

Macroscopic Mixtures¹

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This paper takes up issues related to the notion of chemical substances—henceforth, simply “substances”—arising from their mereological and modal features. Related notions are elements and compounds, into which substances are sub-divided, and the general notion of mixture, which as a special case might involve several substances, but covers other cases too. These are essentially macroscopic concepts. Some of the chemical arguments for this claim have been presented elsewhere.² The present paper is a metaphysical treatment of matter as categorised by the major chemical concepts bearing on distinctions between homogeneous and heterogeneous matter, and between a single and several substances.

Section I introduces some of the central concepts, explaining and illustrating the general notion of mixture, and indicating some connections with issues related to the discussion of natural kinds. Important distinctions between features that are time-dependent and features which aren't are emphasised, as well as a corresponding modal distinction between features that do hold necessarily and those that don't. The argument gets under way in earnest in the section II with a critique of Quine's analysis of the mass predicate character of substance predicates, which he described in mereological terms. Without questioning the basic principles of extensional mereology, I criticise a certain point of his analysis, which raises two issues. One is connected with a technical matter about extending the mereological analysis of mass predicates to deal with relational mass predicates, and leads me to emphasise

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² See Jaap van Brakel, “The Chemistry of Substances and the Philosophy of Mass Terms,” *Synthese*, 69 (1986), 291-324, “The Nature of Chemical Substances,” in N. Bhushan and S. Rosenfeld, eds., *Of Minds and Molecules*, Oxford University Press, New York, 2000, pp. 162-84, *Philosophy of Chemistry: Between the Manifest and the Scientific Image*, Leuven University Press, Leuven, 2000, Paul Needham, “What is Water?” *Analysis*, 60 (2000), 13–21, and “The Discovery that Water is H₂O,” *International Studies in the Philosophy of Science*, 16 (2002), 201–222.

the macroscopic character of the concepts under discussion. The second issue concerns how mixtures are to be understood, which is pursued in section III. I introduce a view of mixture that I call the Stoic view, which I contrast with and defend against a view I call the Aristotelian view. (This will involve an analytic comparison between views which are, as the names suggest, clearly derived from ancient theories. But no attempt is made to justify them as interpretations by detailed examination of the classical texts, and limitations of space preclude any detailed examination of variations on the two central views which the details of the present analysis may well suggest.) The spatial character of mereological relations between quantities is developed here, leading to a distinction between two categories of mass predication, and different conceptions of elements (non-compound substances). The section concludes with some comments about the understanding of cooccupancy from the macroscopic perspective. Modality enters the discussion in section IV with a suggestion for how the notion of potential presence central to the Aristotelian conception of mixture might be understood, and contrasted with modal claims involved in the Stoic view. The rigidity of mereological parthood is introduced here, and necessary preservation or otherwise of properties such as being water iff being H_2O , and being the same substance, is discussed. The final section adds some concluding remarks about potentiality on the two views and summarises some of the main points.

I

Chemists use the term “mixture” on the whole more broadly than philosophers have used the term, but in one sense, more narrowly. For chemists, the term covers *solutions* (i.e. uniform homogeneous mixtures of several substances) and heterogeneous mixtures (comprising matter in distinct homogeneous parts, which chemists call phases, such as the oily and the watery parts of a liquid mixture, or a vapour phase over a volatile liquid), but not single compounds in a uniform homogeneous state (i.e. in a single phase). As used here, “mixture” doesn’t exclude this last case, and in this respect the term is used as it was before a distinction between compounds and solutions was introduced on the basis of the law of constant proportions.

Mixtures are important in chemistry. Features characterising substances such as water are not restricted to properties of pure water, nor even quantities of matter comprising mainly

water.³ What, exactly, is meant by matter comprising mostly some particular substance may depend on whether proportions are measured in gravimetric or mole units, and is not determinable antecedently to theoretical commitment to some understanding of what substances are. But on any account of proportions, the important features of substances are not confined to those exhibited in matter where it is the predominant component. The whole range of composition is important. Consider how composition affects a mixture of water and ether. Adding a little water to ether produces a homogeneous solution of water in ether. Adding more water leads to the formation of two separate layers, i.e. distinct liquid phases, the upper one richer in ether and the lower richer in water. Each phase is homogeneous, but the mixture as a whole is heterogeneous. The composition of each phase is dependent on the temperature and pressure, but not, in accordance with Gibbs' phase rule, on the masses of either component. Accordingly, continuing to add more water at a given temperature and pressure reduces the mass of the upper layer and increases the lower to maintain the concentrations in each layer, until finally the mixture becomes homogeneous again and a single phase remains comprising a solution of ether in water. The example illustrates the general case of a mixture comprising several phases over which distinct substances are distributed in proportions governed by prevailing conditions in accordance with laws of macroscopic theory. Water at its triple point, when solid, liquid and gas are at equilibrium, is a special case of a mixture comprising a single substance simultaneously exhibiting several phases. Solutions such as liquid brine (salt in water) or air (oxygen, nitrogen and other substances in the gaseous state, ignoring dust) are special cases of several substances in a single phase.

Under the conditions which allow for ordinary chemical phenomena, then, matter occurs in the solid, liquid or gas phases, and sometimes in several solid and liquid phases. Distinctions of phase are just as important to the understanding of mixtures as are distinctions of substance, and the corresponding predicates have analogous mereological properties which will be discussed in connection with the two views of mixture. Even single substances exhibit different phases. Putnam's discussion of the property of being the same liquid as this sample of water is misleading in this respect in so far as it suggests that being the same substance is restricted to a particular phase, namely the property of being liquid. It might be. Some substance terms do carry a phase restriction, as do "quartz" (a particular crystalline form of

³ Jessica Brown, "Natural Kind Terms and Recognitional Capacities," *Mind*, 107 (1998), 275-303, and Barbara Abbot, "Water = H₂O," *Mind*, 108 (1999), 145-8.

silicon dioxide) and “rhombic sulphur” (a particular crystalline form of sulphur). But this is not how substances are understood on the general view of mixture outlined above (or when water is said to have a triple point at 0.01°C and 4.58 mm. Hg—a property ascribed not to liquid water, but to water).⁴ Moreover, the usages shouldn’t be confused. If “water” were understood as Putnam evidently understands the term, to refer exclusively to the liquid, then although “Water is H₂O” is true the converse, “H₂O is water” (in the mass quantification sense “All H₂O is water”), would not be. In the general phase-independent sense of the term, however, an equivalence does hold between the terms “is water” and “is H₂O” because they are true of exactly the same things.

Strengthening the equivalence to a necessary equivalence provides a criterion for substitutivity of the predicates for one another in modal contexts. This is not a claim about rigidity. Rigidity is defined in quantified modal logic in terms of identity by the two conditions

$$(R) \quad (i) \quad x = y \supset \Box(x = y) \quad \text{and} \quad (ii) \quad x \neq y \supset \Box(x \neq y),$$

and that is how the term is used here. Predicates cannot meaningfully flank the identity sign, and because substance terms are predicates, it is not meaningful to describe them as rigid. The same goes for complex predicates formed from substance predicates like “is water iff is H₂O”, which are therefore not rigid designators. This doesn’t automatically dissolve the underlying issue since suitable analogues can be formulated. The necessary preservation of being water iff being H₂O yields an analogue of (Ri) which is true, although not a modal theorem.

Neither being the same substance nor its negation, as understood here, are necessarily preserved. The predicate “is water iff is H₂O” does not express the relation of being the same substance. If this is not already apparent from the syntax of the terms, it should be once it is clear that although this predicate is true, indeed necessarily true, of all matter, being the same substance is certainly not. Quantities of water and aluminium oxide are not the same substance. Neither is a quantity of water *always*, and certainly not necessarily, the same

⁴ It is not objected to Lavoisier’s method of determining the composition of water, by boiling the liquid and reducing the steam over heated charcoal, that in decomposing the gas it is not water that is decomposed. Indeed, what sense would it make to say that water is composed of hydrogen gas and oxygen gas? None of the temperature ranges, at any given pressure, over which these three substances are in the solid, liquid and gas phases coincide.

substance. Further, some of what is not water may, and often does, become water. Water, unlike helium, neon and argon, is a typical chemical substance, participating in innumerable chemical reactions in which it is either created or destroyed. The same substance relation involves temporal qualification, which will be accommodated by counting times among its relata. The other relata are the quantities of matter to which substance and phase properties apply.

Quantities are understood in a way which avoids the suggestion of paradox in the formulation of chemical change involving the creation and destruction of matter. Taken literally, this latter notion is certainly foreign to any modern conception of chemical processes because of Lavoisier's principle of the constancy of mass throughout a chemical reaction, which is taken to indicate that matter is neither created nor destroyed. Any appearance of paradox is straightforwardly dispelled by clearly distinguishing the subject of predication from the predicate expressing a substance property: quantities of matter can bear substance properties at one time and perhaps not at another, without gain or loss of parts. The macroscopic character of quantities, which occupy regions of space for intervals of time, is developed in the course of the paper. Quantities will be referred to by singular terms which can meaningfully flank the identity sign, and which will be treated along the lines of Kripke's interpretation of variables in quantified modal logic. They will be treated as mereologically rigid, which implies that they satisfy the rigidity conditions.

II

Substance predicates are apparently mass predicates, applying to all of what they apply to; a quantity which is water is all water. Two general features have been attributed to mass predicates involving mereological concepts. The *distributive condition* applied to water says that any part of what is water is water. It is to be found in Aristotle, who says that when the mixing of ingredients results in the formation of a compound, "the compound *must* be uniform—any part of such a compound is the same as the whole, just as any part of water is water".⁵ Using variables π , ρ , σ , ... to range over quantities of matter⁶ and " \subseteq " for the

⁵ *DG* I.10, 328^a10f. All quotations from Aristotle are taken from *The Complete Works of Aristotle*, ed. Jonathan Barnes, Vol. 1, Princeton University Press, Princeton 1984.

⁶ The formal language is many-sorted, and other styles of variable will be introduced presently.

mereological part relation,⁷ a predicate $\varphi(\pi)$ is distributive iff

$$(1) \quad \varphi(\pi) \wedge \rho \subseteq \pi \supset \varphi(\rho).$$

In recent philosophy, the distributive condition has been supplemented with a *cumulative condition*. Quine says that “mass terms like ‘water’, ‘footwear’, and ‘red’ have the semantical property of referring cumulatively: any sum of parts which are water is water.”⁸ Where “ \cup ” is the dyadic operation of mereological sum, a predicate, φ , is cumulative if it satisfies

$$(2) \quad \varphi(\pi) \wedge \varphi(\rho) \supset \varphi(\pi \cup \rho).$$

More generally, a predicate, φ , is cumulative iff

$$(3) \quad \exists \pi \psi(\pi) \wedge \forall \pi (\psi(\pi) \supset \varphi(\pi)) \supset \varphi(\sum \pi \psi(\pi)),$$

where $\sum \pi \psi(\pi)$ is the general sum of all those things π satisfying ψ . The existential clause, $\exists \pi \psi(\pi)$, stating that there is something satisfying the condition ψ , is necessary to ensure the existence of the sum (in accordance with the general mereological axiom for the existence of sums). Given this, if everything satisfying ψ is φ , then so is the sum of whatever satisfies ψ .

Quine denies the general distributive condition, but suggests a restriction: “In general, a mass term in predicative position may be viewed as a general term which is true of each portion of the stuff in question, excluding only the parts too small to count. Thus, ‘water’ and ‘sugar’ ... are true of each part of the world’s water or sugar, down to single molecules but not to atoms”.⁹ As a restriction on the distributive condition, however, excluding *only* the parts too small to count won’t work. There are mereological sums of “parts too small to count”, such as the hydrogen and the oxygen in a quantity of water, which are not water but may well be larger than some things which are water. Nevertheless, these “large” parts will be overlooked for the time being (they are taken up in the next section), and then it might be said that such counterinstances are not recognised from the macroscopic perspective.¹⁰ Quine’s

⁷ Understood throughout to be governed by the principles of classical extensional mereology such as those of a first-order version (without sets) of H. Leonard and N. Goodman, “The Calculus of Individuals and Its Uses”, *Journal of Symbolic Logic*, 5 (1940), 45-55 or Peter Simons’ system CI, *Parts: A Study in Ontology*, Clarendon Press, New York, 1987, p. 48.

⁸ W. V. Quine, *Word and Object*, MIT Press, Cambridge, Mass. 1960, p. 91, where he mentions Goodman’s earlier use of the term “collective” rather than “cumulative.”

⁹ *Op. cit.*, p. 98.

¹⁰ Peter Simons (*op. cit.*, p. 155) refers to this perspective as logical analysis when he says “although we know that as a matter of fact there are smallest possible portions of gold,

proposal suggests that the cumulative and distributive conditions are independent, allowing the latter to be denied and perhaps replaced by some restricted version, while retaining the former. It might be taken to indicate how a viewpoint integrating macroscopic and microscopic perspectives could be formed by modifying the former. But the issue is not so straightforward.

Water is an ordinary chemical substance with a disposition to react with other substances and a susceptibility to decomposition by physical methods such as heating and electrolysis. It is also formed in many reactions. The possibility of chemical transformation means that the property of being water is something possessed by a quantity of matter at one time or another. Because of this, the predicate “is water” is a two-place relational predicate, and we should write “ π is water at time t ” when we need to be fully explicit. From the macroscopic perspective, what is of relevance is what is true of intervals of time, our observational capacities and instrumental resources telling us what is true of longer or shorter intervals of time. Treated as intervals, times stand, like quantities of matter, in mereological relations of parthood, overlapping and separation, and are subject to the binary mereological operations of sum and product.¹¹ So mass predicates can include relational predicates, and some of the mass predicates ordinarily treated as monadic predicates turn out, in fact, to be relational. Further, a plurality of substances requires a distinction between quantities of matter which are of the same substance and those which aren’t. Accordingly, “is the same kind of substance” is another example of a relational mass predicate. Any of the white solid covering much of Greenland is the same substance as the liquid, or rather what constitutes much of the liquid, in the Atlantic. But this holds for a quantity of the white solid only while it remains water. Most of what was water in the tank yesterday before being decomposed by electrolysis was the same substance then as the matter now in the tank is now. Two quantities of the same kind are not in general indefinitely of the same kind, and the relation to time must be incorporated into a complete representation of the same kind relation, which is a four-place relation “ π is the same kind of substance at time t as ρ is at t' .”

How should the distributive and cumulative conditions be modified to apply to relational predicates? The distributive condition is straightforward; it generalises to

namely gold atoms, this is not something which could be known a priori and should not therefore have any part in the *logical* analysis of such predications.”

¹¹ Paul Needham, “Temporal Intervals and Temporal Order,” *Logique et Analyse*, 93 (1981), 49-64.

$$(4) \quad \varphi(\pi, t) \wedge \rho \subseteq \pi \wedge t' \subseteq t \supset \varphi(\rho, t')$$

in the case of a dyadic predicate $\varphi(\pi, t)$, and predicates of higher arity follow the same pattern.¹² But the cumulative condition raises a problem. Ignoring for simplicity the time-dependency of the “same kind” predicate, a straightforward generalisation of the cumulative condition (2) applied to this predicate would read

(*) If π is the same substance as ρ and π' is the same substance as ρ' , then $\pi \cup \pi'$ is the same substance as $\rho \cup \rho'$.

But the antecedent would be true if π and ρ were each water and π' and ρ' each carbon dioxide, whereas the consequent is false. Here all π is the same substance as all ρ , and all π' is the same substance as all ρ' , but it is clearly not the case that all $\pi \cup \pi'$ is the same substance as all $\rho \cup \rho'$.

Putting it like this may seem to make an appeal to the distributive condition that is questionable at this stage of the argument in view of Quine’s objection. But his objection only concerns micro-constituents below the molecular level. These are not at issue in this counterexample because π is a quantity of macroscopic proportions and obviously a part of $\pi \cup \pi'$, and ρ is similarly a macro-quantity and a part of $\rho \cup \rho'$, but water is not the same substance as carbon dioxide and so π is not the same substance as ρ' . Consequently, it is not the case that all $\pi \cup \pi'$ is the same substance as all $\rho \cup \rho'$, and Quine’s restriction would not save the suggested generalisation of the cumulative condition. In any case, were the consequent true, then reapplying (*) to it together with another same substance claim would entail that larger and more disparate sums were of the same substance, and eventually all distinctions of substance would be eliminated.

It should now be clear that the same substance predicate doesn’t relate mixtures of several substances, but has the sense of “same single substance”. A more general “same stuff” predicate would include the relation of being the same mixture, and perhaps (if some philosophers’ usage is to be followed) be restricted to certain phases. Such a general “same stuff” predicate may not be a mass predicate in any sense distinguished below.

¹² A referee asks why the mereological relation is not relativised to a time. Given that the temporal qualification is put on the substance predicate, relativisation of the mereological relation is not necessary: what undergoes change of substance or phase properties does not thereby become a different quantity. Unlike an individual such as a living organism, which undergoes change in its material constitution, the quantities of material comprising the organism’s material constitution don’t themselves change their parts.

Making the time-dependency of substance predicates explicit yields a similar counterexample, although in this case a condition ensuring the existence of the sum of two intervals must be added. Thus,

$$(\dagger) \quad \text{Water}(\pi, t) \wedge \text{Water}(\rho, t') \wedge \text{Con}(t, t') \supset \text{Water}(\pi \cup \rho, t \cup t'),$$

is not true in general. The clause $\text{Con}(t, t')$, stating that t and t' are connected (abut or overlap), is necessary to ensure the existence of their sum. Suppose that π is a quantity of water at t , and subjected to electrolysis during t' , later than but abutting t , when it gradually dissociates into hydrogen and oxygen. Then the sum, $\pi \cup \rho$, wouldn't be water at $t \cup t'$, even if ρ is water during t' . If the consequent of (\dagger) were true, then reapplying (\dagger) to it together with a further claim of a new quantity being water at a time connected with $t \cup t'$, and so on, would entail that ever larger quantities of shifting composition are water for ever larger intervals. Further counterexamples are to be had with “is as warm as”, “is warmer than”, “has the same pressure as”, and the time-dependent phase properties of being solid, liquid and gas.

A satisfactory formulation of the cumulative condition must reduce to, and so be logically equivalent with, the general formulation (3) in the special case of monadic predicates. But the logically equivalent version for monadic predicates must be generalisable in an acceptable way. A solution has been put forward by Peter Roeper¹³ to the effect that (3) is equivalent with

$$(5) \quad \forall \pi' \subseteq \pi \exists \pi'' \subseteq \pi' \varphi(\pi'') \supset \varphi(\pi).$$

(5) can be generalised into a suitable cumulative condition for relational predicates which is not subject to the counterexamples which tell against the likes of (*). For a dyadic substance predicate like “water”, for example, we have

$$(6) \quad \forall \pi' \subseteq \pi \forall t' \subseteq t \exists \pi'' \subseteq \pi' \exists t'' \subseteq t' \text{Water}(\pi'', t'') \supset \text{Water}(\pi, t).$$

That is, π is water throughout t if some of any part of π during some part of any subinterval of t is water.

But the proof of the equivalence, and more specifically, of the implication of (3) by (5), relies on the distributive condition (1). To show this, the proof is set out in an appendix. In the absence of some alternative deduction of (3) from (5) which doesn't require the unrestricted distributive condition, it would seem that the motivation for a cumulative condition like (6) for relational predicates would be hard to justify without the distributive condition. This falls short of saying that (5) implies (3). The important point is that the quantifiers in the

¹³ “Semantics For Mass Terms with Quantifiers,” *Noûs*, 17 (1983), 251-65; p. 259.

antecedent of (6) range over all the parts of water without restriction, and the feature which Quine found objectionable in the distributive condition has come to roost in the cumulative condition. The domain of a theory expressing substance kinds by mass predicates is confined to macroscopic bodies, and there are no resources for describing microentities.

III

The distributive condition is still in trouble without appealing to parts smaller than any which are, for example, water. As suggested in the last section, the distributive condition is also subject to counterinstances in the form of parts of tangible quantities of water much more massive than the smallest quantities of water. The oxygen composing such a tangible quantity of water is a case in point, as is the hydrogen. It seems that confining the domain of quantification to the macrorealm as required by the cumulative condition is not enough to save the distributive condition. Whether this constitutes a genuine counterexample depends, however, on the conception of mixture which water, as a combination of hydrogen and oxygen, is held to exemplify, and the corresponding understanding of parthood. Two views of mixture are described here, each deriving from ancient sources. On the Aristotelian view, water is construed so as not to offer a counter-instance to the distributive condition, whereas the Stoic view construes water along the lines just indicated. The views differ on how the mereological relations between quantities stand to mereological relations between spatial regions. This leads to a distinction between the general notion of parthood between quantities and the specific notion of spatial parthood on the Stoic view, and a corresponding distinction between distributive and cumulative conditions which leads in turn to a distinction amongst mass predicates between *intensive* and *homogeneous* predicates.

The Aristotelian theory of mixture has it that the elements which go to make up a compound are no longer actually present in the compound. This is expressed by Aristotle's distributive condition, which allows no part of a given compound which is not of that compound kind, just as there is no part of what he took to be the element water which is not water. The elements are, however, potentially present in the compound. The feature of not being actually present serves to contrast the Aristotelian theory with a view here called the Stoic theory of mixture. On this conception, a substance can be separated from a mixture only if it is actually present in the mixture, although the Aristotelian view of the spatial uniformity of compounds, that every spatial part of a compound is of that same compound kind, is upheld. Conflict is avoided by rejecting the Aristotelian view that two quantities of matter cannot occupy the same region of space at the same time, and holding that spatial uniformity

is consistent with the elements composing a compound occupying the same region as the compound and as one another. The Stoic view apparently circumvents the modal notion to which the Aristotelian view appeals. But this will be reconsidered after the modal commitments of the two views are taken up in the next section.

Applied to the post-Lavoisian conception of water as composed of hydrogen and oxygen, the Aristotelian view would have it that neither hydrogen nor oxygen are actually present in any quantity of water, although these elements are potentially recoverable from water. The Stoic view has it that parts of water are hydrogen and others are oxygen, although these are not in general spatial parts. It seems that the distributive condition holds on the Aristotelian view but not on the Stoic, the mereological relation of parthood being held to entail spatial parthood in the one case and not in the other. But a notion of spatial homogeneity is upheld on the Stoic view, which can be distinguished as follows.

Spatial parthood is a matter of parthood relations obtaining between the spatial regions occupied by quantities of matter. Since occupying a region is a time-dependent notion—a body occupies one region at one time, but might move to another at another time—so is the corresponding notion of spatial parthood. Writing $Occ(\pi, p, t)$ for “ π occupies region p for t ”, where p, q, r, \dots are variables ranging over spatial regions, this is understood to mean “ π occupies exactly the region p during t ”, so that it can also be written in functional form $Occ(\pi, t) = p$. Since cooccupancy is a possibility on the Stoic view, $Occ(\pi, p, t)$ doesn’t necessarily imply that π exhausts the occupants of p on that view. A relation which does have this implication can be defined by

Def. $MaxOcc(\pi, p, t) \equiv Occ(\pi, p, t) \wedge \pi = \sum \rho Occ(\rho, p, t)$.

On the Aristotelian view, $Occ(\pi, p, t)$ implies $\pi = \sum \rho Occ(\rho, p, t)$ and no distinction is made. Since the region at issue is still exactly that maximally occupied, $MaxOcc(\pi, p, t)$ can still be written in functional form as $MaxOcc(\pi, t) = p$.¹⁴

¹⁴ It is natural to think of the region occupied by a quantity of matter at a time as the sum of the regions occupied by its parts during subintervals of that time. Laying this down as an axiom,

$$Occ(\pi, p, t) \supset p = \sum q \exists \rho \exists t' (\rho \subseteq \pi \wedge t' \subseteq t \wedge Occ(\rho, q, t')),$$

the exactness of place occupied follows from the uniqueness of sums. $MaxOcc$ is then more appropriately defined by

$$MaxOcc(\pi, p, t) \equiv \pi = \sum \rho \exists q \subseteq p \exists t' \subseteq t Occ(\rho, q, t').$$

Now the Aristotelian maintains that cooccupancy is impossible, so parts are always spatial parts and the distributive condition holds on this view as stated.¹⁵ Not so for the proponent of the Stoic view, who accepts uniformity of substance in the sense

$$(7) \quad \varphi(\pi, t) \wedge t' \subseteq t \wedge \text{MaxOcc}(\rho, t') \subseteq \text{MaxOcc}(\pi, t') \rightarrow \varphi(\rho, t'),$$

where $\varphi(\pi, t)$ is a substance predicate, but not (4). To see whether a simple comparison with (4) is possible, note first that if ρ is the total material content of a region at one time and this region is a proper part of the region exhaustively occupied by π at this *same time*, then ρ is a proper part of π :

$$\text{MaxOcc}(\rho, t) \subset \text{MaxOcc}(\pi, t) \supset \rho \subset \pi.$$

The converse is not in general true on the Stoic view. Moreover, quantities maximally occupying identical regions are identical. Thus,

$$\text{MaxOcc}(\rho, t) \subseteq \text{MaxOcc}(\pi, t) \supset \rho \subseteq \pi.$$

For the purposes of comparing (7) with (4), what must be considered is the more general claim

$$(8) \quad t' \subseteq t \wedge \text{MaxOcc}(\rho, t') \subseteq \text{MaxOcc}(\pi, t) \rightarrow \rho \subseteq \pi,$$

where the times are not necessarily the same, but also stand in the part relation. This generalisation, however, doesn't change matters as far as I can see, and (8) holds too. I don't have a general proof to offer, but it may be instructive to see why an attempted counterexample doesn't succeed. Suppose, then, that ρ is an aqueous solution of which π is the water content exactly filling a sturdy container with a movable lid during some initial subinterval t' of t , and the lid is raised like a piston later on in t so that some of the water in ρ evaporates, entering the vapour phase with the result that ρ occupies a larger region encompassing that which it originally occupied during t' . The water, π , will occupy at t the sum of the regions occupied at subintervals of t , and this will include as a proper part that

¹⁵ If spatial occupancy introduces a time factor, as the above development of the Stoic view suggests, then the claim just made in the text would seem to presuppose that the Aristotelian view subscribes to the equivalence

$$\rho \subset \pi \equiv \forall t \exists p \exists q (Occ(\rho, p, t) \wedge Occ(\pi, q, t) \wedge p \subset q).$$

The universal quantifier can't be placed after the existential quantifiers because quantities can move and change their position. Dropping the universal quantifier would give a 3-place, time-dependent relation. But presumably, a part at any time is a part at every time on the Aristotelian view. At any rate, there is no time at which a sometime part is not a part.

occupied by the solution during the subinterval t' of t . But although ρ and what is the solution at times after the raising of the piston are not parts of π , π doesn't maximally occupy the region in question.

From (8) it follows that the antecedent of (7) is in general stronger than that of (4), making (7) the weaker principle on the Stoic view.

The ancients seem not to have considered the cumulative condition, but an analogous restriction to spatial parts can be introduced there, so that instead of (6) we have

$$(9) \quad \forall \pi' \forall t' \exists \pi'' \exists t'' (MaxOcc(\pi'', t'') \subseteq MaxOcc(\pi', t') \subseteq MaxOcc(\pi, t) \wedge t'' \subseteq t' \subseteq t \supset \varphi(\pi'', t'') \supset \varphi(\pi, t)).$$

The antecedent of the main conditional connective is itself a conditional sentence, whose antecedent is, by (8), stronger than the corresponding antecedent of (6). Consequently, this antecedent conditional is weaker, making the main conditional *stronger* than that of (6). Let us say that predicates satisfying both the distributive and cumulative conditions in the original form (4) and (6) (for whatever sort of variables) are *intensive*, and those satisfying both the explicitly spatial interpretations of these conditions, (7) and (9), are *homogeneous*. On the Aristotelian view this is a distinction without a difference, but not on the Stoic view. The upshot is that, on the Stoic view, the classifications of predicates as intensive and homogeneous cut across one another, since neither of these features implies the other. Accordingly, mass predicates can be divided into three categories, those which are both homogeneous and intensive, those which are homogeneous but not intensive, and those which are intensive but not homogeneous. Non-intensity means either not (4) or not (6), but since (9) implies (6), not (4) is the only possibility in conjunction with homogeneity. Similarly, since (4) implies (7), being intensive but not homogeneous amounts to satisfying (4), (6), (7) and not (9).

To which category does the relational “same substance” predicate belong? As we have seen, on the Stoic view compound substance predicates like “water” are homogeneous but not intensive. The corresponding “same substance” predicate, more specifically the “same compound” predicate, is accordingly homogeneous but not intensive. But elemental substances are both homogeneous and intensive, and the corresponding “same substance” predicate, more specifically the “same element” predicate, is both homogeneous and intensive. Further, in classical (19th century) chemistry where the elements are thought of as non-transmutable, an elemental substance predicate, such as “is oxygen”, can be treated as a monadic predicate. There is no reason to treat it as a dyadic time-dependent predicate if,

unlike water, oxygen never ceases to be oxygen, and what isn't oxygen never becomes oxygen. The corresponding “same element” predicate is therefore also time-independent, and so a dyadic predicate of quantities.¹⁶ This difference is dropped in 20th-century chemistry, which recognises that the elements were once created by intra-stellar fusion processes and are in some cases destroyed by radioactive disintegration. Considered as a homogeneous predicate, however, the disjunctive character of the sameness of substance concept is exclusive. Being either the same element or the same compound is not like being liquid or gas. A quantity is liquid or gas if it is all liquid or gas, i.e. any spatial part is either all liquid, all gas, or partly liquid and partly gas. Two quantities of matter are the same substance if they are each all the same element or each all the same compound; there is no question of them being partly the same element and partly the same compound.

The three categories of mass predicate are not distinguished on the Aristotelian view, and the conception of elements, and how they are contrasted with compounds, is altogether different. Aristotle's own scheme distinguished substances by their degrees of the comparative qualities of being warmer or colder than something, and being moister or dryer than something. He thought that there were maximal degrees of these comparative qualities, which he called “contrary extremes” (*DG* II.8, 335^a8)—what is as warm as can be is Hot, what is as cold as can be is Cold, and similarly for Moist and Dry. The general notion of an element is that of a substance possessing compatible contrary extremes. Particular elements were characterised by the particular pair of compatible extremes which they display in isolation (water is Cold and Moist, earth is Cold and Dry, etc.) but lose when brought together in a mixing process which therefore destroys the elements and creates a compound in their place. Loss of these features means, on the Aristotelian view, that there are no elements in

¹⁶ The spatial distributive condition for the “same element” predicate, *SE*, would then have a spatial parthood relation instead of the parthood between times of (7), thus

$$SE(\pi, \rho) \wedge MaxOcc(\pi', t) \subseteq MaxOcc(\pi, t) \wedge MaxOcc(\rho', t) \subseteq MaxOcc(\rho, t) \supset SE(\pi', \rho')$$

and on this view of the elements, it would be reasonable to maintain that once a spatial part, always a spatial part, i.e. $MaxOcc(\pi', t) \subseteq MaxOcc(\pi, t) \supset \forall t MaxOcc(\pi', t) \subseteq MaxOcc(\pi, t)$ as a general principle. Thermal expansion in the gas phase may well lead to some part of a quantity of an element occupying a more inclusive volume at one time than that quantity at another time. But the principle in question speaks of the same time. Similarly for the spatial cumulative condition.

compounds because no part of what is a compound bears the defining characteristics of any of the elements, which therefore only occur in isolation.

The macroscopic notion of a phase property, such as “is liquid”, is a time-dependent notion. A given quantity of water may be liquid at one time but gas or solid under conditions obtaining at other times. A quantity of water comprising ice floating on the liquid is a two-phase mixture containing just one chemical substance. Aristotle seems to have thought that the notions of phase and substance go hand in hand. Transformation of liquid water to solid or gas would be a transformation of one kind of substance to another. A two-phase mixture would necessarily be a mixture of two substances. Preservation of phase is, on the Aristotelian view, a necessary condition of preservation of substance kind (and Putnam seems to follow Aristotle when he claims that determining whether something is the same kind as a quantity of water is a matter of being the same liquid). But conditions under which a quantity of a single substance can comprise parts of different phase are described by Gibbs’ phase rule, an important theorem of thermodynamics which can be used (amongst other things) to identify the characteristic behaviour of a quantity comprising just one substance. What are called *the* phases in a mixture of ice and liquid water are the maximal quantities of liquid and of solid—the quantity which includes all and only the liquid and the quantity which includes all and only the solid. Now all the spatial parts of what is liquid for a given time t are liquid for all subintervals of t , and any quantity is liquid for t if any spatial part during any subinterval of t has a spatial part for a subinterval of that subinterval which is liquid. Phase properties are thus homogeneous; but they are also intensive. If there is any hydrogen forming part of the water that runs out of the tap, then it is liquid, as is any carbon dioxide when dissolved in chloroform at normal temperature and pressure. The properties of warmth (temperature) and pressure, and the corresponding relational properties of being as warm as, warmer than, having the same pressure as and having higher pressure than, are also homogeneous and intensive.

The Stoic view therefore puts elemental properties and phase properties in the same category. There is no implication that they can’t be distinguished, just that the distinction is not made by the distinction of categories of mass predicates. A given quantity of matter might well be all of a single elemental kind, and yet not all in a single phase; in the extreme case, a mixture of three distinct phases is possible, in accordance with the phase rule, at the triple point. And air, for example, is a solution containing several elements in the same phase. Similarly, on this Stoic conception, predicates describing compounds and solutions belong to the same category of mass predicates, being homogeneous but not intensive. This distinction

was apparently not known to the ancients, who had no conception of the principle of constant proportions which served as the simple criterion differentiating uniform mixtures into these two kinds from the beginning of the 19th century.¹⁷ Modern proponents of what is here called the Stoic view would consider the components of a solution to be actually present in the mixture as distinct parts, just as the elemental constituents are distinct parts of a compound. So solutions are non-intensive although homogeneous. But with the recognition of Berthollide compounds of the transition elements, whose composition, like that of a saturated solution, varies with circumstances, the distinction between compounds and solutions is not as clear-cut as it was once thought to be. Since compounds and solutions belong to the same category, the classification of mass predicates wouldn't be affected should the distinction between them become blurred.

There is much in the Stoic view that accords with modern ways of talking about substances in mixtures, be they heterogeneous, solutions or compounds. Does the cooccupancy claim detract from this? In thermodynamics, the energy of a multi-component system subject to work in the form of change in volume is specified as a function $U(S, V, N_1, \dots, N_r)$ of the entropy, S , the relative amount of each component substance, N_i , and the volume, V . Although the amounts of the several kinds of component substances are specified, only one term for the volume, that of the whole system, appears. This looks very much like cooccupancy. It was certainly the view of Duhem, who studied the matter in detail.¹⁸ And modern textbook writers find it natural to speak of "subsystems which occupy the same region of space".¹⁹

Philosophers have balked at the idea of cooccupancy, claiming, for example, that "impenetrability ... implicitly defined by the principle that no two things can be in the same place at the same time ... is entailed by the occupancy of space".²⁰ But arguments for this stance are thin on the ground. Wiggins claims that distinct objects of the same general kind in the same place cannot be distinguished because "no volume or area of space can be qualified

¹⁷ But see the discussion of "same ratio" following (11) below.

¹⁸ Paul Needham, "Duhem's Theory of Mixture in the Light of the Stoic Challenge to the Aristotelian Conception," *Studies in History and Philosophy of Science*, 33 (2002), 685-708; pp. 705-7.

¹⁹ Roger Bowley and Mariana Sánchez, *Introductory Statistical Mechanics*, Oxford University Press, New York, 1999; p. 189.

²⁰ Anthony Quinton, "Matter and Space", *Mind*, 73 (1964), 332-52; pp. 342-3.

simultaneously by distinct predicates in any range (color, shape, texture and so forth)".²¹ But a gaseous solution such as the air exhibits the different partial pressures of each component substance. Thermodynamics ascribes to each component substance in a mixture its own chemical potential, and they may well have different concentrations (mole fractions) and degrees of ionisation.

From the macroscopic perspective, quantities of matter occupy regions of space for intervals of time. Macroscopic ontology is treated here as in "[t]hermodynamics", which is standardly interpreted to be "... independent of the fine structure of matter".²² The macroscopic perspective is thus neutral towards the details of any view of the nature and behaviour of discrete microstructures ascribed to macroscopic quantities on a microscopic perspective. According to the microscopic perspective, these discrete particles are thought of as moving very fast and, in so far as they can be located, sweeping out a region of space in any given interval of time, as the result of vibrational and rotational motion, and translational motion in fluids. Even on the unrealistic view of micro-particles as impenetrable bodies with definite bounding surfaces, they will sweep out regions which will overlap even during the shortest intervals of macroscopic significance. If cooccupancy is a problem, it is only for localised microparticles with well-defined boundaries at instants of time. Modern chemistry, of course, entertains no such entities, and considers the character of chemical species over different time spans, to use Weininger's²³ expression. But the macroscopic perspective is neutral on the details of microscopic composition, and the stand on cooccupancy implied by the Stoic view as understood here is consistent even with this extraordinary view of the microworld.

IV

The Aristotelian notion of the potential presence of elements in a compound explicitly introduces a modal dimension into the understanding of substance concepts. Applied to a quantity, π , of water, understood as formed from hydrogen and oxygen, it claims something

²¹ David Wiggins, "On Being in the Same Place at the Same Time", *Philosophical Review*, 77 (1968), 90-95; p. 94.

²² Kenneth Denbigh, *The Principles of Chemical Equilibrium*, 4th. ed., Cambridge University Press, New York, 1981, p. 4.

²³ Steven Weininger, "Butlerov's Vision: The Timeless, the Transient, and the Representation of Chemical Structure," in N. Bhushan and S. Rosenfeld, eds., *Of Minds and Molecules*, Oxford University Press, New York, 2000, pp. 143-61; p. 147.

along the lines of “ π is partly possibly hydrogen and partly possibly oxygen”, which might be rendered

$$(10) \quad W(\pi, t) \supset \exists \rho \exists t' (\rho \subset \pi \wedge \diamond(H(\rho, t') \wedge O(\pi - \rho, t'))),$$

where $W(\pi, t)$ abbreviates “ π is water at t ”, $H(\rho, t)$ abbreviates “ ρ is hydrogen at t ”, $O(\rho, t)$ abbreviates “ ρ is oxygen at t ” and “ $-$ ” is the operation of mereological difference. Whether a conjunct saying t is separate from (has no part in common with) t' should be introduced within the scope of the existential quantifiers is left open. (10) is still not satisfactory as it stands, however. But rather than modifying it, perhaps the starting point should be “ π is possibly partly hydrogen and partly oxygen”, and the modal operator in (10) moved to the immediate left of the quantifiers. This point is addressed below. The notion of potentiality may well express something stronger than mere possibility, but that issue is not pursued here. The immediately pressing question following the discussion above is what, if water really is distributive (as maintained on the Aristotelian view) and all the parts of a quantity of water are alike, could distinguish the part which might become hydrogen from that which might become oxygen?

This question doesn't really make sense unless parts are preserved across modal contexts. Other conceptions raise other questions, but that is the mereological conception common to the two conceptions of mixture compared here. It accords with the earlier discussion if it is granted that quantities never gain or lose parts because they can't. To make this fully explicit, it is laid down that a quantity necessarily preserves its structure of parts:

$$(R_{\subseteq}) \quad (i) \quad \pi \subseteq \rho \supset \Box(\pi \subseteq \rho) \quad \text{and} \quad (ii) \quad \pi \not\subseteq \rho \supset \Box(\pi \not\subseteq \rho).$$

Since the identity of quantities is determined by their parts, it will transpire that under reasonable modal assumptions these conditions imply the rigidity conditions (R).

Surely, in answer to the question, there are no appropriate distinguishing features and there shouldn't be any unique division of a quantity of water into two parts, one of which is possibly hydrogen while the other is possibly oxygen. Aristotle seems to have had some such thought in mind when he argued against the idea of a compound held by those who “conceive it as *composition*—just as a wall comes-to-be out of bricks and stones; and this mixture will be composed of the elements, these being preserved in it unaltered but with their small particles juxtaposed each to each”. He continues

That will be the manner, presumably, in which flesh and every other compound results from the elements. Consequently, it follows that Fire and Water do not come-to-be out of any and every part of flesh. For instance, although a sphere might come-to-be out of *this*

part of a lump of wax and a pyramid out of *some other* part, it was nevertheless possible for either figure to have come-to-be out of either part indifferently: *that* is the manner of coming-to-be when both come-to-be out of any and every part of flesh. (*DG* II.7, 334^a29ff.)

Just as a sphere and a pyramid can be obtained from *any* way of dividing a lump of wax into two parts of given relative size, so it must be possible to derive the elements from “any and every part of flesh”. The Aristotelian conception of the potential presence of elements in compounds is one in which a given division into elements is possible, but doesn’t exhaust the possibilities.

There is a question of how to formulate this condition. Immediately preceding the passage from *DG* I.10 quoted at the beginning of section II where Aristotle says that a compound must be uniform, he also requires that “the part exhibit the same ratio between its constituents as the whole” (328^a8-9). Any other division preserving the ratio of the elements is equally possible:

$$(11) \quad W(\pi, t) \supset \exists \rho \exists t' (\rho \subset \pi \wedge \Diamond(H(\rho, t') \wedge O(\pi - \rho, t')) \wedge \forall \sigma (\sigma \subset \pi \wedge \text{SameRatio}(\rho, \pi - \rho, \sigma, \pi - \sigma) \supset \Diamond(H(\sigma, t') \wedge O(\pi - \sigma, t')))).$$

“*SameRatio*(π, ρ, σ, τ)” here abbreviates the relation of the ratio of the amount of matter π to that of ρ being the same as that of σ to τ . Quantitative features are controversial in the interpretation of Aristotle. But without some restriction on the universal quantifier akin to that expressed by the “same ratio” clause here, the Aristotelian view of potential presence would be distinctly less attractive.²⁴

The Aristotelian view won’t strike the modern reader as particularly plausible. The Stoic view, which keeps track of the part of water exhibiting the characteristic features of hydrogen in isolation (density, boiling point, etc.) before and after its involvement with water, and similarly for oxygen, seems more reasonable. This view is not without its modal commitments, however. The Stoic view is not opposed to (10), but instead of (11) would assert the uniqueness of the parts at issue, even the necessary uniqueness:

²⁴ I give a speculative interpretation of this relation along Aristotelian lines in “Aristotle’s Theory of Chemical Reaction and Chemical Substances,” in Davis Baird, Eric Scerri and Lee McIntyre, eds., *Philosophy of Chemistry: Synthesis of a New Discipline*, Boston Studies in the Philosophy of Science, Springer, Dordrecht, 2006; pp. 43-67.

$$(12) \quad W(\pi, t) \supset \exists \rho \exists t' (\rho \subset \pi \wedge \Diamond(H(\rho, t') \wedge O(\pi - \rho, t')) \wedge \Box \forall \sigma (\sigma \subset \pi \wedge \Diamond(H(\sigma, t') \wedge O(\pi - \sigma, t')) \supset \rho = \sigma)),$$

where $\rho = \sigma$ in the last conjunct is taken to imply $\pi - \rho = \pi - \sigma$. The Stoic view in fact goes further than (10), and asserts the corresponding claim with the possibility operator dropped; similarly for (12). Definite ratios of elements in compounds is a matter of comparison between quantities of the same compound, and both views can agree that any quantities of water have the same ratio of elements:

$$(13) \quad W(\pi, t) \supset \exists \rho (\rho \subset \pi \wedge \exists t' \Diamond(H(\rho, t') \wedge O(\pi - \rho, t')) \wedge \Box \forall \sigma \forall t' \forall \tau \forall t'' (W(\sigma, t') \wedge \tau \subset \sigma \wedge \Diamond(H(\tau, t'') \wedge O(\sigma - \tau, t'')) \supset \text{SameRatio}(\rho, \pi - \rho, \tau, \sigma - \tau))).$$

The rigidity of the singular terms is essential for the expression of these views. Clearly, π must refer to the same thing in all its occurrences, both inside and outside the modal operators. The necessary invariance of composition as expressed by (13), and the uniformity expressed by (11), also presuppose that the measure of the amount of matter comprising a quantity, and the relation of *SameRatio* on which it is based, cannot vary. When the relation holds, it is necessarily preserved; in fact, it satisfies the following analogues of the rigidity conditions:

$$(Rratio) \quad \text{SameRatio}(\pi, \rho, \sigma, \tau) \supset \Box \text{SameRatio}(\pi, \rho, \sigma, \tau) \quad \text{and} \\ \sim \text{SameRatio}(\pi, \rho, \sigma, \tau) \supset \Box \sim \text{SameRatio}(\pi, \rho, \sigma, \tau).$$

If such ratios cannot vary, they will not be able to differ from one time to another, and so the *SameRatio* predicate is not time dependent with place-holders for time variables. This stands in contrast to the treatment of the “same substance” predicate.

Both Aristotelian and Stoic views hold that compound properties such as being water are not necessary features of the things that bear them. (It might be thought not possible that what is water at a given time is not water at this same time, as a feature of time-dependent predicates in general rather than any specific feature of substances. But subjunctive conditionals of the kind “If this water had just been subjected to electrolysis, it would now be decomposed into its elements” argue against this general thesis.) In general, writing $S(\pi, t_1, \rho, t_2)$ for “ π is the same substance at t_1 as ρ is at t_2 ”, being the same substance is not necessarily preserved and does *not* satisfy the analogue of the first of the rigidity conditions (R) described in section I,

$$(RSi) \quad S(\pi, t_1, \rho, t_2) \supset \Box S(\pi, t_3, \rho, t_4),$$

nor any special case involving identification of the time variables. Nor does it satisfy the other analogue,

$$(RSii) \quad \sim S(\pi, t_1, \rho, t_2) \supset \Box \sim S(\pi, t_3, \rho, t_4).$$

If π is water and ρ is a mixture of two volumes of hydrogen with one of oxygen, they are not the same substance (π is liquid at normal temperature and pressure, ρ is not; and when both are gases, π has two thirds the volume at the same temperature and pressure) though they could be. Or if π and ρ are isomers with the same compositional formula (deemed different substances because, say, of a difference in some sharp physical property such as boiling point or rotatory power), then we have a counterexample. Now the 19th-century Stoic view, we saw, holds the elements to be incorruptible, and thus exceptions to the present rule. Unlike compound substances which may be transformed into their components or into other substances, they are preserved while composing compounds and not transmutable into one another or anything else. But the 20th-century Stoic view recognises that elements decompose by radioactive decay. No chemical substances constitute essential properties of any matter, and the only essential features of quantities seem to be general ones such as always occupying some place— $\Box \forall t \exists p Occ(\pi, p, t)$.

Although nothing is necessarily water, nothing could be water without being composed solely of hydrogen and oxygen. And conversely, nothing could be H_2O without being water. The composition of water, as distinct from hydrogen peroxide, is given by the compositional formula, H_2O . This can also be written OH_2 (and called oxygen hydride as well as hydrogen oxide) since the compositional formula says nothing about structure, but merely states that hydrogen and oxygen are combined in the proportion of two chemical equivalents to one. Were there isomers of water, with this same compositional formula, it would not be generally true that all H_2O is water, just as some, though not all, C_2H_6O is alcohol. But since water has no isomers the equivalence between being water and being H_2O holds as stated.

There seems to be broad agreement that this equivalence holds of necessity. The claim is clearly compatible with the claim in the preceding paragraph that the “same substance” predicate doesn’t satisfy the analogues of the rigidity conditions, which precludes that anything is necessarily water, or necessarily H_2O , but not that necessarily whatever is water is H_2O and conversely. As distinct from the same substance predicate, relating quantities and times, this latter condition just gives a characterisation of the “is water” predicate by providing another which can be substituted for it in modal contexts. Many other predicates

serve in the same way; a good example is that expressing the triple point of water (0.01°C and 4.58 mm. Hg)—that a quantity of the pure substance at equilibrium at this temperature and pressure is partly solid, partly liquid and partly gas.

Although solutions are usually described by names of their constituents, such as a solution of copper and nickel, there are a few expressions in common use for kinds of matter which are solutions. Brine, for example, is a solution of salt and water, and brass is a solution of copper and zinc. These instantiate a schema along the lines of: necessarily, for specified conditions of temperature and pressure, material at equilibrium has composition falling within specific ranges iff it is a certain kind of solution. Compounds called Berthollides are, in a sense, intermediate cases. Their composition is specified by a given formula, albeit a non-stoichiometric one (in which the indices are non-integral), but only for a fixed temperature and pressure. Ordinary usage contains many other terms which philosophers²⁵ may be pleased to call kinds of stuff, with tighter or looser criteria of application, but which do not apply to single substances. Advertising hype calls the recipe for coca cola a secret formula. But the specification of a solution as a kind of stuff iff its components fall within very narrow ranges of concentration, without any reference to the laws governing the chemist’s use of the term “formula”, is just an arbitrary stipulation which can be, and I believe has been, changed. Following the same line of thought, fogs and smogs of the past have no doubt satisfied certain general conditions with varying chemical constitution, and there is no saying how the chemical constitution of future smogs will continue to vary. Failure of terms from this assorted ragbag to stand in necessary equivalences detailing a definite chemical formula does not show that terms appropriated to name matter satisfying systematic (thermodynamic or other) criteria for being a single substance fail to do so. There are, in fact, “chemical kinds” (discussed in physical chemistry textbooks) which are not characterised by component substance kinds. Colloids, for example, may take the form of smokes, foams, emulsions, gels, sols, etc., containing small (but macroscopic) particles of a dispersed phase (which may be liquid or solid) in a dispersing phase (gas, liquid or solid). Necessarily, whatever is smoke is a heterogeneous mixture of small solid particles dispersed in a gas.

Another question raised by (10) is the order of the existential quantifiers with respect to the possibility operator. Would there be any difference if the above discussion started with

$$(10a) \quad W(\pi, t) \supset \diamond \exists \rho \exists t' (\rho \subset \pi \wedge H(\rho, t') \wedge O(\pi - \rho, t'))$$

²⁵ For example, Jerry Fodor, Review of Christopher Hughes, *Kripke: Names, Necessity and Identity*, *London Review of Books*, 26 (21 Oct., 2004).

rather than (10)? Rigidity ensures that π maintains the same reference in its occurrences outside and within the scope of \diamond . But can the existential quantifier introduce new possible but non-actual quantities?

The general case of the relation between the existential and possibility operators raises the question of the Barcan formula, $\exists\pi\diamond\varphi \supset \diamond\exists\pi\varphi$, according to which quantities which actually exist cannot fail to do so, and the converse Barcan formula, $\diamond\exists\pi\varphi \supset \exists\pi\diamond\varphi$, according to which there is no possibility that quantities which might exist actually don't. For quantification over times and regions, these seem to be very natural principles indeed. As for quantities, it is natural to recall Lavoisier's classical principle of the constancy of mass during chemical transformation. This has usually been understood to express an idea of the non-creatability and indestructibility of matter (i.e. the conservation of matter) after the introduction of mass as the universal measure of the amount of matter in Newtonian mechanics.²⁶ In the present context, this is naturally understood in terms of a principle of the conservation of quantities jointly expressed by the Barcan formula and its converse, namely that it is not possible that quantities which actually exist may not, nor that quantities may exist which actually don't.

Lavoisier's conservation thesis does not, it seems, entail that the elements endure (are necessarily preserved), though he and 19th century chemists thought they did.²⁷ It seems, however, that transmutation of elements in the modern sense conflicts with the conservation principle as classically understood: there is a certain seepage of old matter with radioactive disintegration and the concomitant loss of binding energy, and a trickle of new matter with nuclear fusion. The classical conception of the conservation of mass is not strictly true, then, holding only as an excellent approximation for virtually all chemical and biochemical processes under accessible conditions. But the principle of the conservation of quantities may be able to accommodate these effects of transmutation by understanding quantities to be bearers of the property of mass-energy rather than simply mass. If this is acceptable, then (10)

²⁶ Lavoisier's own formulation reads "in all operations of art and nature, nothing is created; an equal quantity of matter exists both before and after the experiment; ..." Antoine-Laurent Lavoisier, *Elements of Chemistry*, trans. by Robert Kerr, Dover reprint, New York, 1965; p. 130.

²⁷ Lavoisier continues the passage in the last footnote: "the quality and quantity of the elements remain precisely the same; and nothing takes place beyond changes and modifications in the combination of these elements".

and (10a) would be equivalent, quite apart from considerations of the rigidity of parts, in virtue of a quantifier shift law encompassing the Barcan formula and its converse applied to quantity variables as well as space and time variables. Similarly, necessarily whatever is water iff it is H₂O implies that being water iff being H₂O is an essential feature of matter.

Which principles of sentential modal logic are appropriate for this modality? Given the basic principles of normal modal logic (system K), the rigidity conditions (Ri) and (Rii) follow from (R \subseteq i) and (R \subseteq ii) and the mereological identity axiom, $\pi \subseteq \rho \wedge \rho \subseteq \pi \cdot \supset \pi = \rho$. The proofs, given in the appendix, rely on the law that \Box distributes over \wedge as an equivalence. \Diamond does not, and $\Diamond\varphi \wedge \Diamond\psi \cdot \supset \Diamond(\varphi \wedge \psi)$ is invalid. This can be illustrated in Aristotelian terms.²⁸ Earth, for example, may be worked up into wood and into clay, but presumably it is not possible for what is earth at one time to be both wood and clay at some one other time. Similarly, on the Stoic conception, with a modern example, a quantity π which is hydrogen at t might be possibly part of some water and possibly part of some methane (M); i.e., such that

$$\Diamond\exists\rho W(\pi \cup \rho, t) \wedge \Diamond\exists\rho M(\pi \cup \rho, t).$$

But this doesn't imply

$$\Diamond(\exists\rho W(\pi \cup \rho, t) \wedge \exists\rho M(\pi \cup \rho, t)).$$

What of the converse principle, which does hold in K? Gill²⁹ argues that it is not possible for earth to be (the material of) a statue because it is dry and crumbly, and doesn't retain the shape into which it is moulded. If it is first worked up into bronze, however, which does have the appropriate dispositions, then the production of a statue is possible. Thus, some quantity, π , which is earth might be both (part of) the material of a statue and bronze, and so (in accordance with the principle now at issue), it might be the material of a statue. This state of affairs is not in conflict with what Gill says since the conclusion, that π might be the material of a statue, carries no implication that π might be both the material of a statue and (yet remain) earth. What, following Gill, is not possible is that some quantity be both the material of a statue at some time and earth at the same time. Observing the distinction between subject and predicate in this way, it is difficult to see that K gets the combination of modality and sentential connectives wrong.

²⁸ Cf. Mary Louise Gill, *Aristotle on Substance: The Paradox of Unity*, Princeton University Press, Princeton, 1989; p. 150.

²⁹ *loc. cit.*.

Moving on to consider stronger principles, it seems difficult to deny the characteristic schema of system T, namely $\varphi \supset \Diamond\varphi$. Admittedly, actuality is often contrasted with potentiality, as though potentiality presupposes deprivation. Some such distinction is readily accommodated by understanding the contrast to stand between what is actual and what is *merely* possible, with the latter defined along the lines of $\sim\varphi \wedge \Diamond\varphi$. But manifestations of, say, dissolving in water do not imply absence of the potential to do so. As Gill³⁰ puts it, potentialities are grounded in the nature of the subject, which endows them with the appropriate dispositions, and not in a state of deprivation. Beyond this, the question is whether iterated modalities have any special significance or should be eliminated, along with nested modalities, in favour of the innermost modality as in S5. I am inclined towards the latter. Accordingly, to say, for example, that water necessarily can boil at 100°C, $\Box(W(\pi, t) \supset \Diamond B(\pi, t))$, is equivalent to saying $\Box\sim W(\pi, t) \vee \Diamond B(\pi, t)$, or possibly being water implies possibly boiling at 100°C. With these modal principles, (R \subseteq ii) follows from (R \subseteq i).

V

On the Aristotelian view, the elements are not actually present in a compound. They are characterised by features exhibited only in isolation, and by the distributive condition, no part of the compound has these features. The Stoic view rejects this, and maintains that the elements are present in compounds. This underlies the reason for maintaining (12). Both views are agreed on the conservation of the parts involved. What distinguishes them is the substance properties they ascribe to the parts when the whole is a compound. Although Aristotle's actual definitions of the elements are antiquated, the general manner of characterising elements by listing properties exhibited in isolation along with the kinds of compounds they can form does resemble what is found in a modern textbook description of the elements. This raises the question of how the elements are to be characterised on the Stoic view, and given, as hitherto suggested, that this view corresponds more closely to the modern conception of substance, how modern descriptions of the elements are to be understood. It seems doubtful that elemental properties can be defined, or characterised by a range of criteria, entirely in terms of non-modal properties. Being oxygen, for example, is having the disposition to be a gas when not chemically combined and above -183°C at standard pressure, and to be more stable as a component of water when mixed with hydrogen at standard temperature and pressure. No quantity of oxygen could actually display both features at the

³⁰ *Op. cit.*, pp. 179-80.

same time. The very idea of keeping track of the parts of water exhibiting the characteristic features of hydrogen and oxygen in isolation before and after their combination in water suggests that what is elemental stuff exhibits the potential, both to display these characteristic features of the isolated state when in a state of combination, and to display the characteristic features of its compounds when in isolation. The Aristotelian view incorporates modality into its conception of mixture, and gives a non-modal account of the features characterising the elements. The Stoic view doesn't circumvent modal notions altogether, but incorporates them into the characterisation of the elements.

The two views of mixture were introduced as alternative responses to the distributive condition on mass predicates, the Aristotelian view preserving this condition and the Stoic view rejecting it. Spatial homogeneity is retained on the latter view, however, by accepting the possibility of cooccupancy, which it was argued can be understood as a reasonable principle of macroscopic ontology. In general, mixtures comprise different substances or different phases (the "or" being inclusive). The traditional Aristotelian view that phase and substance coincide remains the vulgar conception, but this feature needn't be incorporated into the Aristotelian view presented here. It is this general conception of mixture which sanctions the necessary equivalence between being water and being H_2O , which I have contrasted with the non-necessary relations of being the same substance and its negation.

Appendix

The proof that (5) implies (3) follows by the deduction theorem from the deduction of the consequent of (3), $\varphi(\sum \pi \psi(\pi))$, from its antecedents and (5), which can be set out as follows:

$$(i) \quad \exists \pi \psi(\pi) \quad [\text{Assumption}]$$

$$(ii) \quad \forall \pi (\psi(\pi) \supset \varphi(\pi)) \quad [\text{Assumption}]$$

Put $\pi = \sum \pi' \psi(\pi')$, which exists since by (i) there are ψ -ers. Now by general mereology, anything which is a part of the sum of ψ -ers overlaps some ψ -er; i.e.

$$(iii) \quad \forall \pi' \subseteq \pi \exists \sigma \subseteq \pi' (\psi(\sigma) \wedge \pi' \bullet \sigma).$$

But by the definition of overlap, the conjunct $\pi' \bullet \sigma$ here implies that there is a part, π'' , common to π' and σ , and by the distributive condition (1), the first conjunct then implies $\psi(\pi'')$. Accordingly,

$$(iv) \quad \forall \pi' \subseteq \pi \exists \pi'' \subseteq \pi' \psi(\pi'').$$

Then, in view of (ii),

$$(v) \quad \forall \pi' \subseteq \pi \exists \pi'' \subseteq \pi' \varphi(\pi'')$$

$$(vi) \quad \varphi(\pi) \quad [\text{from (v) by (5)}]$$

Since $\pi = \sum \pi' \psi(\pi')$, this is $\varphi(\sum \pi' \psi(\pi'))$. Removing the assumptions (i) and (ii) yields (3). Q.E.D.

Turning now to the proofs of rigidity of quantities, it is shown that, given the principles of modal system K, the rigidity conditions (Ri) and (Rii) applied to quantity variables follow from the rigidity of their parts, conditions (R \subseteq i) and (R \subseteq ii). Thus, the mereological identity axiom, $\pi \subseteq \rho \wedge \rho \subseteq \pi \supset \pi = \rho$, yields

$$(i) \quad \Box(\pi \subseteq \rho \wedge \rho \subseteq \pi) \supset \Box(\pi = \rho) \quad [\text{Necessitation, distribution of } \Box \text{ over } \supset]$$

$$(ii) \quad \Box(\pi \subseteq \rho) \wedge \Box(\rho \subseteq \pi) \supset \Box(\pi = \rho) \quad [\text{distribution of } \Box \text{ over } \wedge]$$

$$(iii) \quad \pi \subseteq \rho \wedge \rho \subseteq \pi \supset \Box(\pi \subseteq \rho) \wedge \Box(\rho \subseteq \pi) \quad [(\text{R}\subseteq \text{i}), \text{sentential logic}]$$

$$(iv) \quad \pi = \rho \supset \pi \subseteq \rho \wedge \rho \subseteq \pi \quad [\text{Leibniz' law, } \pi \subseteq \pi, \text{sentential logic}]$$

$$(v) \quad \pi = \rho \supset \Box(\pi = \rho) \quad [(iv), (iii), (ii)]$$

Similarly, (Rii) follows from (R \subseteq ii); again the mereological identity axiom yields

$$(vi) \quad \pi \neq \rho \supset \pi \not\subseteq \rho \vee \rho \not\subseteq \pi$$

$$(vii) \quad \pi \not\subseteq \rho \vee \rho \not\subseteq \pi \supset \Box(\pi \not\subseteq \rho) \vee \Box(\rho \not\subseteq \pi) \quad [(\text{R}\subseteq \text{ii}), \text{sentential logic}]$$

(viii) $\Box(\pi \not\subseteq \rho) \vee \Box(\rho \not\subseteq \pi) \supset \Box(\pi \not\subseteq \rho \vee \rho \not\subseteq \pi)$ [distribution of \Box over \vee]

(ix) $\pi \neq \rho \supset \Box(\pi \neq \rho)$ [(vi), (vii), (viii); (iv), K],

where the last step is obtained by substitution of equivalents, which is sanctioned by the principles of K.