1 Generalized Linear Models

- GLM extend the range of application of linear statistical models by accomodating response variables with non-normal conditional distributions.

- Except for the error, the right hand side of a GLM is essetially the same as for a linear model.

A GLM cosists of three components:

1. A random component. The conditional distribution of the response variable, Y, given the predictors. For the usual linear model it is normal - but it need not to be in GLM.

2. An index function - called the linear predictor, i.e.,

$$\eta_i = \alpha + \beta_1 X_{i1} + \beta_2 X_{i2} + ... + \beta_k X_{ik},$$

   on which the expected value μ_i of Y_i depends.

3. An invertible link function

$$g(\mu_i) = \eta_i,$$

   which transforms the expectation of the response to the linear predictor. Mean function:

$$g^{-1}(\eta_i) = \mu_i.$$

Link	$\eta_i = g(\mu_i)$	$\mu_i = g^{-1}(\eta_i)$
Identity	μ_i	η_i
Log	$\ln \mu_i$	e^{η_i}
Inverse	μ_i^{-2}	$\eta_i^{-1/2}$
Inverse-square	$\sqrt{\mu_i}$	η_i^2
Logit	$\ln \frac{\mu_i}{1-\mu_i}$	$\frac{1}{1+e^{-\eta_i}}$
Probit	$\Phi^{-1}(\mu_i)$	$\Phi(\eta_i)$
Complementary log-log	$\ln[-\ln(1-\mu_i)]$	1-exp$[-\exp(\eta_i)]$

GLMs are typically estimated by maximum likelihood.

Using GLM in R

Command in R:
logit.model <- glm(depvar ~ indepvar, family=binomial)
summary(logit.model)

Link function is given by:

Family	Default link	Range of Y	$V(y_i \mid \eta_i)$
gaussian	identity	$(-\infty, \infty)$	ϕ
binomial	logit	$\frac{0,1,\ldots,n_i}{n_i}$	$\mu_i(1-\mu_i)$
poisson	log	0,1,2,....	μ_i
Gamma	inverse	$(0, \infty)$	$\phi\mu_i^2$
inverse.gaussian	1/mu^2	$(0, \infty)$	$\phi\mu_i^3$

You can change the default function to e.g. probit, then: binomial(link=probit).

2 Newton-Raphson

Stegvis metod:

1. Gissa b_0.
2. Räkna ut $g(b_0)$.
3. Räkna ut $g'(b_0)$.
4. Dra en linje genom $(b_0, g(b_0))$. Formel: $g(b_0) + g'(b_0)(b - b_0) = 0 \Longrightarrow b_1 = b_0 - \frac{g(b_0)}{g'(b_0)}$.
5. Upprepa 1-4 för b_1.

Samma princip för vektorer och matriser

Gissa $\mathbf{b}_0$.

$\mathbf{b}_1 = \mathbf{b}_0 - \mathbf{g}\mathbf{H}^{-1}$ (där $\mathbf{g}$ är gradientvektorn och $\mathbf{H}$ är hessianen). $\mathbf{b}_1 = \mathbf{b}_0 + g\left(-\mathbf{H}^{-1}\right)$. ($-\mathbf{H}$ är positivt semidefinit eftersom $\mathbf{H}$ negativt semidefinit).

Repetera.

3 Numerisk derivering

Förstaderivatan:

$f'(x_0) = \frac{df}{dx}|_{x_0} = \lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h}$

$f'(x_0)$ approximeras med $\frac{f(x_0+h)-f(x_0)}{h}$.

Andraderivatan:

$f''(x) = \frac{f'(x+h/2)-f'(x-h/2)}{h}$,
substituera för $f(x)$
$f''(x) = \frac{f(x+h)-2f(x)+f(x-h)}{h^2}$