Compositionality, Understanding, and Proofs

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Abstract

The principle of semantic compositionality, as Jerry Fodor and Ernie Lepore have emphasised, imposes constraints on theories of meaning that are hard to meet by psychological or epistemic accounts. Here I argue that this general tendency is exemplified in Michael Dummett's account of meaning. On that account, the so-called manifestability requirement has the effect that the speaker who understands a sentence *s* must be able to tell whether or not *s* satisfies central semantic conditions. This requirement is not met by truth conditional accounts of meaning. On Dummett's view, it *is* met by a *proof conditional* account: understanding amounts to knowledge of *what counts as a proof of a sentence*. A speaker is supposed always to be capable of deciding whether or not a given object is a proof of a given sentence she understands. This requirement comes into conflict with compositionality. If meaning is compositionally determined, then all you need for understanding a sentence is what you get from combining your understanding of the parts according to the mode of composition. But that knowledge is not always sufficient for recognising any proof at all of a given sentence. Dummett's proof-theoretic argument to the contrary is mistaken.

1. Compositionality and meaning theoretical constraints

In their comment on Paul Horwich's claim in 1998 that semantic compositionality is a trivial property of natural language, Jerry Fodor and Ernie Lepore sum up the constraints on lexical meaning that compositionality offers, in their view:

> Over the last few years, we have just about convinced ourselves that compo-

> sitionality is the sovereign test for theories of lexical meaning. So hard is this test to pass, we think, that it filters out practically all of the theories of lexical meaning that are current in either philosophy or cognitive science. Among the casualties are, for example, the theory that lexical meanings are statistical structures (like stereotypes); the theory that the meaning of a word is its use; the theory that knowing the meaning of (at least some) words requires having a recognitional capacity for (at least some of) the things it applies to; and the theory that knowing the meaning of a word requires knowing the criteria for applying it. Indeed, we think that only two theories of the lexicon survive the compositionality constraint: viz., the theory that all lexical meanings are primitive and the theory that some lexical meanings are primitive and the rest are definitions (Fodor and Lepore 2001, pp. 43-44).

The present author agrees in spirit with the stance of Fodor and Lepore in this passage, especially regarding the requirement of recognitional capacities. I shall in fact here take the same stance to *sentence meaning* as Fodor and Lepore take on lexical meaning. In particular I shall be concerned with the requirement of recognitional capacities on sentence understanding that Michael Dummett has imposed. Presenting and discussing Dummett's view will make up the main part of this paper.

Before turning to that, I shall make some more general comments about the idea of compositionality as constraint on theories of meaning. A standard informal formulation of the principle of compositionality is e.g.

(PC) The meaning of a complex expression is a function of the meanings of its parts and its mode of composition.

For the formal counterpart, I shall call a function $[\![\cdot]\!]$ that maps syntactic items on meanings (irrespective of what entities serve as meanings) a *semantic function*. I shall call a function ρ_i that for some n maps meanings $m_1, \ldots, m_n$ on a meaning m a (meaning) *composition function*. A *generalised composition function* ρ is then a function such that, given a language L, for any syntactic operator α in L, $\rho(\alpha)$ is an ordinary composition function. Then, a semantic function $[\![\cdot]\!]$ for a language L is *compositional* just in case there is a generalised composition function ρ such that for each operator σ in L and any relevant syntactic items $e_1, \dots, e_n$ (with $[\![\cdot]\!]$ defined for $\sigma(e_1, \dots, e_n)$) it holds that

(PC′) $[\![\sigma(e_1, \dots, e_n)]\!] = \rho(\sigma)([\![e_1]\!], \dots, [\![e_n]\!])$.

Intuitively, (PC′) says that the meaning of the complex is a function (ρ) of the meanings of the parts and the mode of composition (σ).[1]

Now, suppose we have an account of meaning which entails claim of the following form:

(F) For any expression e (of category X), $[\![e]\!] = m$ iff $G(e, m)$

where G is some relation having to do with the use of the expression, or the mental states of the speakers in using it, etc. The claim that the requirement (F) is in conflict with compositionality is then a claim that for at least some syntactic construction there is *no* composition function (in the sense above) that meets the requirement (F). More formally spelt out:

(NC) There is syntactic operator σ defined for many n-tuples $\langle e_1, \dots, e_n\rangle$, but no function ρ such that for all n-tuples $\langle e_1, \dots, e_n\rangle$ for which σ is defined it holds both that $[\![\sigma(e_1, \dots, e_n)]\!] = \rho(\sigma)([\![e_1]\!], \dots, [\![e_n]\!])$ and that $G(\sigma(e_1, \dots, e_n), \rho(\sigma)([\![e_1]\!], \dots, [\![e_n]\!]))$.

Some examples of the format are given in Fodor and Lepore 1996. Concerning the noun-noun compound 'pet fish' in relation to the *prototype theory* of concepts, Fodor and Lepore say

> Now, according to prototype theory, to have a concept is to have its prototype together with a measure of the distance between the prototype and an arbitrary object in the domain of discourse. *Prima facie*, however, the distance of an arbitrary object from the prototypical pet fish *isn't* a function of

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[1] The advantage of this format is that if σ is just the first argument of ρ, the arity of ρ would vary if there are syntactic operators of different arities.

To exemplify: for the usual conjunction connective, we have the syntactic operation $\sigma_\wedge(s, s') = s^\frown`\wedge'^\frown s'$ and the classical semantics $[\![\sigma_\wedge(s, s')]\!] = \rho(\sigma_\wedge)([\![s]\!], [\![s']\!]) = 1$ iff $[\![s]\!] = 1$ and $[\![s']\!] = 1$, and 0 otherwise.

> its distance from the prototypical pet and its distance from the prototypical fish (Fodor and Lepore 1996, p. 35, italics in the original).

Here the *G* relation simply is the relation that holds between an expression and its meaning *only if* that meaning is a measure of distance to a prototype. In this case, according to Fodor and Lepore, we can look at one single complex expression, 'pet fish', to see that the (NC) refutation condition is satisfied. However, if '...isn't a function of' amounts to the same as 'there is no function f such that ...is f of', then the claim cannot be right, unless much stricter conditions are placed on appropriate functions. For there clearly are infinitely many functions that map the distance to the prototypical pet and the distance to the prototypical fish on the distance to the prototypical pet fish, just as there are infinitely many functions that map 5 and 7 on 11. Functionality cannot be violated in one single argument–value pair.

A little further on it becomes clear that Fodor and Lepore have stronger conditions in mind. They discuss an appeal to context dependence and consider the analogy with the expression 'big ant'. That expression has a standard reading according to which it means *big for an ant*. According to Fodor and Lepore, when this reading is combined with prototype theory, the result is that the standard for *size* has to be "re-calibrated" with respect to the prototype in question. Hence, one needs to know the prototype, and for that one must "go to the world". This is taken to be against the spirit of compositionality:

> Whereas, the whole point of assuming compositionality is to explain how somebody who understands the vocabulary and the syntax of a complex expression is *thereby* able to understand the complex expression itself; viz., *without relying upon further information from the world* (Fodor and Lepore 1996, p. 38, italics in the original).

As I understand Fodor and Lepore here, a further condition is imposed (as part of the relevant *G* relation), viz. that the relevant composition function be identified independently of any features of the world. And this condition is not met here, for the composition function applies a recalibration operation on *big* that depends on the prototype of *ant*. This provides then a second example of applying the (NC) refutation format. I shall not pursue the question of whether in this case it is applied successfully.

The rest of the paper will be concerned with a third example, Michael Dummett's account of linguistic meaning, according to which *a*) the meaning of a sentence A is given by what counts as a *canonical proof* of A, and *b*) a speaker who understands A is able, for an arbitrary object x, to determine whether or not x is a proof of A. I shall argue that to the extent that the understanding of a sentence is supposed to be derived compositionally from the mode of composition of the sentence and the meaning of its immediate parts, no compositional semantics can be given that meets both requirements. I shall begin by sketching the background of requirement *b*).

2. Dummett's manifestation argument

In the 1970's Michael Dummett launched a meaning theoretic argument against truth conditional semantics.[2] The most central text is his celebrated 'What is a theory of meaning? (II)' (Dummett 1976). It proceeded from setting out conditions on successful communication to requirements on theories of linguistic meaning, with the conclusion that truth conditional semantics doesn't meet those requirement, or cannot be part of a more general theory of meaning that does. The path from communication to semantics went via knowledge of meaning and, most importantly, *manifestation* of knowledge of meaning. Below, I set out Dummett's argument in condensed form:

1. Knowledge of the meaning of a sentence is publicly manifestable.
2. Public manifestation of knowledge of the meaning of a sentence consists in exercising the ability to tell whether the central semantic concept applies to that sentence or not.
3. If the central semantic concept is the concept of truth (and therefore knowledge of meaning is knowledge of truth conditions) and the sentence is not effectively

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[2] I do not here intend to criticise this argument. It has been extensively criticised over the past 30 years. I did so myself in my dissertation (1987). Here it is presented as a background for showing that Dummett's own positive account falls victim to the same argument.

decidable, then there is no ability to tell whether that concept applies to the sentence.

4. Hence knowledge of truth conditions is not (always) publicly manifestable.
5. Hence knowledge of meaning is not knowledge of truth conditions.
6. Hence meaning is not truth conditions.

I shall briefly expand on the ideas of manifestability and undecidability.

2.1. Manifestability

The underlying reason for premise 1 is Dummett's view that successful communication requires that the communicators *know* that they understand the linguistic expressions the same way. It is not enough that you and I *in fact* mean the same by the same expressions, for then we cannot make sure that we understand each other. In a characteristic passage Dummett says

> If language is to serve as a medium of communication, it is not sufficient that a sentence should in fact be true under the interpretation placed on it by one speaker just in case it is true under that placed on it by another; it is also necessary that both speakers should be aware of the fact (Dummett 1980*a*, p. 132).

The idea that successful communication cannot simply rest on an act of faith, that it requires knowledge of mutual understanding, not just belief, is what motivates the manifestability requirement.[3] The two communicators should be able to reveal to each other how they understand an expression. Because of that, they should also be able to put each other to the understanding test. And since Dummett generally writes with the assumption that there is a shared public language rather than a set of overlapping idiolects, the form that the testability requirement takes is that of manifesting understanding of the shared public language itself.

2.2. Undecidability

Part of the requirement of manifestability is that the speaker that is asked to show her understanding can do so at the time of being asked. The means of showing is

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[3] In another place I have argued that the knowledge requirement is too strong (cf Pagin 2008), but that is not the topic of the present discussion.

to be available. In some cases this might be difficult because the sentence is too long to be grasped or because of other practical problems, but that is taken to be beside the point. The idea is that the means of manifestation of understanding is available at the time “in principle”. And the only principled obstacle to do it is that the speaker *does not know how*.

In case the central semantic concept is the concept of truth, the way of manifesting understanding of a particular sentence is by deciding whether the sentence is true or false. Dummett points to three kinds of sentences for which the speaker might not know how to decide the truth value:

1) sentences that include reference to inaccessible regions of space-time
2) sentences involving quantification over infinite domains
3) counterfactual conditionals.

For none of these three kinds of sentence is there any general *method*, i.e. a decision method, for determining whether some arbitrary sentence of that kind is true or false. These kinds are in that sense *undecidable*. By contrast, e.g. the class of arithmetical equations is decidable, since there are algorithms for computing the values of arithmetical expressions. If a sentence A belongs to a decidable class (and the speaker knows the decision method), then it is always in principle possible for a speaker to manifest her understanding by carrying out the method. By carrying out the method she will find either a proof or a disproof, but need not know which in advance.

In case A does not belong to a decidable class, this does not hold. For any given sentence, there might exist a proof or disproof that the speaker happens to know, e.g. a proof of the sentence

(1) Every prime number is smaller than some prime number

which does involve quantification over the infinite domain of the natural numbers. But in case A does not belong to a decidable class and the speaker does not already possess a proof or disproof, she will not know how to get one. And then she does not know how to manifest her understanding of A, at least not in accordance with Dummett's requirements. Dummett calls such a sentence A 'undecidable'.

By hypothesis, there are sentences that we do understand, even though we (collectively) do not know how to prove or disprove them. Fermat's Last Theorem was until recently such a sentence. At present, to my knowledge, Goldbach's conjecture, i.e.

(2) Every even number greater than 2 is the sum of two primes

is a sentence of that kind. We understand it but don't know how to prove or disprove it. That is, we know that if it is false, we will find a counterexample by trying number after number, but we don't know categorically that by going through the numbers we will find one.

The rest of Dummett's argument, consisting of steps 4, 5, and 6, follows from the three premises.

3. Recognition and canonical proofs

The positive part of Dummett's meaning theory is the proposal that some form of verificationist semantics should take the place of truth conditional semantics, and the idea is that such a semantics would meet the manifestation requirement. Such a semantics would be suitable for justifying intuitionistic logic, but not classical logic.

The constructive part has been worked out by Dummett himself and by Dag Prawitz in a number of works starting from the early-mid seventies, and here the concept of *proof* plays the central role. The Dummett-Prawitz intuitionistic alternative to truth as the central semantic concept is, however, not provability, since it is not in general *decidable* whether a sentence is provable. Instead, the unary concept of being provable is replaced by the relation *x is a proof of A*. It is normally assumed that when presented with a particular object or construction a and a sentence A, we are able to tell whether or not a is a proof of A. Under this assumption a speaker can always be tested for understanding of a sentence A by being asked whether some particular objects are proofs of A. The conception that the proof relation is decidable is captured by Georg Kreisel's dictum 'We can recognise a proof when we see one' (Kreisel 1962, p. 202). Following established practise, I shall refer to a semantics that is based on this idea as *proof-theoretic* semantics.[4]

The very idea that proofs are recognisable turns out to be a problem, however. Prawitz himself points to an aspect of it involved in the definition of →: A proof of an implication $A \to B$ is to be a function that transforms any proof of A into a proof of B, and it may clearly be difficult to determine of a particular construction that it has this property (Prawitz 1998).

This difficulty does in fact pose a threat to the program. For even if it *is* in principle decidable whether or not a particular construction a is a proof of a sentence A, it is not so clear that the *knowledge* of a speaker who understands A is enough for deciding the issue. We can assume that with every well-defined class of proofs $\mathscr{P}$ and any well-defined class of interpreted sentences S there is a decision procedure by which it is possible to determine, for any $a \in \mathscr{P}$ and any $A \in S$, whether a is a proof of A. It is not, however, a consequence of this assumption that a speaker who does understand all the sentences in S also has the knowledge required to recognise all the proofs in $\mathscr{P}$. Worse, there need not be any general method for acquiring such knowledge.

How serious is this problem? Well, it is hardly very serious if all the proofs of sentences of S belong to particular class $\mathscr{P}_S$ of proofs such that the speaker who does understand all the sentences in S also grasps all the proofs of $\mathscr{P}_S$. But this is clearly not the case, since whenever there is a proof a of A we can enrich a into a proof a' by adding extra material that is not in fact needed. The speaker may not be able to tell whether a' is a proof or not.

What we need to handle this problem is a way of mapping sentences on proofs that do not contain such unnecessary detours, but are direct. In particular, we would want to be able to tell, from the syntax of the sentence itself, what such a direct proof would be like.

This problem was addressed in the early seventies by Dummett and Prawitz, who made the distinction between *canonical* and *non-canonical* (or *direct* and

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[4] Cf. the special issue of *Synthese* on proof-theoretic semantics, vol 148, No 3, 2006.

indirect) proofs (Dummett 1975, pp. 240-47, Prawitz 1974, pp. 63-77). Although a proof in general of a sentence A can have any form, a *canonical* proof of A reflects the form of A itself:

- $\wedge$ A canonical proof of $A \wedge B$ is a pair $\langle a, b\rangle$ where a is a proof of A and b is a proof of B.
- $\vee$ A canonical proof of $A \vee B$ is a pair $\langle a, b\rangle$ where either a is l and b is a proof of A or a is r and b is a proof of B.
- $\rightarrow$ A canonical proof of $A \rightarrow B$ is a function f that for any proof of A as argument gives a proof of B as value.
- $\forall$ A canonical proof of $\forall x A x$ is a function f that for any term t in the language as argument gives as value a proof of $A(t/x)$.
- $\exists$ A canonical proof of $\exists x A x$ is a pair $\langle t, b\rangle$ where t is a term in the language and b is a proof of $A(t/x)$.

As usual, $\neg A$ is defined as $A \rightarrow \bot$.

This is the modern version of the Brouwer-Heyting explanation of the meanings of the logical constants. A proof in general of a sentence A is then seen as a *method* which, when applied, yields a canonical proof of A.

A speaker is assumed to understand a sentence A *compositionally*, from understanding it parts and the way they are put together, and this idea of understanding is realised with the recursive specification of canonical proofs. For a sentence A of arbitrary complexity the speaker can compute the condition for being a proof of A from the syntactic build-up of A.

This can help with the recognition problem to the extent that a speaker who understands a provable sentence A is not required to be able to recognise all non-canonical proofs of A, only the canonical ones. It does not by itself help very much, however, since a canonical proof may contain parts that are not canonical and even contain unknown material. For instance, a proof of $A \rightarrow B$ may have the following form:

(3)
$$\dfrac{\begin{array}{c}\dfrac{\dfrac{[A] \qquad \begin{array}{c}\mathscr{D}_1\\ C\end{array}}{A \wedge C}}{A}\\ \mathscr{D}_2\\ B\end{array}}{A \to B}$$

Here C is proved by the derivation $\mathscr{D}_1$. C may be any provable sentence and $\mathscr{D}_1$ may contain any proof material, provided it does prove C. The proof as a whole is still canonical: its immediate subargument is a derivation from A to B, with A as the only undischarged assumption. However, the derivation (3) *reduces* to a derivation that does not contain the extra material:

(4)
$$\dfrac{\begin{array}{c}\dfrac{\dfrac{[A] \qquad \begin{array}{c}\mathscr{D}_1\\ C\end{array}}{A \wedge C}}{A}\\ \mathscr{D}_2\\ B\end{array}}{A \to B} \qquad \triangleright \qquad \dfrac{\begin{array}{c}[A]\\ \mathscr{D}_2\\ B\end{array}}{A \to B}$$

In (4) $A \wedge C$ is a so-called *maximum formula* (Prawitz 1965, p. 34), i.e. a formula where the main operator has been introduced in an introduction step, and then (immediately) is eliminated in the following elimination step, without having been used for anything of consequence for what is proved. The maximum formula, together with subsidiary derivations needed for its unused subformulas, is taken out of the derivation in a Gentzen-Prawitz *reduction step*.

This reduction step is available to the speaker. That is, the speaker has the capacity to inspect the proof (3), and although she does not understand C or the $\mathscr{D}_1$ derivation, she realises that it is not needed, and that she can extract from it by means of the reduction step a proof that she does understand (we may that assume she does understand $\mathscr{D}_2$).

A proof without maximum formulas is a so-called *normal* proof. As was shown in Prawitz 1965, Natural Deduction proofs in first-order logic can be *normalised*, i.e. reduced to normal proofs. Reduction is a method available to the speaker that transforms a proof in first-order logic into a normal proof. As also shown by Prawitz, normal proofs obey a *sub-formula principle*. In the intuitionistic case, the sub-formula principle says that every formula occurring in a normal derivation is a sub-formula either of an assumption or of the conclusion (Prawitz 1965, p. 53).[5] This means that in a normal proof of a sentence A no formula will occur that is not a sub-formula of A. Since normalisation is itself a mechanical procedure, we have a guarantee that any part of a proof of A that requires more understanding will be removed during the normalisation procedure. In first-order logic, understanding a sentence A through its canonical-proof conditions guarantees that if a proof of A is valid, the speaker who understands A can recognise the validity of at its normalised equivalent. That is, she understands that what she is given is a method for arriving at canonical proof, and hence she *does* understand that it is a proof, in the *non*-canonical sense. As far as first-order logic goes, this takes care of the recognition problem.

4. Compositional proof-theoretic semantics

Normalisation does not, however, offer a general solution to the recognition problem, for the sub-formula principle does not in general hold outside first-order logic. When that happens, grasp of the general conditions for being a canonical proof of a sentence A does not always provide the knowledge for recognising individual items as proofs. For, on the one hand, recognising a particular object a as a proof involves recognising the *parts* of a as valid parts, i.e. valid *subarguments*. And, on the other hand, the conditions for being a proof of A are not in general concerned with properties of *parts* of proofs of A. In short, parts of conditions on being a proof of A are not in general conditions on being part of a proof of A.

In some cases, this does nevertheless hold. It is true in the case of $\wedge$, $\vee$ and $\exists$. For instance, in accordance with the explanation ($\exists$) of the existential quantifier, if a speaker knows a canonical proof of $\exists x Ax$ she also knows the immediate subargument of that proof.

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[5] In the classical case, it is just slightly more complicated, because of double negation elimination.

But this does not hold of ¬, → , or ∀. A proof of a conditional $A \to B$, is a function f taking any proof of A as argument to give a proof of B as value. This corresponds to a proof of the form

$$\frac{\begin{matrix}[A]\\ \mathscr{D}\\ B\end{matrix}}{A \to B} \tag{5}$$

with the immediate subargument $\mathscr{D}$ from A to B. The problem is that the condition that $\mathscr{D}$ shall constitute a function from proofs of A to proofs of B does not impose any particular restriction on $\mathscr{D}$, except that of being valid and leading from A to B. If the sub-formula principle holds, this is not a problem, but if it doesn't hold, it is.

In Dummett 1977, pp. 394-95, Dummett attempts to impose a restriction on proofs of the sub-sentences A and B in setting out the conditions for being a proof of $A \to B$. The assumption is that if $A \to B$ can be proved at all, it can be proved by means of a proof that has a complexity bound given by the complexity of A. Dummett writes

> Thus, when we have to decide whether to accept a given construction as a proof of a conditional statement $A \to B$, we need to judge whether, when applied to an arbitrary proof of A, it will yield a proof of B. In so doing, we shall need to to consider only those possible proofs of A that have a complexity below a certain bound determined by the complexity of A, and hence of a lower complexity then is required for a proof of $A \to B$ itself (Dummett 1977, pp. 394-95).

This indeed holds in first-order logic. But first-order logic is the exception rather than the rule. When we move into arithmetic, higher-order logic or set theory, things change.[6] For instance, proofs by arithmetical induction often involve proofs of a stronger claim from which the intended theorem follows as a corollary. The induction schema, in the preferred Natural Deduction format is given by

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[6] Prawitz points out in 1985, p. 166, as well as in 1987, p. 147 and in 1994, p. 394, that we cannot in general expect an extension from a language $\mathscr{L}$ to a language $\mathscr{L}'$ to be *conservative*. The extension is conservative if any sentence of $\mathscr{L}$ provable in $\mathscr{L}'$ is also provable in $\mathscr{L}$.

(IS)
$$\dfrac{\begin{matrix}\mathscr{D}_1\\ F(0)\end{matrix}\qquad\begin{matrix}[F(x)]\\ \mathscr{D}_2\\ F(s(x))\end{matrix}}{F(y)}$$

where the immediate output is a free variable formula, from which a ∀-introduction can be made (cf. Prawitz 1973, pp. 243-44).

In some cases, however, the induction cannot be performed directly on the predicate of the conclusion, since the induction hypothesis is too weak. In that case a stronger formula must be proved. Where G is the stronger predicate, we then have

(6)
$$\dfrac{\begin{matrix}\dfrac{\begin{matrix}\mathscr{D}_1\\ G(0)\end{matrix}\qquad\begin{matrix}[G(x)]\\ \mathscr{D}_2\\ G(s(x))\end{matrix}}{G(y)}\\ \mathscr{D}_3\\ F(y)\end{matrix}}{\forall y F(y)}$$

A proof of this kind still conforms to the canonical proof format for $\forall y F(y)$, since it ends with an ∀-introduction. However, there is no bound to the possible complexity of the stronger formula $G(y)$. Also, the sub-formula principle does not hold. So G may contain formulas and expressions not occurring in F.[7]

If understanding the meaning of a provable sentence $\forall x F(x)$ involves being able to recognise at least one proof of it, and the only type of proofs are of the form of (6), then it seems we need to understand more complex formulas in order to understand a simpler one.

However, it is important to keep separate the threat of circularity or regress in the semantic theory, and the threat of circularity in the *understanding* of sentences, insofar as understanding involves judging the validity of proofs. The for-

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[7] Prawitz considers Dummett's proposed complexity restriction in Prawitz 1987, pp. 156-63. He points out that if → is defined by means of quantification over a restricted domain of proofs and we then expand the domain after discovering new proof methods, the meaning of → will not be stable.

mer problem can be dealt with by the appeal to canonical proofs. Prawitz writes

> [...] If, on the other hand, what constitutes a proof depends on the meaning of the sentences involved in the proof, and, as is often the case, they are of greater complexity that that of the sentence proved, we get into a vicious circle that seems to endanger the project of taking the correctness of assertions as a central feature in the theory of meaning.
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> The notion of canonical argument offers hope of getting out of this circle, given that the canonical arguments for a sentence A can be specified in terms of the constituents of A.
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> [...] What we need is thus an explanation of what a proof of a sentence A is that does not depend on knowing what a proof is for all the sentences involved in the proof of A but only on knowing what a proof is for the constituents of A.
>
> Such an explanation is now forthcoming, provided that knowledge of a valid closed argument – i.e., to have a closed valid argument $\mathscr{D}$ and a set of justifying procedures $\mathscr{J}$, and, in addition, to know that $(\mathscr{D}, \mathscr{J})$ is valid – is the right analysis of what it is to be in possession of a proof (Prawitz 1985, p. 167).

The idea of what validity amounts to for an open argument is given in the definitions earlier (Prawitz 1985, pp. 164-65). There validity is defined for a pair $(\mathscr{D}, \mathscr{J})$ of an argument $\mathscr{D}$ and an assignment of justifying operations $\mathscr{J}$. The justifying operation are essentially of the kind illustrated in the reduction step (4). This reduction justifies the elimination rule for $\wedge$ by showing that any sentence that can be proved by way of a final step of $\wedge$-elimination can also be proved without relying on that step, by using part of the proof of the conjunction that is the premise of the step. Hence, we don't get out more of a conjunction by the elimination rule than what justifies the conjunction in the first place.

The definition runs as follows:

(PV) When $\mathscr{D}$ is a *closed argument*, $(\mathscr{D}, \mathscr{J})$ is *valid* if and only if either

(i) $\mathscr{D}$ is in canonical form and each immediate subargument $\mathscr{D}'$ of $\mathscr{D}$ is valid with respect to $\mathscr{J}$, or

(ii) $\mathscr{D}$ is not in canonical form, but by successively applying the operations

> in $\mathscr{J}$, $\mathscr{D}$ is transformed to an argument for which (i) holds.
>
> When $\mathscr{D}$ is an *open argument*, $(\mathscr{D},\mathscr{J})$ is *valid* if and only if all closed instances $\mathscr{D}'$ of $\mathscr{D}$ that are obtained by substituting for free parameters closed terms and for free assumptions closed arguments for the assumptions valid with respect to an extension $\mathscr{J}'$ of $\mathscr{J}$

Applied to the subargument $\mathscr{D}_3$ for $F(y)$ in (6), which has a free variable but no open assumption, it amounts to the condition that when substituting a closed term t for the variable y throughout the argument, the resulting argument is valid (with respect to some justifying operations). In this case, it amounts to first proving $G(t)$ and then $F(t)$ from $G(t)$.

According to Prawitz's idea of recursive meaning explanations, we need not know this particular proof of $F(t)$. What we need to know is what counts as a *canonical* proof of $F(t)$ and that again is determined only by the form of $F(t)$ itself. If $F(t)$ is logically complex, what we need is specified by the relevant clause of the meaning explanations of the logical constants. If it is atomic and arithmetical, we need to understand the recursion equations for the main operator, so that we understand how $F(x')$ is defined in terms of $F(x)$. Recognition of an actual proof is not required. Our understanding of the conclusion $\forall yF(y)$ itself will then consist in knowing what counts as a canonical proof of the main sentence, and in knowing what counts as canonical proofs of closed counterparts to subformulas, and so on, without needing to take the complexity of actual proofs into account.

As Prawitz points out (1985, p. 165), provided that the premises of an introduction and the assumptions bound by it (if any) are always of lower complexity than that of its conclusion, the definition can be understood as proceeding by simultaneous induction, (over open and closed arguments). Then, the validity of an argument for a formula A will always be defined in terms of arguments for formulas that are either arguments for formulas of lower syntactic complexity when clause (i) can be applied, or for formulas of the same complexity a finite number of steps until clause (i) can be applied again.

With such a definition of *valid argument*, and hence of *proof*, i.e. of what it is to be valid *closed* argument, the appeal to canonical proofs allows a recursive specification of proof conditions of a sentence. We get a semantics that is compositional in the respect that the condition for being a proof of a sentence A is a function of the conditions for being a proof of the sub-sentences of A and the mode of composition (main operator) of A.

As a matter of illustration, we can present it in a set-abstraction format as follows. Assume that we are giving a semantics for a language L, and have now arrived at the clauses for the logical constants. Let $[\![A]\!]$ be the meaning of A, i.e. the set of canonical proofs of A. Let $A(t/x)$ be the formula that results from Ax by substituting t for x throughout.

Let $\mathscr{H}$ be a function that, when given a *method* f as argument, as value gives the result of applying the method f.[8] Then the set of proofs of A, i.e. the set of methods of getting canonical proofs of A is

$$\{f : \mathscr{H}(f) \in [\![A]\!]\}$$

Let us abbreviate this as '$([\![A]\!])$'. Then we can set down the following:

$$\text{(PS)} \qquad \begin{aligned}
&[\![\bot]\!] = \varnothing \\
&[\![A \wedge B]\!] = \{\langle a, b\rangle : a \in ([\![A]\!]) \text{ and } b \in ([\![B]\!])\} \\
&[\![A \vee B]\!] = \{\langle a, b\rangle : a = l \text{ and } b \in ([\![A]\!]) \text{ or } a = r \text{ and } b \in ([\![B]\!])\} \\
&[\![A \rightarrow B]\!] = ([\![B]\!])^{([\![A]\!])} \\
&[\![\forall x Ax]\!] = \{f : \text{ for any } t \in L(f(t) \in ([\![A(t/x)]\!]))\} \\
&[\![\exists x Ax]\!] = \{\langle t, b\rangle : t \in L \text{ and } b \in ([\![A(t/x)]\!])\}
\end{aligned}$$

(PS) clearly gives a compositional semantics for logically complex formulas, relative to the semantics for the atomic formulas. The semantics proceeds by recursion over syntactic complexity in a manner parallel to Prawitz's definition of validity. For instance, a valid closed argument a for $\forall x Ax$ has as its immediate subargument an open argument such that each substitution instance, where t replaces x, is a valid argument of the corresponding formula At.

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[8] That is, the application of $\mathscr{H}$ corresponds to applying the Prawitz justifying operations.

We can think of $[\![A]\!]$ as an extensional semantic entity. In order to understand A the speaker does not need to know the set extensionally. What the speaker needs to know are the *conditions for belonging* to the set, i.e. "what counts as a canonical proof of A". Hence, if understanding a sentence A consists in knowing what counts as a canonical proof of A, it is clear that there is a compositional specification of what needs to be known, as desired.

In particular, even if an actual proof does contain formulas of greater complexity than its conclusion, there is no need to understand those formulas for understanding the conclusion itself. But this contrasts with the demand of *recognising* the validity of that particular proof. We now turn to this question.

5. Semantics-transcendent proofs

When we are concerned with recognising proofs, the recursive definition of validity does not solve all problems. When judging a *particular* proof to be valid, I have to check the details of the proof, and for each subargument of the proof either verify that it is correct, or else verify that it is dispensable. Even if I can understand $\forall x A(x)$ and the conditions for the validity of a proof of it from its syntactic construction, I may still not be able to recognise a particular proof, or indeed any proof, of $\forall x A(x)$, since the proof will involve other formulas, both of greater complexity and not being constructed only from constituents of $\forall x A(x)$. We can call such proofs *semantics-transcendent*, since they involve constructions that are not specified in an intuitively adequate compositional semantics for the language in question. We are not in a possession of a method for determining whether an arbitrary semantics-transcendent construction is or is not a proof of a sentence A for which we do have a compositional semantics.

If it so happens that we have to do with proofs of a *formal system*, then it is decidable whether a particular construction is a proof in the system or not, provided that it is required that each derivation step in the proof be an instance of a formal rule of that system. In that case, a proof in the system of a formula in the language of the system is simply a proof in the system that ends with that formula. Then the set of proofs of a formula A is a decidable set, and learning the system involves learning a finite vocabulary, a finite set of syntactic construction rules and a finite set of derivation rules. So in this case, the decidability condition is met, although at the cost of defining proofs in terms of construction rules rather than in terms of their semantic import.

But even at that cost, not much is gained, and precisely for the reason that if we have to do with a formal system $\mathscr{F}$ that contains arithmetic or something of equal or higher expressive power, then we know from Gödel's first incompleteness theorem that the language of $\mathscr{F}$ will contain sentences that are neither provable nor refutable by any derivation constructable in $\mathscr{F}$ itself. A Gödel sentence for $\mathscr{F}$ is of the form $\forall x G(x)$, where G is a decidable predicate. Since G is decidable, every sentence Gt, where t is a numeral, is either provable or disprovable in the system. If for some numeral u, $G(u)$ is disprovable, then $\exists x \neg G(x)$ is provable, and this leads to a contradiction. Hence, we can conclude that $G(t)$ is provable for each numeral t. We can then conclude that the Gödel sentence is true. That is, we have proved it. The proof of the Gödel sentence makes use of concepts and principles belonging to a meta-level. Grasp of those concepts and acceptance of the principles of reasoning are required for recognising the Gödel reasoning as a proof. Clearly, the explanations of the meanings of the arithmetic sentences of the system do not involve explanations of the meta-level expressions used in the Gödel proof itself. Hence, knowledge of meaning is not in this case enough for being able to recognise a proof.[9]

A more vivid example is that of Fermat's Last Theorem. The theorem says that the equation

$$\text{(F)} \qquad x^n + y^n = z^n$$

does not have any non-zero solution for $n > 2$. The theorem is stateable in the language of arithmetic with exponentiation. The proof that was eventually given by

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[9] In Cozzo 1994, pp. 137-41, Cesare Cozzo discusses these issues and concludes that because proofs may contain previously unknown methods and concepts, one cannot in general require the global argument role an expression has in a language be reducible to the *immediate* argument role which defines its meaning. In itself, this agrees with the conclusion of the present paper. According to Cozzo, however, the theory of immediate argumental role satisfies Dummett's manifestability requirement, as Cozzo interprets it. There is unfortunately not space here for discussing this issue further.

Andrew Wyles includes as part a proof of a theorem about elliptical curves, belonging to geometry. It is not reasonable to assume that grasp of the theory of elliptical curves was needed for understanding Fermat's Last Theorem or indeed is needed today (and indeed I don't understand it). What seems to be needed is only grasp of (Peano, or Heyting) arithmetic, and the clauses for exponentiation.

And it is not a matter of isolated examples. A lesson to be drawn from Gödel's first incompleteness theorem is that we cannot code the entire domain of intuitively acceptable proofs by any finite set of rules. It is Dummett's own conclusion that the domain of proofs is essentially open, or indefinitely extensible (cf. Dummett 1980*b*). A further conclusion, although not Dummett's, is that we will always be able to *understand* sentences that are intuitively provable (there is a proof to be discovered), but whose proofs we are not *yet* equipped to recognise.[10]

6. Dummett on understanding and recognition

In conclusion, there is conflict between on the one hand the requirement that understanding a sentence consists in the ability to recognise proofs of it, and on the other that it consists in understanding it compositionally, i.e. by understanding its parts and its syntactic construction. This same conflict apparently manifests itself as a tension in Dummett's own thinking about the matter. On the one hand, Dummett endorses the idea of compositional understanding:

> [...] but the principle of compositionality in no way demands this; all that is essentially presupposed for the understanding of a complex sentence is the understanding of the subsentences (Dummett 1991, p. 258).

On the other hand, Dummett also takes it as a matter of course that recognition capacity goes hand-in-hand with such understanding:

> Our definitions escape circularity because, in order to judge the validity of a canonical argument of degree n, we need only to be able to recognise the validity of arbitrary arguments of degree $< n$, while, to

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[10] I discuss the theme of the totality of proofs and its relation to intuitionism also in Pagin 1998 and in Pagin *forthcoming* 2008.

> judge the validity of an arbitrary argument of degree n, we need only only to be able to recognise the validity of canonical arguments of degree $\leq n$ (Dummett 1991, p. 263).

The *degree* of an argument is the maximum number of logical constants occurring in some initial premise or in the conclusion (1991, p. 261). Dummett's comments on circularity relate to the definition given on the pages preceding the passage quoted. It is one thing, however, that the definition of validity escapes circularity, and another thing whether *judging* the validity of a particular argument is guaranteed to be achievable without having to check the validity of more complex arguments. I shall first illustrate the problem using Prawitz's and standard terminology.

In order to see where Dummett's reasoning goes wrong with respect to his own proof theoretic definitions, we need to look at an argument form that is a bit more complex than that of (6). The derivation (7), where $G(y)$ is derived by induction

$$\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{c}\mathscr{D}_1\\ G(0)\end{array}\qquad\begin{array}{c}[G(x)]\\ \mathscr{D}_2\\ G(s(x))\end{array}}{\begin{array}{c}G(y)\\ \mathscr{D}_3\\ F_1(y)\rightarrow F_2(y)\end{array}}\qquad [F_1(y)]}{F_2(y)}}{F_1(y)\rightarrow F_2(y)}}{\forall y(F_1(y)\rightarrow F_2(y))} \tag{7}$$

and where the first occurrence of $F_1(y) \rightarrow F_2(y)$ is derived by an elimination rule (and so is not a maximum formula), is, in the standard sense, in canonical form, since it ends with an $\forall$-introduction. Assume that it is a valid argument, and hence, by the standard definition, a canonical proof.

Its immediate subargument has (the second occurrence of) $F_1(y) \rightarrow F_2(y)$ as conclusion. Let's call it $\mathscr{D}_4$. It is itself an open derivation in canonical format, since it ends with an $\rightarrow$-introduction. Any closed instance, or (in Dummett's terminology) supplementation, of it is a canonical proof, in the standard sense. Such a closed instance is a derivation $\mathscr{D}_4[t/y]$, with the closed term t uniformly substituted for the free variable y. $\mathscr{D}_4[t/y]$ is valid, according to Prawitz's definition, just in case its immediate subargument is valid. That subargument is a derivation of $F_2(t/y)$ from $F_1(t/y)$. Call this derivation $\mathscr{D}_5$.

$\mathscr{D}_5$ is valid just in case any proof of $F_1(t/y)$ can be transformed into a proof of $F_2(t/y)$, and by assumption that condition is met. In Dummett's own terms, it is valid just in case "we can effectively transform any supplementation of it into a valid canonical argument with the same final conclusion and initial premises" (1991, p. 261). The crux is just that for *recognising* that we can effectively transform it in this way requires recognising the validity of the proof of $G(t/y)$, and there is no restriction on what can be involved in that proof. This is where the complexity of the *recognition* of validity in this case transcends the definition of validity itself.

The matter is somewhat complicated, although not essentially changed, by Dummett's own definitions of canonical argument and validity. For these definition, the notions of *boundary rule*, *main stem*, *critical subargument* and of *supplementation* are needed:

> We assume that we are given certain rules of inference, which we recognise as valid, for deriving atomic sentences from one or more other atomic sentence; we may call these 'boundary rules' (1991, p. 254).
>
> [...] an (occurrence of) a sentence belongs to the 'main stem' of an argument if it, and every sentence intervening between it and the final conclusion, depend only on (some or all of the) initial premises of the argument (1991, p. 260).
>
> A subargument of an argument (α) will be said to be 'critical' if its conclusion stands, in (α), immediately above a closed sentence in the main stem of (α), but is itself either an open sentence or a closed sentence not in the main stem (1991, p. 261).
>
> Given an arbitrary argument—whose initial premises may of course be complex—we define a 'supplementation' of it to be the result of replacing a complex initial premiss by a canonical (sub)argument having that premiss as its final conclusion (1991, p. 255).

I shall assume here that Dummett, like Prawitz's notion of a *closed instance* above, includes under the notion of supplementation that free variables are replaced by closed terms (since otherwise the definition is incomplete).

Then, Dummett's 1991 definition of a canonical argument is as follows:

(DC) An argument is *canonical* if:

- (a) its final conclusion is a closed sentence;
- (b) all its initial premises are closed atomic sentences;
- (c) every atomic sentence in the main stem is either an initial premise or is derived by a boundary rule;
- (d) every closed complex sentence in the main stem is derived by means of one of the given set of introduction rules.

Dummett also acknowledges that his definition 'places no restrictions on the derivations of open sentence, [...], or of sentences not in the main stem, such as the premiss of an if-introduction rule' (1991, p. 261). Because of this, as exemplified in the case of induction, an argument may contain subarguments of higher degree and still be canonical.

Dummett's definition of validity (1991, p. 261) then runs:

(DV) An arbitrary argument will be said to be 'valid' if we can effectively transform any supplementation of it into a valid canonical argument with the same final conclusion and initial premisses. A canonical argument will be said to be 'valid' just in case every critical subargument it contains is valid.

Given these definitions, (7) is a canonical argument, as before. However, $\mathscr{D}_4[t/y]$ need *not* be a canonical argument in Dummett's sense. For $G(t/y)$ is proved by the induction rule, which is a boundary rule in Dummett's sense, and if it belongs to the main stem, which it may, and if it is logically complex, which it may be, clause (d) of Dummett's definition of canonical argument is violated. It is a bit unclear how to regard such an argument from Dummett's point of view. It is clear, however, that the induction argument whose conclusion $G(t/y)$ is, does not belong to the main stem, and, as Dummett points out, *no restriction* is placed on that subargument. Still, for recognising the validity $\mathscr{D}_4[t/y]$, the validity of the induction argument must be recognised as well.

7. Conclusion

With both truth conditional and proof-theoretic semantics there is a gap between the semantic theory and Dummett's required manifestability of understanding. In the case of truth theoretic semantics, there is a gap between knowledge of meaning and the manifestability of knowledge of meaning, but there is no gap between what meaning the semantic theory assigns to a sentence and what is supposed to be known.

In the case of proof-theoretic semantics, where the gap is depends on what linguistic meaning is supposed to be. If linguistic meaning is what is specified by the semantic theory, then there is a similar gap as in the truth conditional case, between knowledge of meaning and manifestability of knowledge. For manifesting may require knowledge of proof methods that transcend the semantic theory.

If, on the other hand, linguistic meaning is precisely what corresponds to the manifestability requirement, i.e. the property of being a proof of the sentence, then it is the other way around. There is no gap between knowledge of meaning and the manifestability of knowledge of meaning, but there is a gap between the meaning that the intuitively correct proof theoretic semantics assigns to the sentence and the meaning that is supposed to be known. In that case, the proposed compositional semantics cannot provide the meaning that is asked for.

The combination proof theoretic semantics with the requirement of recognisability of proofs comes into conflict with compositionality. For assume we have a semantic function $[\![\cdot]\!]$ for a language L. A generalised composition function ρ for $[\![\cdot]\!]$ must then meet two conditions: a) it must be possible to know the meaning of any complex expression in L by knowing ρ, the modes of composition and the meaning of simple expressions; and b) the condition of being a canonical proof must, for every provable sentence A, be met by some proof that is recognisable by any speaker who understands A.[11] Then, if we meet condition a) by means of specifying inductively a fixed domain of (recognisable) proofs that can known in

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[11] In terms of the set-theoretic construction (PS), this would mean intersecting the set of proofs meeting the recursive conditions with some set of recognisable proofs.

advance, then condition b) will not be met, for there will be sentences that have proofs only outside that domain.

If, on the other hand, we meet condition b) by letting the domain of (recognisable) proofs for a sentence A *depend* on A, then condition a) will not be met. For what domain of proofs is needed for a sentence A cannot always be known in advance of having found a proof or a disproof. Hence, it can be known only partially what the function ρ is (i.e. only insofar as it gives meaning to sentences whose proofs are in known domains). So the speaker will not always understand a new sentence in advance of knowing a proof or a disproof, even if it is constructed from familiar parts in familiar ways. Since this is unacceptable, condition b) should be dropped.

The final upshot seems to be this: *representational* properties of a sentence can be compositionally determined, while its *epistemic* properties in general can not. It is because we cannot in general, except in compositional outline, determine from its meaning what the proof of a sentence must be like that we *can* understand a sentence without knowing how to decide it.[12] [13]

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[12] In a related vein, I argue in Pagin 1994 that the requirement that proofs be knowable induces the so-called *knowability paradox*. The paradox is not induced by the accompanying claim alone that a sentence is true only if it has a proof, but needs the further requirement that proofs are knowable. But I also stress that without the epistemic requirement, it is less clear how to distinguish proofs in particular from truth makers in general.

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