

1 The restriction matrix for linear tests

We have the linear model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

J linear restrictions

$$\begin{aligned} r_{11}\beta_1 + r_{12}\beta_2 + \dots + r_{1k}\beta_k &= q_1 \\ r_{21}\beta_1 + r_{22}\beta_2 + \dots + r_{2k}\beta_k &= q_2 \\ &\cdot \\ &\cdot \\ r_{J1}\beta_1 + r_{J2}\beta_2 + \dots + r_{Jk}\beta_k &= q_J \end{aligned}$$

This can be represented as

$$\mathbf{R}\boldsymbol{\beta} = \mathbf{q}$$

This framework can be used in order to test a great variety of hypotheses.

Examples:

1. $H_0 : \beta_2 = 0$

$$\mathbf{R} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \end{bmatrix}; \mathbf{q} = 0.$$

2. $H_0 : \beta_3 = c$

$$\mathbf{R} = \begin{bmatrix} 0 & 0 & 1 & \dots & 0 \end{bmatrix}; \mathbf{q} = c.$$

3. $H_0 : \beta_{k-i} = \beta_{k-i+1} = \dots = \beta_k = 0$

$$\mathbf{R} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \end{bmatrix}; \mathbf{q} = \mathbf{0}.$$

4. $H_0 : \beta_2 = 5\beta_3; \beta_4 = 3$

$$\mathbf{R} = \begin{bmatrix} 0 & 1 & -5 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 \end{bmatrix}; \mathbf{q} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}.$$