Strategic Risk and Coordination Failure in Blame Games

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Abstract

Within a class of games that we call Blame Games, we discuss how strategic risk may discourage play of a unique and efficient (strictly) dominance solvable equilibrium.

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1 Introduction

Let us define *coordination failure* broadly as the play of any strategy profile yielding expected payoffs that are different from the best available Nash equilibrium payoffs. This definition covers both the play of an inferior Nash equilibrium and the play of non-equilibrium strategy profiles.[1] The two types of coordination failures are often treated separately in the literature. Play of inferior Nash equilibria is typically discussed in terms of equilibrium selection, whereas non-equilibrium play is attributed to bounded rationality or doubts that others are rational. In this paper we propose a class of games that highlights that both types of coordination failures are due to strategic risk, i.e. uncertainty about what other players will do.[2]

Strategic risk would appear to be least important in games in which only one strategy profile satisfies iterated elimination of strictly dominated strategies (IESDS). IESDS corresponds to iterated belief in rationality (Tan and Werlang, 1988), so rational players with sufficiently high degree of iterated belief that others are rational will play the IESDS outcome. Experimental subjects, however, do not always play iteratively undominated strategies. This has been shown most clearly in Beauty Contests (e.g., Nagel, 1995, Grosskopf and Nagel, 2008), in which each player chooses an integer from the set $\{1, ..., 100\}$ and the winner is the player that comes closest to two thirds of the average of all players' choices. This game is solvable through IESDS, and the unique solution is for all players to choose 1, resulting in all players sharing the fixed prize.

We here devise a class of games—that we call Blame Games—in which only one strategy profile survives IESDS, and this profile yields the maximum attainable payoff for all players. In contrast to Beauty Contests, all players in Blame Games prefer the equilibrium outcome to any other outcome of the game. Yet, a rational player should be wary of playing an iteratively undominated strategy because the own payoff is low in case some opponent fails to do so.

Blame Games are most closely related to Weak-link Games (a.k.a. Minimum Effort Games) which constitute a paradigmatic example of coordination failures of the first type, i.e. play of inferior Nash equilibria. The Weak-link Game is a symmetric simultaneous-move n-player game in which players choose their action from a set of numbers and each player's best response is to play the minimum of the other players' chosen numbers (van Huyck et al., 1990). Weak-link Game have as many pure strategy Nash equilibria as there are pure strategies. It is well known that players frequently fail to coordinate on the efficient equilibrium in Weak-link Games, especially when the number of players is large (e.g., van Huyck et al., 1990, Knez and Camerer, 1994, Goeree and Holt, 2001, 2005).

In a Blame Game, the set of actions is also a set of numbers, but each player's unique best response is to play the smallest number *above* the minimum of the other players' chosen numbers (and if all the others choose the maximum number, the best response is the maximum number). In a Blame Game, the unique iteratively undominated outcome is thus to play the

[1] In what follows, payoffs correspond to players' von Neumann-Morgenstern utilities. For example, to the extent that players have social preferences, this is already taken into account in the description of the payoffs.

[2] Non-equilibrium play might also be due to failures to best respond, see for instance Grosskopf and Nagel (2008).

maximum number. However, just as it may be risky to play the equilibrium strategy in a Weak-link Game, it may be risky to play the equilibrium strategy in a Blame Game. Indeed, each player may know that all the opponents are rational, but if some player thinks that some player thinks...that some player does not know that all players are rational, then this limits the admissible number of rounds of deletion of strictly dominated strategies. We conjecture that inexperienced players will fail to coordinate on the equilibrium in the Blame Game with many players and high costs of choosing high numbers.[3] Since Blame Games represent slightly perturbed Weak-link Games, we think that coordination failures in the Weak-link Game are *not* due to multiplicity of equilibria per se, but to the riskiness of the (best) equilibrium strategy.

We believe that Blame Games can be a useful tool to identify different sources of coordination failure, in particular since Blame Games are flexible with respect to the number of players and strategies as well as the magnitude of strategic risk. Blame Games also serve as one of few examples of strictly dominance solvable games in which players are unlikely to coordinate on an equilibrium that is strictly preferred by all players. (Other related games are discussed in the concluding section.) In addition, Blame Games are interesting from an applied perspective. Weak-link Games have been used as a model of strategic interaction within organizations (Knez and Camerer, 1994, Camerer, 2003, Weber, 2006, Brandts and Cooper, 2006).[4] One typical example of organizations with this structure are airlines where workers have to prepare an airplane for departure and the plane cannot leave until everybody have finished their task (Camerer, 2003). To the extent that play will converge to equilibrium in Blame Games, the punishment technology of Blame Games provide a simple solution to coordination failures in Weak-link Games.[5] Such small punishments are also considerably cheaper than the incentives considered by Brandts and Cooper (2006) and Hamman et al. (2007).

2 Blame Games

A continuous Weak-link Game is a non-cooperative simultaneous move game with $N \geq 2$ players. Each player i is free to choose actions (henceforth called effort levels) e_i anywhere in the real interval $[1, K]$. Let $\boldsymbol{e} := (e_1, ..., e_N)$ denote a (pure) strategy profile, and define

[3] It is straightforward to demonstrate that standard behavioral alternatives to Nash equilibrium such as logit QRE (McKelvey and Palfrey, 1995) and level-k (Stahl and Wilson 1994, 1995 and Nagel, 1995) often imply effort level distributions below the efficient outcome in both Weak-link Games and Blame Games when the number of players is large. See also Crawford (1995) for a model of adaptive dynamics (focusing on limiting outcomes) that can account for many of the empirical regularities observed in Weak-link Games without referring to a theory of equilibrium selection.

[4] Weak-link Games were originally put forward and studied experimentally by van Huyck et al. (1990) as a strategic form representation of the simple Keynesian model of Bryant (1983).

[5] Since Blame Games have strategic complementarities, a very broad class of adaptive dynamics converge to the unique equilibrium (Milgrom and Roberts, 1990). For example, the Cournot best response dynamic requires only as many rounds as there are effort levels to converge to equilibrium (given that all players update their strategy choices in each round).

$\underline{e} := \min\{e_1, e_2, ..., e_N\}$. The payoff function of player i is

$$\pi_i^W(e_i, \underline{e}) := \underline{e} - ce_i,$$

where $c \in (0,1)$. As is well known, such continuous Weak-link Games have a continuum of pure strategy equilibria (e.g., Anderson et al., 2001). In each pure strategy equilibrium all players choose the same effort level. However, note that the smallest effort level $e_i = 1$ yields a safe payoff to player i of $1-c$, whereas any higher effort level yields a payoff anywhere in the interval $[1 - ce_i, e_i - ce_i]$, depending on the lowest effort of the other players. Thus, minimum effort is associated with minimal downside risk.

We now introduce the slight modification of the Weak-link Game that turns it into a *Blame Game*. Let there be a cost $\varepsilon > 0$ associated with choosing the minimum effort level as long as the minimum is below the highest possible effort level. This cost may be either monetary—for example because there is a manager that punishes workers that provide minimum effort—or a psychological cost of being the one to blame for the low production level. Defining the indicator function

$$I_i := \begin{cases} 1 \text{ if } e_i = \underline{e} \text{ and } \underline{e} < K; \\ 0 \text{ otherwise,} \end{cases}$$

the payoff function of each player in a Blame Game can thus be written

$$\pi_i^B(e_i, \underline{e}) := \underline{e} - ce_i - \varepsilon I_i.$$

One application of the Blame Game is the case of meetings that cannot start before the last participant arrives. No participant wants to come long before the others, since the meeting cannot start without them. At the same time, participants dislike being guilty of holding up the start of the meeting.

It is straightforward to prove that all players choose the highest effort level in the Blame Game's unique pure strategy equilibrium.

Proposition 1 *The unique pure strategy Nash equilibrium of a continuous Blame Game is* $\boldsymbol{e} = (K, K, ..., K)$.

Proof. Let $\eta < \varepsilon/c$. For any strategy profile with $\underline{e} < K$, the payoff to a player from playing $e_i = \underline{e} + \eta$ is $\pi_i^B(\underline{e} + \eta, \underline{e}) = \underline{e} - c(\underline{e} + \eta)$, which exceeds $\pi_i^B(\underline{e}, \underline{e}) = \underline{e} - c\underline{e} - \varepsilon$. For strategy profiles with $\underline{e} = K$, the unique best response for all players is to play K, so the unique pure strategy Nash equilibrium is $\boldsymbol{e} = (K, K, ..., K)$. ■

As defined above, Blame Games have one slightly unattractive feature, namely that players experience the same cost ε irrespective of how many players there are to blame. However, the main insight holds more generally: As long as the cost is positive, it does not matter if the size of the payoff reduction is decreasing in the number of blameworthy players.

In discrete versions of Blame Games, the condition for a unique equilibrium is a little bit more restrictive. On the other hand, if the condition is fulfilled, we can relax the solution concept. Consider in particular the finite strategy set $\{1, 1+k, ..., K-k, K\}$ with $k > 0$. We call this the discrete Blame Game.

Proposition 2 *Suppose $\varepsilon > ck$. In a discrete Blame Game, the only strategy profile to survive iterated elimination of strictly dominated strategies is $\mathbf{e} = (K, K, ..., K)$.*

Proof. The proof proceeds by showing that the lowest effort level is strictly dominated by the second lowest effort level. Once the lowest effort level is eliminated, the second lowest effort level is strictly dominated by the third lowest, and so on. To see this, suppose that $e < K$ is the lowest effort level that is not yet eliminated. We want to show that playing $e_i = e$ is strictly dominated by playing $e_i = e^* = e + k$. Playing e yields payoff $\pi_i^B(e, \underline{e}) = e - ce - \varepsilon$ irrespective of what the other players do (since effort levels below e have been eliminated). Playing e^* gives payoff $\pi_i^B(e^*, \underline{e}) = e - c(e+k)$ if $\underline{e} = e$ and $\pi_i^B(e^*, \underline{e}) = e + k - c(e+k) - \varepsilon$ if $\underline{e} = e^*$. Clearly, playing e^* strictly dominates playing e as long as $\varepsilon > ck$. This process of elimination continues until all effort levels except $e = K$ are eliminated. ■

As an example, consider the game in Figure 1. The game has $c = 0.5$, and $k = 1$. Consequently the game is a Weak-link Game and has seven Pareto-ranked pure strategy equilibria when $\varepsilon < 0.5$, but is a Blame Game with only one equilibrium whenever $\varepsilon > 0.5$.

		Smallest effort level						
		7	6	5	4	3	2	1
Your effort	7	3.5	2.5	1.5	0.5	-0.5	-1.5	-2.5
	6		$3-\varepsilon$	2	1	0	-1	-2
	5			$2.5-\varepsilon$	1.5	0.5	-0.5	-1.5
	4				$2-\varepsilon$	1	0	-1
	3					$1.5-\varepsilon$	0.5	-0.5
	2						$1-\varepsilon$	0
	1							$0.5-\varepsilon$

Figure 1. Discrete Weak-link/Blame Game (c = 0.5)

3 Concluding Discussion

We are not the first to suggest that actual people may lack the strategic sophistication to iteratively eliminate strictly dominated strategies, nor do we pioneer the idea that such lack of sophistication is a source of coordination failures. Fudenberg and Tirole (1991, Figure 1.4) consider the game depicted in Figure 2.

	L	R
U	$8, 10$	$-100, 9$
D	$7, 6$	$6, 5$

Figure 2. Fudenberg and Tirole's game

If players are selfish materialists, Fudenberg and Tirole's game has a unique outcome that survives IESDS, and this outcome yields the maximum payoff to both players, just as in a Blame Game. Indeed, we see Blame Games as symmetric n-player relatives of Fudenberg and Tirole's asymmetric 2×2 game. The symmetry admits flexibility with respect to the number of strategies and players, as well as a simple way to parametrize payoffs.

Fudenberg and Tirole (1991, page 8) report a classroom survey in which about half the students report that they would choose D if they were in the position of the row player, in apparent contradiction with two rounds of elimination of strictly dominated strategies. However, we are not completely convinced that choosing D contradicts iterated strict elimination. Suppose that there is a more than one percent chance that the column player is spiteful enough to give up one cent in order to impose an opportunity cost of 108 cents on the opponent. Then it is better for a purely selfish row player to play D than to play U. To get around the problem of spitefulness, one might reduce the column player's payoff in the (U, R) cell. For example, if the payoff is reduced from 9 to -98, a spiteful player has no reason to prefer R to L. In Blame Games, the parameter ε can similarly be used to weaken or strengthen the spitefulness motive.

As discussed in the introduction, Blame Games have some family resemblance with Beauty Contests (e.g., Nagel, 1995, Grosskopf and Nagel, 2008). Although Beauty Contests are solvable through IESDS, it is hard to apply iterated strict dominance. For example, the first elimination round only eliminates the choice of 100, and entails the use of a particular mixed strategy that puts positive weight on all integers $\{0, ..., 67\}$. In the Beauty Contest, iterated elimination of weakly dominated strategies (IEWDS) is both faster and easier, but IEWDS is a criterion that generally cannot easily be justified on epistemic grounds (see Asheim and Dufwenberg, 2003 and the references therein).

Blame Games are also related to Traveler's Dilemmas (Basu, 1994, Capra et al., 1999 and Goeree and Holt, 2001). In a Traveler's Dilemma, two players choose integers between 2 and 100. The payoffs are determined by the minimum number plus a fixed reward $R > 1$ to the player that picked a lower number and a penalty $R > 1$ to the player that picked a higher number. Blame Games resemble an upside-down Traveler's Dilemma with an added cost of picking high numbers, but there are important differences. The equilibrium of Traveler's Dilemmas is not efficient and altruism on the part of the players will, if anything, lead away from the "selfish" equilibrium in Traveler's Dilemmas, but towards the ("selfish") equilibrium in Blame Games. In Blame Game experiments, inferences about non-equilibrium play are thus relatively less likely to be confounded by social preferences. Furthermore, analogues to our observations regarding IESDS and IEWDS in Beauty Contests apply also to the Traveler's Dilemma.

To conclude, we assert that Blame Games provide a simple yet flexible vehicle for understanding the roles of limited cognition and strategic risk in the creation of coordination failures.

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References

Anderson, S. P., Goeree, J. K. and Holt, C. A., 2001. Minimum-Effort Coordination Games: Stochastic Potential and Logit Equilibrium. Games and Economic Behavior 34(2), 177–199.

Asheim, G. B. and Dufwenberg, M., 2003. Admissibility and Common Belief. Games and Economic Behavior 42(2), 208–234.

Basu, K., 1994. The Traveler's Dilemma: Paradoxes of Rationality in Game Theory. American Economic Review 84(2), 391–395.

Brandts, J. and Cooper, D. J., 2006. A Change Would Do You Good... An Experimental Study on How to Overcome Coordination Failure in Organizations. American Economic Review 96(3), 669–693.

Bryant, J., 1983. A Simple Rational Expectations Keynes-Type Model. Quarterly Journal of Economics 98(3), 525–528.

Camerer, C. F., 2003. Behavioral Game Theory. Princeton University Press, Princeton.

Capra, C. M., Goeree, J. K., Gomez, R. and Holt, C. A., 1999. Anomalous Behavior in a Traveler's Dilemma? American Economic Review 89(3), 678–690.

Crawford, V. P., 1995. Adaptive Dynamics in Coordination Games. Econometrica 63(1), 103–143.

Fudenberg, D. and Tirole, J., 1991. Game Theory. MIT Press, Cambridge.

Goeree, J. K. and Holt, C. A., 2001. Ten Little Treasures of Game Theory and Ten Intuitive Contradictions. American Economic Review 91(5), 1402–1422.

Goeree, J. K. and Holt, C. A., 2005. An Experimental Study of Costly Coordination. Games and Economic Behavior 51(2), 349–364.

Grosskopf, B. and Nagel, R., 2008. The Two-Person Beauty Contest. Games and Economic Behavior 62(1), 93–99.

Hamman, J., Rick, S. and Weber, R. A., 2007. Solving Coordination Failure with 'All-or-None' Group-Level Incentives. Experimental Economics 10(10), 285–303.

Knez, M. and Camerer, C. F., 1994. Creating Expectational Assets in the Laboratory: Coordination in 'Weakest-Link' Games. Strategic Management Journal 15(8), 101–119.

McKelvey, R. D. and Palfrey, T. R., 1995. Quantal Response Equilibria for Normal Form Games. Games and Economic Behavior 10(1), 6–38.

Milgrom, P. and Roberts, J., 1990. Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities. Econometrica 58(6), 1255–1277.

Nagel, R., 1995. Unraveling in Guessing Games: An Experimental Study. American Economic Review 85(5), 1313–1326.

Stahl, D. O. and Wilson, P. W., 1994. Experimental Evidence on Players' Models of Other Players. Journal of Economic Behavior and Organization 25(3), 309–327.

Stahl, D. O. and Wilson, P. W., 1995. On Players' Models of Other Players: Theory and Experimental Evidence. Games and Economic Behavior 10(1), 33–51.

Tan, T. C.-C. and Werlang, S. R. d. C., 1988. The Bayesian Foundations of Solution Concepts of Games. Journal of Economic Theory 45(2), 370–391.

van Huyck, J. B., Battalio, R. C. and Beil, R. O., 1990. Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure. American Economic Review 80(1), 234–248.

Weber, R. A., 2006. Managing Growth to Achieve Efficient Coordination in Large Groups. American Economic Review 96(1), 114–126.