{ht}^*.$$

Together with (2.7) and (A.3), this gives

$$Y_t = C_t \left(\frac{P_t}{P_t^c}\right)^{-\eta}.$$

Similarly, world market equilibrium for foreign final goods gives

$$Y_t^* = C_t \left(\frac{P_t^f}{P_t^c}\right)^{-\eta}.$$

The log of these two expressions give (2.19) and (2.20).

The first-order condition for optimal real balances can be written

$$\frac{V'(\frac{M_t}{P_t^c})}{C_t^{-1/\sigma}} = 1 - \mathrm{E}_t Q_{t+1,t},$$

which by (A.1) results in (2.54).