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Job search and mobility in developing countries. Theory and policy implications ☆

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Abstract

A labor market model is developed in which the formal sector is characterized by search frictions whereas the informal sector is competitive. We show that there exists a unique steady-state equilibrium in this dual economy. We then consider different policies financed by a tax on firms' profits. We find that reducing the unemployment benefit or the firms' entry cost in the formal sector induces higher job creation and formal employment, reduces the size of the informal sector but has an ambiguous effect on wages. We also find that an employment/wage subsidy policy and a hiring subsidy policy have different implications. In particular, the former increases the size of the informal sector while the latter decreases it.

JEL classification: D83; J41; J64; O17
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1. Introduction

The informal sector is a pervasive and persistent economic feature of most developing economies, contributing significantly to employment creation, production, and income generation. Recent estimates of the size of the informal sector in developing countries in terms of its share of non-agricultural employment range roughly between one-fifth and four-fifths. In terms of its contribution to GDP, the informal sector accounts for between 25% and 40% of annual output in developing countries in Asia and Africa.[1]

Some researchers in this area have tried to understand how an informal sector emerges. The usual argument puts forward is that firms and workers join the informal sector in order to avoid taxation or any formal regulation from the government (see e.g. Rauch, 1991, or Loayza, 1996). Other researchers have focused on the implications of the existence of the informal sector on the economy. In particular, they have studied how the informal sector generates externalities (both positive and negative) for the formal economy (see e.g. Marcoullier and Young, 1995; Dessy and Pallange, 2003; Fugazza and Jacques, 2003).

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[1] For more empirical evidence and literature surveys on this issue, see Schneider and Enste (2000) and Schneider (2005).

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In the present paper, we focus on the labor-market aspects of the informal sector, and show that its emergence is mainly due to search-matching frictions in the formal sector. We determine the steady-state equilibrium and then evaluate different policies aiming at reducing unemployment. We show that, even if the informal sector is unregulated and cannot be directly targeted by a government's policy, the latter affects indirectly the wage, the employment and thus the size of the informal sector.

Our paper is related to the literature on rural–urban migration, initiated by the two seminal papers of Todaro (1969) and Harris and Todaro (1970). One of the main issues raised in this literature is that creating urban jobs may increase rather than decrease urban unemployment because of the induced negative effect on rural migration, which may outweigh the positive effect of creating jobs (Todaro, 1976). This is referred to as the Todaro paradox.[2] Even though this is not the main issue of this paper, we also investigate the Todaro paradox in the context of formal and informal sectors.

We consider a search-matching model. There is a tradition of search models in the migration literature. The early models were using the old search approach where only one side of the market (the workers) was modeled (see e.g. Fields, 1975; 1989; Banerjee, 1984; Mohtadi, 1989). There is also a more recent literature, which incorporates the search-matching approach a la Mortensen–Pissarides (Mortensen and Pissarides, 1999; Pissarides, 2000) in a Harris–Todaro model (see Coulson et al., 2001; Ortega, 2000; Sato, 2004; Laing et al., 2005; Satchi and Temple, 2006). These authors model the urban/rural areas or formal/informal sectors in different ways and focus on different issues. Coulson et al. (2001) analyze the effect of location and commuting costs on wages and unemployment in a search model with two different locations. Ortega (2000) and Sato (2004) propose a similar analysis but focus on the welfare effects of migration. Laing et al. (2005) analyze how a huku system (which prevents workers from freely moving between rural and urban areas) in China can affect workers' outcomes of a labor market characterized by search frictions. Finally, Satchi and Temple (2006) focus on the impact of growth on the labor-market outcomes of workers in both the formal and informal sectors, where the former is characterized by search frictions. Although all these approaches are in some way related to our model, none of them analyzes the policy implications of their model but rather characterizes an equilibrium that matches certain empirical facts. To the best of our knowledge, this is the first paper that evaluates the consequences of different policies on workers' labor-market outcomes in the formal and informal sectors where there is free mobility between the two sectors and search frictions in the formal sector.

To be more precise, we develop a model where there are search frictions in the formal sector whereas the informal sector is competitive. In the formal sector, the wage is determined by a bargaining between workers and firms and, because of search frictions, unemployment emerges in equilibrium. In the informal sector, wages are paid at the marginal productivity of workers and there is full employment. The informal sector is fully accessible for everybody while there is an entry cost both for firms and workers in the formal sector. We characterize the steady-state equilibrium of the economy and show that the equilibrium exists and is unique but not efficient because of search externalities. We then consider different policies financed by a tax on firms' profits. We find that reducing the unemployment benefit or the firms' entry cost in the formal sector induces higher job creation and formal employment, reduces the size of the informal sector but has ambiguous effect on wages. We also find that an employment/wage subsidy policy and a hiring subsidy policy have different implications. In particular, the former increases the size of the informal sector while the latter decreases it.

2. Model and notations

There are two sectors in the economy: the formal sector and the informal one. All workers and firms are ex ante totally identical. There is a continuum of workers whose mass is N. Among the N workers, N^{F} and N^{I} work in the formal and informal sector, respectively (F and I stand for formal and informal). We have $N=N^{\mathrm{F}}+N^{\mathrm{I}}$, and

$$N^{\mathrm{F}} = L^{\mathrm{F}} + U^{\mathrm{F}}$$

$$N^{\mathrm{I}} = L^{\mathrm{I}}$$

where L^{F}, L^{I}, and U^{F} are respectively the employment levels in the formal and informal sectors, and the unemployment level in the formal sector. Since there is no unemployment in the informal sector, U^{F} is also the unemployment level in the economy. Thus, by combining these two equations, we obtain:

$$U^{\mathrm{F}} = N - L^{\mathrm{F}} - L^{\mathrm{I}}. \tag{2.1}$$

[2] See, in particular, the papers by Zarembka (1970), Blomqvist (1978), Arellano (1981), Takagi (1984), Nagakome (1989), Brueckner (1990), Stark et al. (1991), Raimondos (1993), Brueckner and Zenou (1999), Brueckner and Kim (2001).

2.1. The formal sector

It is assumed that there are search frictions[3] in the formal sector and we use the standard search matching framework (Mortensen and Pissarides, 1999; Pissarides, 2000) to model these frictions. The starting point is the following matching function

$$M^{\mathrm{F}} = M(U^{\mathrm{F}}, V^{\mathrm{F}}) \tag{2.2}$$

where U^{F} and V^{F} are respectively the total number of unemployed workers and vacancies in the formal sector, and M^{F} is the total number of matches. Because we assume that time is continuous, this is true during a small interval of time. This matching function captures the frictions that search behaviors of both firms and workers imply. It is assumed that M (.) is increasing in its arguments, concave and homogeneous of degree 1. In the absence of frictions, $M^{\mathrm{F}} \to +\infty$, so that workers will instantaneously find a job and firms a worker.

At a micro level, a matching function (2.2) can be derived from specific specifications of the meeting process. What is crucial is that even with ex ante identical agents (i.e. workers and firms) and no wage distribution, there can exist search frictions captured by the matching function. The traditional micro-foundation behind the aggregate matching function is the so-called urn-ball model and coordination failures can explain the emergence of search frictions. Let us give the intuition (borrowed from Petrongolo and Pissarides, 2001) that gives the reasons for the existence of a well-behaved matching function. Think of firms as urns and workers as balls. If all workers and firms are ex ante identical and if only one worker can occupy each job, an uncoordinated application process by workers will lead to overcrowding in some jobs and to no applications in others. The search frictions that lead to the existence of unemployment and vacancies in equilibrium are here the lack of information about other workers' actions (i.e. to which firm workers send their job application). To be more precise, there are U workers who know exactly the location of V job vacancies and send one application each. If a vacancy receives one or more applications, it selects an applicant at random and forms a match. The other applicants are returned to the pool of unemployed workers to apply again. The matching function is derived by writing down an expression for the number of vacancies that do not receive any applications. It is easy to show that, as V becomes large, the matching function is equal to (Butters, 1979; Hall, 1979; Blanchard and Diamond, 1994; Smith and Zenou, 2003):

$$M^{\mathrm{F}} = V^{\mathrm{F}}(1 - e^{U^{\mathrm{F}}/V^{\mathrm{F}}}).$$

This matching function is a particular case of our general matching function and exhibits the same properties. Of course, other specific meeting processes can give rise to a similar matching function (see Mortensen and Pissarides, 1999; Petrongolo and Pissarides, 2001). A common story is that workers know where vacancies are, but do not know which particular vacancies other workers will visit, allowing for the possibility that some workers are unable to fill vacancies because they were 'second in line'. This structure reduces the aggregate meeting process to again an 'urn-ball' process in which the labor market is visualized as V 'urns' (i.e. vacancies) and U 'balls' (i.e. workers), each ball having a probability $1/V$ of being directed to any given urn. More generally, as soon as it is assumed that it takes time to find a partner then it can coexist in equilibrium both unemployment and vacancies, even if both workers and firms are ex ante identical.[4]

Given the matching function (2.2), we can determine the rate at which vacancies are filled. It is equal to: $M(U^{\mathrm{F}},V^{\mathrm{F}})/V^{\mathrm{F}} = M(1/\theta^{\mathrm{F}},1) \equiv q(\theta^{\mathrm{F}})$, where

$$\theta^{\mathrm{F}} = \frac{V^{\mathrm{F}}}{U^{\mathrm{F}}} \tag{2.3}$$

is the labor market tightness in the formal sector and $q(\theta^{\mathrm{F}})$ is a Poisson intensity. Similarly, the rate at which an unemployed worker leaves unemployment (job acquisition rate) is now given by

$$a^{\mathrm{F}} = \frac{M(U^{\mathrm{F}}, V^{\mathrm{F}})}{U^{\mathrm{F}}} \equiv \theta^{F} q(\theta^{\mathrm{F}}). \tag{2.4}$$

2.2. The informal sector

The informal sector is assumed to be frictionless, i.e. whoever decides to work in this sector finds a job instantaneously, so that there is no unemployment. If we

[3] As defined by Mortensen and Pissarides (1999), "market friction is the costly delay in the process of finding trading partners and determining the terms of trade." In other words, search frictions imply that it takes time and other resources for a worker to obtain a job and for a firm to fill a vacancy.

[4] Another way to look at search frictions is through heterogeneity so that the matching between workers with specific skills and firms with specific skill requirements is not perfect and involves search frictions (as in Marimon and Zilibotti, 1999). This could be introduced in the present model but would complicate substantially the analysis since there will be a wage distribution in equilibrium and thus a reservation rule needs to be introduced.

believe that coordination failures give rise to search frictions then why are they significantly more coordination failures in the formal than in the informal sector? When one looks at the literature on formal and informal sectors in developing countries, it is striking to see that, in the informal sector, either people create their own business (self-employed, entrepreneur) or work for friends and relatives. Since most firms are family related, coordination failures and thus search frictions should not be too large. For example, based on data from South American countries (Mexico, El Salvador, and Peru), Yamada (1996), Marcouiller et al. (1997), and Maloney (1999) show that self-employment and workers employed in family enterprise are prevalent in the informal sector. Moreover, Maloney (2004) presents evidence of an informal sector that is an unregulated micro-entrepreneurial sector. In other words, self-employment represents the bulk of informality in many economies. On the other hand, in the formal sector, people formally apply to jobs. For example, in Egypt, nearly 70% of workers obtain a formal job through formal methods (see Wahba and Zenou, 2005). This means that coordination failures and thus search frictions are more likely to occur there.[5] There are of course search frictions in the informal sector but they are much lower than in the formal sector. Thus, for the sake of simplicity, we only assume search frictions in the formal sector.

Observe that, in the present model, the fact that there are more search frictions in the formal sector than in the informal sector is independent of the size of the firm. Indeed, in the informal sector, either people are self-employed or work with relatives or friends and thus do not apply formally for jobs posted in newspapers or in employment agency. They mainly find jobs through word-of-mouth communication. We do not model this feature explicitly but this is what we have in mind. In the formal sector, it is assumed that it takes time to find a job because one has to go through a formal process. Firms have first to advertise jobs and then workers have to apply for these jobs. Firms then have to select workers for interviews and interview them. Finally, firms have to decide which worker to hire. This takes time and creates frictions. These are captured by the matching function (2.2). In the informal sector, this process is much quicker since there is no need for advertising jobs and screening workers. People know each other and rely on reputation.

Observe also that it is because there are search frictions in the formal sector that an informal sector arises. Indeed, since in the present model all workers prefer to work in the formal sector (see below), without search frictions in the formal sector (i.e. $M^{\mathrm{F}} \to +\infty$), all workers would instantaneously find a job in the formal sector and the informal sector will not exist. Satchi and Temple (2006) calibrate a search-matching model for Mexico and show that the observed informal sector employment of 30% of the urban workforce can be explained solely in terms of matching frictions provided either that workers receive a relatively large share of the match surplus, or that recruitment costs are significant.

Everybody can thus obtain a job in the informal sector and it is assumed that the wage in the informal sector is flexible enough to guarantee that there is full-employment; this wage is denoted by $w_{\mathrm{L}}^{\mathrm{I}}$. In the informal sector, we have the following production function: $F(L^{\mathrm{I}})$, with $F'(L^{\mathrm{I}})>0$ and $F''(L^{\mathrm{I}})\leq 0$. The price of the good is taken as a numeraire and, without loss of generality, normalized to 1. As stated above, in the informal sector, wages are flexible and equal to marginal product. We thus have:

$$w_{\mathrm{L}}^{\mathrm{I}} = F'(L^{\mathrm{I}}). \tag{2.5}$$

We assume that workers in the formal sector have a higher productivity than those working in the informal sector. Indeed, formal sector firms have a higher productive capacity than those in the informal sector because of more extensive infrastructure and greater accessibility to those production factors, and thus a better technology. As pointed out by Straub (2005), complying with costly registration procedures[6] allows formal firms to benefit from key public goods (which make production possible because of police and judicial protection against crime and enhance productivity because of public infrastructure), enforcement of property rights and thus can participate to the formal credit market. The latter is crucial because financial markets are at the heart of productive activities, both by sustaining medium and long run investments and by smoothing exchanges through short term credits.

There are evidence showing the link between informality and productivity. For example, Fajnzylber et al. (2006) study the dynamics of micro firms in Mexico using propensity score matching techniques to contrast formal firms with access to credit, participating in a business association, paying taxes and receiving training, with informal firms that decide not to participate in these societal institutions. They show that less than 10% of the informal firms have received credit or training and less than 17% participate in a business association. Using data on Kenya, Hjalmarsson et al. (2001) and Bigsten et al.

[5] By estimating a matching function, Rama (1998) finds that there are substantial search frictions in the formal sector in Tunisia.

[6] which are captured in our model by the entry costs γ (see below).

(2004) show that informal firms are less technically efficient than formal ones and exhibit lower rates of labor productivity (see in particular Table 3 in Bigsten et al., 2004).

2.3. Lifetime expected utilities

Workers are risk neutral and live forever. We assume that changes in employment status are governed by a Poisson process in which a^F is the (endogenous) job acquisition rate in the formal sector and δ the (exogenous) destruction rate. Let us denote by r the common discount rate of all workers. Then, the standard steady-state Bellman equations for the employed and unemployed workers in the formal sector are respectively given by:

$$rI_L^F = w_L^F - \delta(I_L^F - I_U^F) \tag{2.6}$$

$$rI_U^F = w_U^F + \theta^F q(\theta^F)(I_L^F - I_U^F) \tag{2.7}$$

where w_L^F and w_U^F are the wage and the unemployment benefit in the formal sector (subscripts L and U stand respectively for employed and unemployed). By combining Eqs. (2.6) and (2.7), we obtain:

$$I_L^F - I_U^F = \frac{w_L^F - w_U^F}{r + \delta + \theta^F q(\theta^F)}. \tag{2.8}$$

Combining Eqs. (2.8), (2.6) and (2.7) yields:

$$rI_L^F = \frac{\delta w_U^F + [r + \theta^F q(\theta^F)]w_L^F}{r + \delta + \theta^F q(\theta^F)} \tag{2.9}$$

$$rI_U^F = \frac{(r+\delta)w_U^F + \theta^F q(\theta^F)w_L^F}{r + \delta + \theta^F q(\theta^F)}. \tag{2.10}$$

The discounted lifetime utility of an employed (I_L^F) and unemployed worker (I_U^F) is thus a convex combination of unemployment benefits and wages. The weights are determined by the exit rates, which are different for the employed and unemployed workers. For firms with a filled job (subscript J) and a vacancy (subscript V), we have the following steady-state Bellman equations:

$$rI_J = y^F - w_L^F - \delta(I_J - I_V) \tag{2.11}$$

$$rI_V = -\gamma + q(\theta^F)(I_J - I_V) \tag{2.12}$$

where γ is the entry costs for the firm and y^F is the product of the match. Observe that, as it is usual in search-matching models (Pissarides, 2000), each firm employs only one worker. However, as shown by Cahuc and Wasmer (2001), under the assumption of a perfect capital market and constant returns to scale in all factors, the wage bargaining solution (see below) is identical whether one considers large firms that employ more than one worker or firms employing only one worker (as in the present model).

2.4. Job creation

Firms with vacancies enter in the labor market until their expected profits are equal to zero, i.e. $I_V=0$. From Eq. (2.12) and using $I_V=0$, the value of a job is equal to:

$$I_J = \frac{\gamma}{q(\theta^F)}. \tag{2.13}$$

Indeed, firms enter the labor market until the expected benefit I_J of a job is equal to its expected cost $\gamma/q(\theta^F)$ (remember that, in a Poisson process, the inverse of the exist rate $q(\theta^F)$ expresses the average duration of a vacant job). Finally, plugging Eq. (2.13) into Eq. (2.11) and using $I_V=0$, we obtain the following decreasing relation between labor market tightness and wages in equilibrium:

$$\frac{\gamma}{q(\theta^F)} = \frac{y^F - w_L^F}{r+\delta}. \tag{2.14}$$

In words, the value of a job is equal to the expected search cost, i.e. the cost per unit of time multiplied by the average duration of search for the firm. So, firms' job creation is endogenous and is determined by Eq. (2.14).

2.5. Wages

At each period, the total intertemporal surplus is shared through a generalized Nash-bargaining process between the firm and the worker. The total surplus is the sum of the surplus of the workers, $I_L^F - I_U^F$, and the surplus of the firms $I_J - I_V$. At each period, the wage is determined by:

$$w_L^F = \arg\max_{w_L^F} (I_L^F - I_U^F)^\beta (I_J - I_V)^{1-\beta} \tag{2.15}$$

where $0 \le \beta \le 1$ is the bargaining power of workers. First order condition gives:

$$\frac{\beta}{1-\beta}\left(\frac{\partial I_L^F}{\partial w_L^F} - \frac{\partial I_U^F}{\partial w_L^F}\right) I_J + (I_L^F - I_u^F)\frac{\partial I_J}{\partial w_L^F} = 0. \tag{2.16}$$

Since the wage is negotiated at each period, I_U does not depend on the current wage w_L^F and so $\frac{\partial I_U}{\partial w_L^F} = 0$. Since by

Eq. (2.6), $\frac{\partial I_L^F}{\partial w_L^F} = 1/(r+\delta)$, by Eq. (2.13), $I_J = \gamma/q(\theta^F)$ and by Eq. (2.11), $\frac{\partial I_J}{\partial w_L^F} = -1/(r+\delta)$, Eq. (2.16) can be written as:

$$I_L^F - I_U^F = \frac{\beta}{1-\beta}\frac{\gamma}{q(\theta^F)}. \quad (2.17)$$

Then, using Eqs. (2.8) and (2.14), we finally obtain the following wage:

$$w_L^F = (1-\beta)w_U^F + \beta(y^F + \gamma\theta^F). \quad (2.18)$$

This is the wage-setting curve (a relation between wages and the state of the labor market, here θ^F) that replaces, in search-matching models, the traditional labor-supply curve.

3. Steady-state equilibrium

In steady-state, flows in and out unemployment have to be equal and we obtain the following relationship:

$$L^F = \frac{\theta^F q(\theta^F)}{\delta + \theta^F q(\theta^F)}(N - L^I). \quad (3.1)$$

We assume that a worker in the informal sector cannot search directly a job in the formal sector but must first be unemployed in the formal sector. One way to justify this assumption is the fact that, in developing countries, a large fraction of jobs (at least for the uneducated) are found through word-of-mouth communication and social networks (see, e.g. Wahba and Zenou, 2005). So one has first to be in the formal sector to gather information about jobs. Another justification is that formal and informal sectors are usually not located in the same part of the city. So one has first to move to the location where formal jobs are, and then, while unemployed, searches for a formal job. In some sense, the informal sector plays a buffer role in the transitional stage of a search for a formal sector job.

Observe that this assumption is consistent with the way we wrote the matching function Eq. (2.2) where only the unemployed workers are looking for a job. If one could search while employed in the informal sector, then this should be reflected in the matching function. Observe also that, since Eq. (2.8) implies that the lifetime expected utility of being employed in the formal sector is strictly higher than that of being unemployed, workers employed in the formal sector never want to work in the informal sector. When they lose their job (at rate δ), they stay unemployed until they obtain a new formal job. So we assume that formal workers never look for an informal job. A way to justify this assumption is that the labor market in the informal sector is not that open since most people work with their friends and relative.

[7]We could, for example, assume that there is a cost of switching from the formal sector to the informal sector. Because of Eq. (3.2) below, this would imply that a "formal" worker would always be better off being unemployed than working in the informal sector.

To be more precise, the equilibrium mobility condition can thus be written as:

$$I_U^F = \int_0^{+\infty} w_L^I e^{-rt}\mathrm{d}t = \frac{w_L^I}{r}. \quad (3.2)$$

Using Eqs. (2.10) and (2.5), this equality is equivalent to:

$$\frac{(r+\delta)w_U^F + \theta^F q(\theta^F) w_L^F}{r+\delta+\theta^F q(\theta^F)} = F'(L^I). \quad (3.3)$$

Definition 1. A Harris–Todaro equilibrium with search externalities and bargained wages in the formal sector is a 7-tuple (w_L^{F*}, θ^{F*}, w_L^{I*}, L^{F*},U^{F*},V^{F*}, L^{I*}) such that Eqs. (2.18), (2.14), (2.5), (3.1), (2.1), (2.3) and (3.3) are satisfied.

Here is the way the equilibrium is calculated. The system is recursive. First, by combining Eqs. (2.18) and (2.14), we obtain a unique θ^{F*} that is only function of parameters and given by:

$$y^F - w_U^F = \frac{\gamma}{q(\theta^F)}\left[\frac{r+\delta+\beta\theta^F q(\theta^F)}{1-\beta}\right]. \quad (3.4)$$

Second, by combining Eqs. (2.18) and (3.3), we obtain:

$$\frac{[r+\delta+(1-\beta)\theta^F q(\theta^F)]w_U^F + \beta\theta^F q(\theta^F)(y^F+\gamma\theta^F)}{r+\delta+\theta^F q(\theta^F)} = F'(L^I) \quad (3.5)$$

which using θ^{F*} gives a unique L^{I*} as a function of parameters only. Furthermore, by plugging θ^{F*} and L^{I*} in

[7] On the contrary, informal workers search for a job in both sectors. We assume that they accept a formal job only if the expected lifetime utility of being unemployed is strictly higher than that of working in the informal sector. In case of equality, they always prefer to stay in the informal sector.

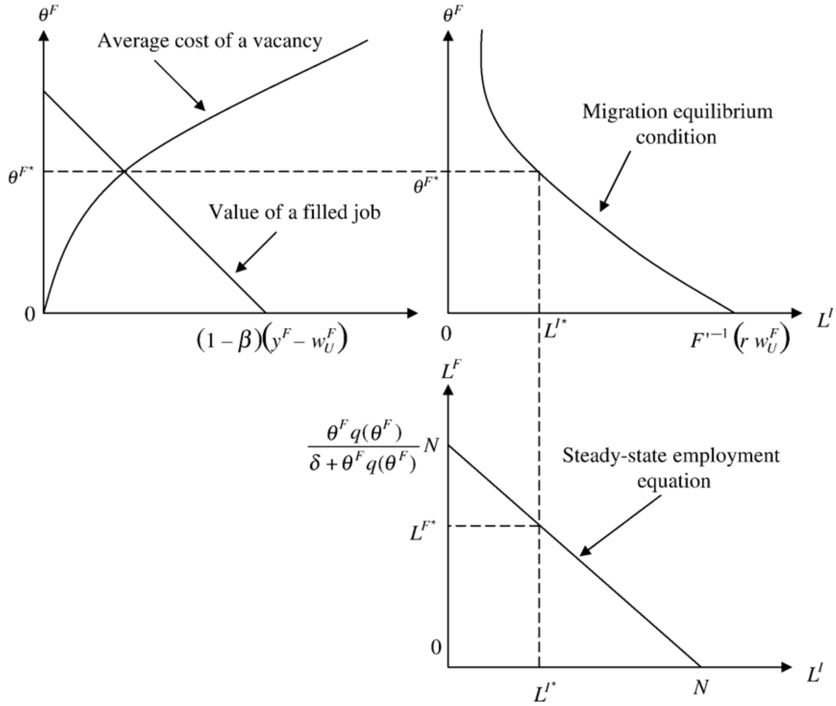

Fig. 1. Harris–Todaro equilibrium with search externalities.

Eq. (3.1), we obtain a unique L^{F*}. Fig. 1 illustrates the way the equilibrium is calculated.

Finally, by plugging L^{F*} and L^{I*} in Eqs. (2.5) and (2.1), we obtain respectively w_L^{I*} and U^{F*} and by plugging θ^{F*} in Eq. (2.18), we obtain w_L^{F*}. Also, using the values of θ^{F*} and U^{F*} in Eq. (2.3), we obtain the equilibrium number of vacancies in cities, V^{F*}.

There exists thus a unique steady-equilibrium and it is not efficient because of search externalities (as in Pissarides, 2000). So we would like now to consider different policies that aim at reducing unemployment. The policies that we consider are implemented in the formal sector.

4. Unemployment benefit policy

We consider a first simple policy where the government reduces the unemployment benefit w_U^F in the formal sector. In order to study the unemployment benefit policy, we first give some analytical results for the case when w_U^F is not financed and thus exogenously determined. Then, we extend the model to incorporate the government budget's constraint. Since the model becomes quite complicated, we resort to numerical simulations to analyze the unemployment benefit policy.

4.1. Decreasing unemployment benefits

Consider the model developed in the previous section. We have then the following result:[8]

Proposition 1. *In a Harris–Todaro model with search externalities and bargained wages in the formal sector, a decrease in unemployment benefits* w_U^F *leads to:*

(i) an increase in job creation in the formal sector θ^{F*}*;*

(ii) a decrease in employment in the informal sector L^{I*} *and an increase in the informal wage* w_L^I *if*

$$\frac{1}{\beta}\left[1+\frac{y^{\mathrm{F}}-w_{\mathrm{L}}^{\mathrm{F}}}{\gamma\theta^{\mathrm{F}}}\right]+\gamma\frac{\partial\theta^{\mathrm{F}}}{\partial w_{\mathrm{U}}^{\mathrm{F}}}<1 \tag{4.1}$$

holds.

(iii) an increase in employment in the formal sector L^{F*} *if* Eq. (4.1) *holds.*

When the government reduces the unemployment benefit, it has a direct positive effect on job creation in the formal sector θ^{F*}. Indeed, firms holding a vacant job are forward looking and thus there are more of them that enter

[8] The proofs of all propositions can be found in the Appendix.

the labor market because they anticipate that it will be less costly in terms of wages to hire a worker. Because θ^{F*} increases following a decrease in unemployment benefits, the equilibrium wage will not necessarily decrease because of the indirect effect of θ^{F*}. Indeed, since θ^{F*} increases, it becomes easier to find a job for workers and thus the latter can increase their wages during the bargaining process because their outside option is better. It is easy to verify that:

$$\frac{\partial w_L^F}{\partial w_U^F} = \underbrace{1-\beta}_{\text{direct negative effect}} + \beta\gamma \underbrace{\frac{\partial \theta^F}{\partial w_U^F}}_{\text{indirect positive effect}}. \tag{4.2}$$

Concerning the effect of w_U^F on both formal and informal employment, the intuition is as follows. There are again two effects. Remember that Eq. (3.5) was determining the mobility between the two sectors, and it was given by $rI_U^F = w_L^I$. So, when w_U^F decreases, there is a *direct positive (resp. negative) effect on* L^{I*} (resp. L^{F*}) since I_U^F, the lifetime expected value of being unemployed in the formal sector, decreases, and thus more individuals are willing to work in the informal sector. There is also an *indirect negative (resp. positive)* effect on L^{I*} (resp. L^{F*}) since the increase in job creation θ^{F*} yields a higher value of I_U^F it becomes easier to find a job. As a result, the net effect is ambiguous. However, if Eq. (4.1) holds, which expresses the fact that the indirect effect is much stronger than the direct effect, then L^{I*} decreases and L^{F*} increases following a decrease in the unemployment benefit. Of course, because wages in the informal sector are competitive and equal marginal productivity, an increase in the unemployment benefit will have the opposite effect on w_L^{I*}. The effect of w_U^F on the unemployment U^{F*} is ambiguous even if Eq. (4.1) holds because the effects go in the opposite directions. Observe that here, there is a possibility for a Todaro paradox, that is a decrease in unemployment benefit can increase both formal employment and unemployment if both conditions (4.1) and

$$-\frac{\partial L^F}{\partial w_U^F} > \frac{\partial L^I}{\partial w_U^F} \tag{4.3}$$

hold.

4.2. Steady-state equilibrium with a government budget constraint

We have now some intuition on the unemployment benefit policy. We would like to go further by introducing a government budget constraint. For that, we assume that the unemployment benefit w_U^F is financed by a tax t^F on firms. This means that when a firm hires a worker, its instantaneous profit is equal to: $y^F - w_L^F - t^F$. The government's budget constraint can be written as:

$$t^F L^F = w_U^F (N^F - L^F). \tag{4.4}$$

The fiscal policy is such that taxes are kept constant and the budget adjustment is realized through a decrease or increase in unemployment benefit taxes w_U^F. Using Eq. (2.1), this means that, for a constant value of t^F, the unemployment benefit level that balances the budget is given by:[9]

$$w_U^F = \frac{t^F L^F}{N - L^I - L^F}. \tag{4.5}$$

One can see that, for a given tax level t^F, a higher formal employment level or a higher informal employment level are associated with an increase in unemployment benefits. Indeed, when L^F or L^I increases, then less workers are unemployed and thus a higher w_U^F will balance the budget. The steady-state equation on flows is still given by Eq. (3.1). However, the free-entry condition (2.14) and the wage Eq. (2.18) are now given by:

$$\frac{\gamma}{q(\theta^F)} = \frac{y^F - t^F - w_L^F}{r+\delta}$$
$$w_L^F = (1-\beta)\left[\frac{t^F L^F}{N - L^I - L^F}\right] + \beta(y^F - t^F + \gamma\theta^F).$$

By combining these equations, we obtain the equilibrium job creation equation:

$$y^F - \left[\frac{t^F(N - L^I)}{N - L^I - L^F}\right] = \frac{\gamma}{q(\theta^F)}\left[\frac{r+\delta+\beta\theta^F q(\theta^F)}{1-\beta}\right]. \tag{4.6}$$

This is a new equation and it implies, in particular, that now θ^{F*} is a function L^F and L^I, which was not the case before (see Eq. (3.4)). Thus, Eqs. (4.6) and (3.1) cannot be solved independently. By using the implicit function theorem, we easily obtain that: $\frac{\partial \theta^F}{\partial L^F} < 0$.

Indeed, when L^F increases, more workers are employed in the formal sector and, given that the taxes are held constant, the unemployment benefit w_U^F has to increase to balance the budget (see Eq. (4.5)). Because unemployment benefits are higher, wages increase, and thus less firms enter the labor market. Therefore, less jobs are created and thus θ^F decreases. Similarly, one can see

[9] An alternative fiscal policy would be that the unemployment benefits are kept constant so that taxes are adjusted to balance the budget. In this case, it is well-known that multiple equilibria emerge (Rocheteau, 1999). Since the focus of this paper is on policy issues and not on multiple equilibria, we focus on the other fiscal policy. Of course, these two fiscal policies are strictly equivalent but in one of them (the one we propose here), workers and firms can coordinate on only one equilibrium whereas this is not the case in the other policy.

that $\frac{\partial \theta^F}{\partial L^I} < 0$ for exactly the same reason since higher L^I implies that w_U^F has to increase to balance the budget.

Finally, using Eq. (4.5) and the value of w_L^F, the equilibrium mobility condition (3.3) is now given by:

$$\frac{\left(\frac{t^F L^F}{N-L^I-L^F}\right)[r+\delta+(1-\beta)\theta^F q(\theta^F)] + \beta\theta^F q(\theta^F)(y^F - t^F + \gamma\theta^F)}{r+\delta+\theta^F q(\theta^F)} = F'(L^I). \tag{4.7}$$

Compared to Eq. (3.5), L^I is now a function of θ^F and L^F and not only of θ^F. In particular, for a given θ^F, an increase in L^F leads to a decrease in L^I. Indeed, for a given level of job creation θ^F, when there is a higher level of formal employment L^F, the unemployment benefit w_U^F has to increase to balance the budget. This, in turn, increases the expected utility to be unemployed in the formal sector, which induces more workers to work in the formal sector and thus reduces employment in the informal sector L^I.

The steady-state equilibrium is 3-tuple (θ^{F*}, L^{F*}, L^{I*}) such that Eqs. (3.1), (4.6), and (4.7) are satisfied. The model is much more complicated since each equation involves the three endogenous variables θ^{F*}, L^{F*}, and L^{I*}. This is why we resort now to numerical simulations.

4.3. Unemployment policy with a government budget constraint

We calibrate the model to obtain reasonable values of the employment and unemployment rates in the formal sector, the size of the informal sector, and the job creation rate. As it is usual, we use the following Cobb–Douglas function for the matching function:

$$M(U^F, V^F) = K(U^F)^{0.5}(V^F)^{0.5}$$

where $K>0$ is a scale parameter of the matching function. This implies that $q(\theta^F)=K(\theta^F)^{-0.5}$, $\theta^F\ q(\theta^F)=K(\theta^F)^{0.5}$ and, the elasticity of the matching rate (defined as $\eta(\theta^F)= -q'(\theta^F)\ \theta^F/q(\theta^F)$) is equal to 0.5. The production function

Table 1
Parameter values

$N=100$ Total population	$r=0.05$ Pure discount rate
$\delta=1$ Job-specific shock arrival rate	$\beta=0.5$ Workers' share of surplus
$A=60$, $a=0.5$ Parameters of the production function	$\eta(\theta)=0.5$ Search elasticity of matching
$y^F=10$ General productivity	$\gamma=2$ Cost of a vacant job
$t^F=2$ Lump-sum tax	$K=2$ Parameter of the matching function

Table 2
Steady-state equilibrium

L^{F*}	49.39
L^{I*}	25.67
θ^{F*}	0.98
U^{F*}	24.94
u^{F*}	0.34
V^{F*}	24.45
v^{F*}	0.33
w_L^{F*}	6.96
w_U^{F*}	3.96
w^{I*}	5.92
w_L^{F*}/w^{I*}	1.18
$I_L^{F*}-I_U^{F*}$	1.47
I_J^*	1.09

in the informal sector is also a Cobb–Douglas function and it is defined as:

$$F(L^I) = A(L^I)^a$$

where $0<a<1$ and $A>0$ is a scale parameter of the production function. The values of the parameters (in yearly terms) are the following. The total population N is normalized to 100. The relative bargaining power of workers is equal to $\eta(\theta)$, i.e. $\beta=\eta(\theta)=0.5$ and the parameter of the matching function is $K=2$. The lump-sum tax on firms' profits t^F has a value of 2 and the costs of maintaining a vacancy γ are equal to 2 per unit of time while the formal productivity y^F is 10. The discount rate is $r=0.05$, whereas the job-destruction rate is $\delta=1$, which means that, on average, a job is destroyed every year. Table 1 summarizes these different values.

Let us calculate the steady-state equilibrium using the parameter values given in Table 1. The numerical results of the steady-state equilibrium are displayed in Table 2.

We have calibrated the model to obtain an economy where roughly 50% of workers are employed in the formal sector and 25% in the informal sector. The rest of the workers are unemployed. So the unemployment rate in the economy U^{F*} is nearly 25% whereas the one in the formal sector, i.e. u^{F*}, as measured by the number of unemployed workers over the active population in the formal sector (and not in the entire population), is 34%. These features are in accordance with what we observe in cities of the developing world, like for example Mexico (see Gong and van Soest, 2002). Moreover, nearly 25% of formal jobs are vacant and the number of vacant job per formal worker (employed and unemployed) is 33%. The wage difference between the formal and informal sectors is 1.18. Finally, for a tax level t^F of 2, the equilibrium unemployment benefit w_U^{F*} that balances the budget is equal to 3.96, which is lower than both the

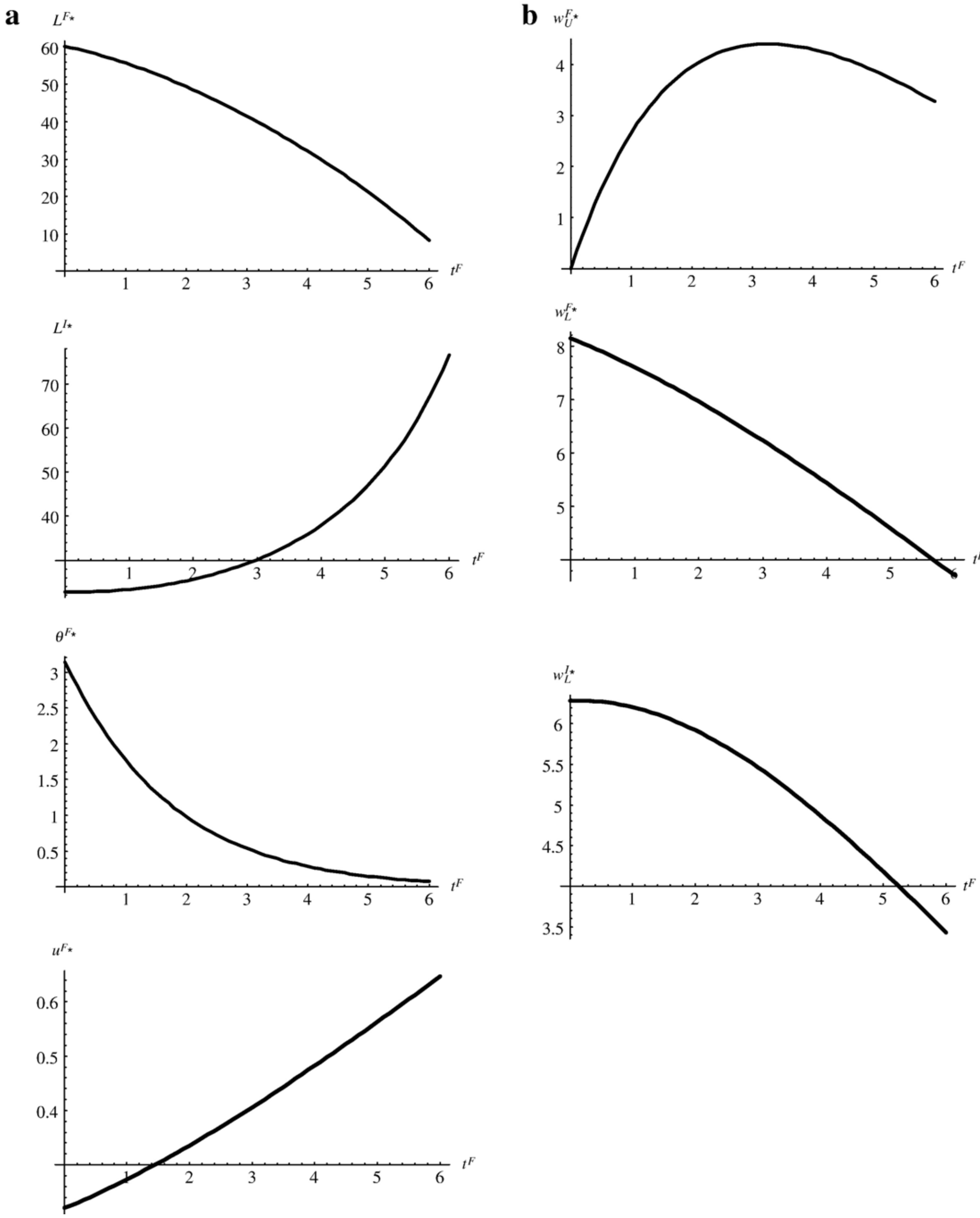

Fig. 2. a. Unemployment benefit policy: General equilibrium effects on employment and unemployment b. Unemployment benefit policy: General equilibrium effects on wages.

formal and informal wages. So when workers want to leave the informal sector, they trade off an income loss of $w^{I*} - w_U^{F*} = 1.96$ in the short run with a possible income gain of $w_L^{F*} - w^{I*} = 1.04$ in the long run.

Let us now study an unemployment benefit policy with general equilibrium effects. For that, we evaluate the impact of a change in the tax t^F on the different equilibrium values. Fig. 2a and b display the results. If we consider that w_U^{F*} and t^F are positively correlated, then the results in Proposition 1 are similar, given that Eq. (4.1) holds. Indeed, when firms are more taxed, there is less job creation (θ^{F*} decreases) and thus formal employment L^{F*} also decreases. This implies that formal employment is less attractive and thus more workers stay in the informal sector, which, in turn, increases L^{I*}. The effects are however more complex here because of the government budget constraint Eq. (4.4), which implies that w_U^{F*} and t^F are not always positively related. Indeed, looking at Fig. 2b, one sees that there is a non-monotonic relationship between w_U^{F*} and t^F. For low levels of t^F, formal unemployment is quite low so the level of w_U^{F*} that needs to balance the budget has to increase following an increase in t^F. On the other hand, for high levels of taxes (say $t^F=4$), unemployment is already quite high and more and more unemployed workers need to be financed. As a result, increasing even more taxes reduces the level of unemployment benefit that balances the budget. The non-monotonic relationship between w_U^{F*} and t^F reflects the trade off between higher proceeds from higher taxes and more unemployed workers to be financed as a result of the increased unemployment rate in the formal sector (see the lowest panel in Fig. 2a).

Let us now focus on wages (Fig. 2b). Higher taxes lead to lower formal and informal wages, w_L^{F*} and w_L^{I*}, respectively. The first relationship is not obvious since, as showed by Eq. (4.2), there are two opposite effects: (*i*) higher t^F implies an increase in w_U^F, and thus higher w_L^{F*}; (*ii*) higher t^F decreases job creation and thus the rate at which workers find a job, which decreases w_L^{F*}. The second relationship is easier to understand since wages in the informal sector are paid at workers' marginal product. So when taxes increase, L^{I*} increases and, since $F''(L^I)<0$, informal wages decrease.

5. Subsidizing firms' entry cost

Another interesting policy to be considered is to reduce the entry cost γ for firms in the formal sector. As in the previous section, we first implement this policy when there is no budget constraint, and then when the policy is financed by taxes on firms.

5.1. Decreasing firms' entry cost

Define $\eta \equiv -\frac{\partial\theta^F}{\partial\gamma}\frac{\gamma}{\theta^F}>0$, which is entry-cost elasticity of job creation. We have the following result.

Proposition 2. *In a Harris–Todaro model with search externalities and bargained wages in the formal sector, a decrease in γ, the firms' entry cost in the formal sector, leads to:*

- (i) *an increase in job creation in the formal sector θ^{F*};*
- (ii) *a decrease in employment in the informal sector L^{I*} and an increase in the informal wage w_L^{I*} if $\eta>1$.*
- (iii) *an increase in employment in the formal sector L^{F*} if $\eta>1$.*

As with the unemployment benefit, when the government reduces firms' entry cost in the formal sector, more firms holding a vacant job enter the labor market and therefore more jobs are created (θ^{F*} increases). The effect on the *equilibrium wage* is however ambiguous because of the indirect negative effect of θ^{F*}. Furthermore, when γ increases, contrary to the unemployment benefit policy, there is no direct effect on L^{I*} or L^{F*} since I_U^{F*}, the lifetime expected value of being unemployed in the formal sector, is not directly affected by γ. There is however an indirect effect through the increase of the job creation θ^{F*}, which affects both the rate at which workers find a job in the formal sector and their wage in case of a match. The net effect is ambiguous because, as we have seen above, the effect on wages is ambiguous.

If however $\eta>1$, which means that (in absolute value) the effect of γ on θ^{F*} is important, then quite naturally, a decrease in the entry cost increases employment in the formal sector and decreases it in the informal sector.

Overall, the effect of γ on the different equilibrium values is similar to that of w_U^{F*}, with the difference that γ does not affect directly I_U^{F*}, the mobility decision between the two sectors, but indirectly through job creation.

5.2. Steady-state equilibrium with a government budget constraint and policy implications

We now assume that the government subsidizes the entry cost of all firms, so that firms pay $(1-\sigma^F)\gamma$ instead of γ, where $0<\sigma^F<1$ is the ad valorem subsidy. Both the unemployment benefit w_U^F and the entry-cost subsidy are now financed by a tax t^F on firms.

The local government's budget constraint can thus be written as:

$$t^F L^F = w_U^F(N-L^F-L^I)+\sigma^F\gamma V^F.$$

Since $\theta^F=V^F/U^F=V^F/(N-L^F-L^I)$, we obtain:

$$w_U^F=\frac{t^F L^F}{N-L^F-L^I}-\sigma^F\gamma\theta^F. \tag{5.1}$$

The model is exactly as before but we now replace w_U^F by Eq. (5.1) and γ by $(1-\sigma^F)\gamma$. When $\sigma^F=0$, we are obviously back to the previous model described in Section 4.2. Using the same parameters as in Table 1, let us now see how an increase in σ^F affects the equilibrium outcomes. Fig. 3 reports the results.[10] They are similar to

[10] To save some space, we do not report the results on wages and unemployment, but we comment them in the text.

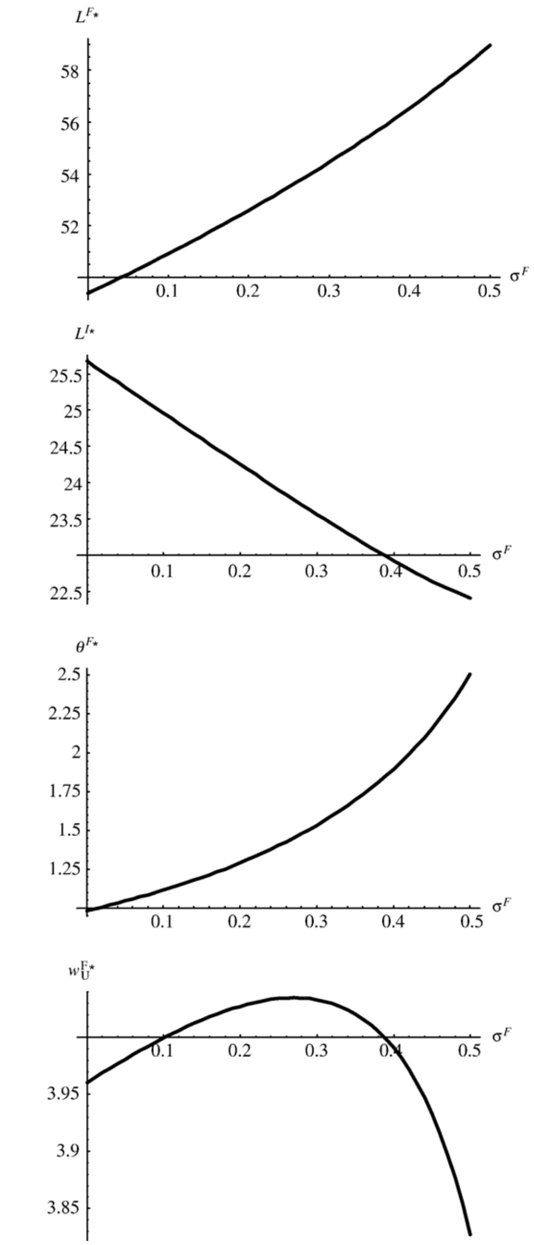

Fig. 3. Subsidizing firms' entry costs: General equilibrium effects.

that of Proposition 2. Indeed, increasing the subsidy σ^{F} induces more firms to enter in the labor market, which increases both $\theta^{\mathrm{F}*}$ and $L^{\mathrm{F}*}$. This, in turn, reduces informal employment since more workers are willing to work in the informal sector. As in the unemployment benefit policy and for the same reason, we also find that there is a non-monotonic relationship between σ^{F} and $w_{\mathrm{U}}^{\mathrm{F}*}$. The effects on wages are as expected (results not reported here). Increasing σ^{F} increases the formal wage $w_{\mathrm{L}}^{\mathrm{F}*}$ because there is more job creation so that workers leave unemployment at a faster rate. Observe that the negative effect of unemployment benefit on formal wages is here quite weak and becomes positive when roughly 30% of the entry cost is subsidized (see the lowest panel of Fig. 3). The positive effect of σ^{F} on the informal wage $w_{\mathrm{L}}^{\mathrm{F}*}$ is due to the fact that $L^{\mathrm{I}*}$ decreases following a rise in the subsidy. Interestingly, all these effects are not negligible. For example, an increase of the subsidy from zero to 50% reduces the size of the informal sector by nearly 15% (from 25.67 to 22.41), decreases the unemployment rate by more than 41% (from 0.34 to 0.24), increases formal employment by 19% (from 49.39 to 58.97), and increases the number of vacancies posted per formal worker by more than 80% (from 0.33 to 0.60).

6. Employment/wage subsidies

We now consider a policy where employment is subsidized at a rate $S^{\mathrm{F}}>0$ per job and the employment subsidy S^{F} is paid to firms throughout the duration of the job. In that case, Eq. (2.11) is changed and becomes

$$rI_{\mathrm{J}} = y^{\mathrm{F}} - w_{\mathrm{L}}^{\mathrm{F}} + S^{\mathrm{F}} - \delta(I_{\mathrm{J}} - I_{\mathrm{V}}).$$

Observe that it is the firm who receives the subsidy and not the worker so that

$$rI_{\mathrm{L}} = w_{\mathrm{L}}^{\mathrm{F}} - \delta(I_{\mathrm{L}} - I_{\mathrm{U}}).$$

It is clear here that S^{F} can be interpreted as either an employment or a wage subsidy. Let us now solve the model with the subsidy. The value of a job is still given by

$$I_{\mathrm{J}} = \frac{\gamma}{q(\theta^{\mathrm{F}})} \tag{6.1}$$

but the job creation equation is modified and now given by:

$$\frac{\gamma}{q(\theta^{\mathrm{F}})} = \frac{y^{\mathrm{F}} - w_{\mathrm{L}}^{\mathrm{F}} + S^{\mathrm{F}}}{r+\delta}. \tag{6.2}$$

It is easy to verify that the wage is now equal to:

$$w_{\mathrm{L}}^{\mathrm{F}} = (1-\beta)w_{\mathrm{U}}^{\mathrm{F}} + \beta(y^{\mathrm{F}} + S^{\mathrm{F}} + \gamma\theta^{\mathrm{F}}). \tag{6.3}$$

By combining these two last equations, we have the following job creation equation:

$$y^{\mathrm{F}} - w_{\mathrm{U}}^{\mathrm{F}} + S^{\mathrm{F}} = \frac{\gamma}{q(\theta^{\mathrm{F}})}\left[\frac{r+\delta+\beta\theta^{\mathrm{F}}q(\theta^{\mathrm{F}})}{1-\beta}\right] \tag{6.4}$$

which implicitly determines θ^{F}. Furthermore, L^{I} is given by:

$$\frac{[r+\delta+(1-\beta)\theta^{\mathrm{F}}q(\theta^{\mathrm{F}})]w_{\mathrm{U}}^{\mathrm{F}}+\theta^{\mathrm{F}}q(\theta^{\mathrm{F}})\beta(y^{\mathrm{F}}+S^{\mathrm{F}}+\gamma\theta^{\mathrm{F}})}{r+\delta+\theta^{\mathrm{F}}q(\theta^{\mathrm{F}})}=F'(L^{\mathrm{I}}). \tag{6.5}$$

Finally, the employment L^{F} and unemployment U^{F} in the formal sector are defined as before by Eqs. (3.1) and (2.1), respectively.

As before, let us examine the effects of increasing the employment/wage subsidy S^{F} without and with the government budget constraint.

6.1. Increasing employment subsidies

We have the following result:

Proposition 3. *In a Harris–Todaro model with search externalities and bargained wages in the formal sector, an increase in the employment/wage subsidy S^F in the formal sector, leads to:*

(i) an increase in job creation in the formal sector θ^{F*};
(ii) a decrease in employment in the informal sector L^{I*} and an increase in the informal wage w_L^I.
(iii) a increase in employment in the formal sector L^{F*}.
(iv) an increase in wages w_L^{F*}.

The employment/wage subsidy policy is quite different from the previous ones. First, it has a direct positive effect on job creation since forward-looking firms are more willing to create new jobs because the cost of hiring a worker is lower. Second, contrary to the previous policies, the effect of S^{F} on wages $w_{\mathrm{L}}^{\mathrm{F}*}$ is unambiguously positive. Indeed, when S^{F} increases, there is a direct positive effect on wages since workers have a better outside option given that the value of a job is less costly for a firm. There is also an indirect positive effect since increasing S^{F} increases job creation $\theta^{\mathrm{F}*}$, which means that workers find more easily a job, and thus their outside option in the bargaining process is increased. Third, since wages and job creation increase following an increase in the subsidy, both the surplus $I_{\mathrm{L}}^{\mathrm{F}}-I_{\mathrm{U}}^{\mathrm{F}}$ of being employed and $\theta^{\mathrm{F}*}\ q(\theta^{\mathrm{F}*})$ the rate at which workers find a job increase, which imply that $I_{\mathrm{U}}^{\mathrm{F}}$, the lifetime expected value of being unemployed in the formal sector, increases. This means that workers are more willing to work in the formal sector than in the informal sector, and thus $L^{\mathrm{F}*}$ increases while $L^{\mathrm{I}*}$ decreases. Finally, the effect of S^{F} on the unemployment $U^{\mathrm{F}*}$ is ambiguous because of the opposite effects on $L^{\mathrm{F}*}$ and $L^{\mathrm{I}*}$.

6.2. Steady-state equilibrium with a government budget constraint and policy implications

Let us write the (local) government's budget constraint. It is given by:

$$t^{\mathrm{F}}L^{\mathrm{F}}=w_{\mathrm{U}}^{\mathrm{F}}(N-L^{\mathrm{F}}-L^{\mathrm{I}})+S^{\mathrm{F}}L^{\mathrm{F}}.$$

Observe that, as in the previous sections, we have written this budget constraint at the steady-state, which means that we do not take into account the fact that the government has paid the subsidy for the duration of each job. We just look at what happens at the steady-state and we know at that date that L^{F} workers are employed and receive S^{F}. We obtain:

$$w_{\mathrm{U}}^{\mathrm{F}}=\frac{(t^{\mathrm{F}}-S^{\mathrm{F}})L^{\mathrm{F}}}{N-L^{\mathrm{F}}-L^{\mathrm{I}}}. \tag{6.6}$$

The steady-state equilibrium is as follows. The steady-state equation on flows is still given by Eq. (3.1). The wage Eq. (6.3) and the job-creation Eq. (6.4) can now be written as:

$$w_{\mathrm{L}}^{\mathrm{F}}=(1-\beta)\frac{(t^{\mathrm{F}}-S^{\mathrm{F}})L^{\mathrm{F}}}{N-L^{\mathrm{F}}-L^{\mathrm{I}}}+\beta(y^{\mathrm{F}}-t^{\mathrm{F}}+S^{\mathrm{F}}+\gamma\theta^{\mathrm{F}}). \tag{6.7}$$

$$y^{\mathrm{F}}-\left[\frac{t^{\mathrm{F}}(N-L^{\mathrm{I}})-S^{\mathrm{F}}L^{\mathrm{F}}}{N-L^{\mathrm{F}}-L^{\mathrm{I}}}\right]=\frac{\gamma}{q(\theta^{\mathrm{F}})}\left[\frac{r+\delta+\beta\theta^{\mathrm{F}}q(\theta^{\mathrm{F}})}{1-\beta}\right]. \tag{6.8}$$

Finally, the equilibrium mobility condition is given by Eq. (6.5), where $w_{\mathrm{U}}^{\mathrm{F}}$ is replaced by its value in Eq. (6.6) and y^{F} by $y^{\mathrm{F}}-t^{\mathrm{F}}$.

Using the same parameters as in Table 1, let us now see how an increase in S^{F} affects the equilibrium outcomes. Fig. 4 displays the results.[11] Compared to Proposition 3, some effects are the same (i.e. the positive impact of S^{F} on $\theta^{\mathrm{F}*}$ and $w_{\mathrm{L}}^{\mathrm{F}*}$), some are the opposite (the impact of S^{F} on $L^{\mathrm{I}*}$ and $w_{\mathrm{L}}^{\mathrm{I}*}$), and the impact of the subsidy on $L^{\mathrm{F}*}$ is now non-monotonic. Let us explain these different features. Whether $w_{\mathrm{U}}^{\mathrm{F}}$ is exogenous or financed by taxes, higher S^{F} leads to more job creation $\theta^{\mathrm{F}*}$ because the value of a job is higher and higher formal wages $w_{\mathrm{L}}^{\mathrm{F}*}$. However, for the impact on formal

[11] Again, to save some space, we do not report the results on wages and unemployment, but we comment them in the text.

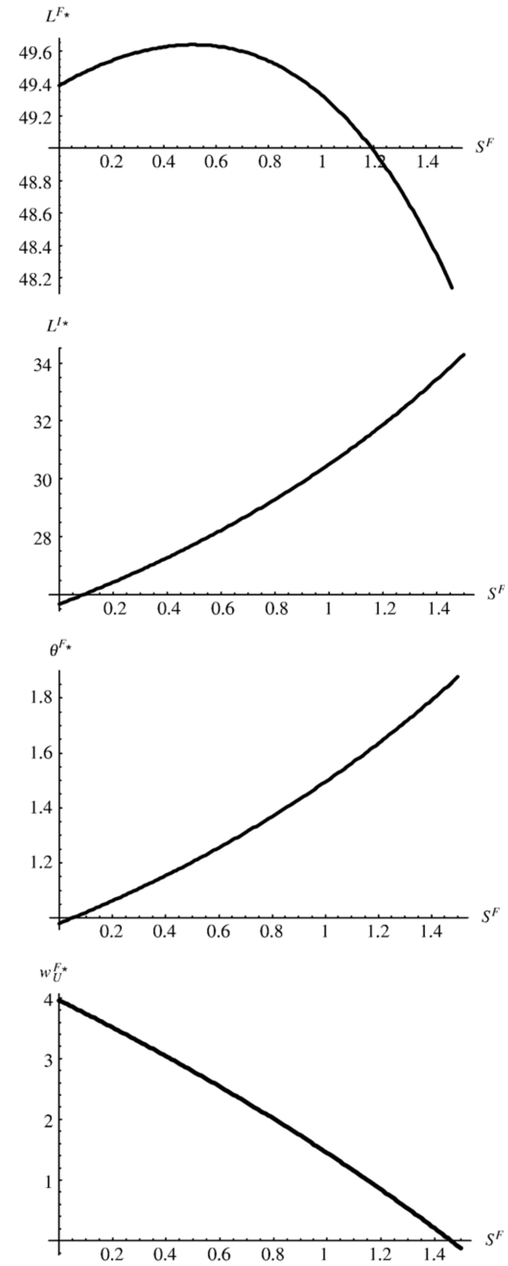

Fig. 4. Subsidizing employment: General equilibrium effects.

employment L^{F*}, this is not true. When w_U^F is exogenous, because θ^{F*} increases, L^{F*} also increases. When w_U^F becomes endogenous, this is true for low values of the subsidy S^F because t^F, who is fixed to 2, is much higher (Eq. (6.6)). When S^F becomes higher, even if more formal jobs are created, the policy becomes much more costly and the gap between t^F and S^F is reduced. Since formal wages also increase, the net effect becomes negative. In fact, the increase of L^{I*} is over a small range of values for S^F, between 0 and 0.6, while after this last value, it always decreases. The effects are quite different since increasing S^F from 0 to 0.6 has nearly no impact on L^{F*} (which increases from 49.39 to 49.63) while the same increase of S^F from 2 to 2.6 decreases L^{F*} from 45.53 to 39.13, which is a little bit more than 16%. The effect on informal employment is now easy to understand. When S^F increases, informal employment is reduced when w_U^F is exogenous. But, when it is financed by taxes, as we have seen, formal employment decreases quite rapidly so it becomes less attractive to work in the informal sector and thus both L^{I*} and w_L^{I*} decrease. Finally, observe that we have here a Todaro paradox for a sufficiently high level of the subsidy. Indeed, in the top panel of Fig. 4, one sees that, after $S^F=0.6$, an increase in S^F reduces employment in the formal sector while it reduces the unemployment rate in the formal sector (results not reported here).

7. Hiring subsidies

We finally consider a different policy that consists in giving to firms a hiring subsidy $H^F>0$ when a worker is hired. In that case, Eq. (2.12) that gives the value of a vacant job is now equal to:

$$rI_V = -\gamma + q(\theta^F)(I_J + H^F - I_V). \tag{7.1}$$

Contrary to the previous policy of employment/wage subsidy, here the hiring subsidy H^F is not paid to firms throughout the duration of the job but only once when they hire a new worker. So after the worker is hired, the benefit to the firm from continuing of hiring the worker is only I_J and not I_J+H^F, since no further subsidies are received. As a result, I_J is still given by Eq. (2.11) and the value of a job following the free-entry condition $I_V=0$ is now equal to:

$$I_J = \frac{\gamma}{q(\theta^F)} - H^F. \tag{7.2}$$

Using this value, we obtain the following decreasing relation between labor market tightness and wages in equilibrium:

$$\frac{\gamma}{q(\theta^F)} - H^F = \frac{y^F - w_L^F}{r+\delta}. \tag{7.3}$$

The wage will also be modified since the value of a filled job has changed. The wage solution is now given by:

$$w_L^F = \arg\max_{w_L^F} (I_L^F - I_U^F)^{\beta}(I_J + H^F - I_V)^{1-\beta}.$$

Solving this program, we obtain:

$$w_L^F = (1-\beta)w_U^F + \beta(y^F + \gamma\theta^F) \tag{7.4}$$

which is Eq. (2.18), that is the wage without the hiring subsidy policy. This is not surprising because the subsidy has already been received by the firm before it negotiates with the worker.

Plugging this wage into Eq. (7.3) gives the equilibrium job creation θ^F, which is implicitly defined as:

$$y^F - w_U^F + H^F = \frac{\gamma}{q(\theta^F)}\left[\frac{r+\delta+\beta\theta^F q(\theta^F)}{1-\beta}\right]. \tag{7.5}$$

The other equilibrium values are determined by exactly the same equations as in the case with no policy, i.e. L^I, L^F, and U^F are defined by Eqs.), (3.1), and (2.1).

7.1. Increasing hiring subsidies

Let us analyze the properties of the model when both the unemployment benefit and the subsidies are not financed. We have the following result:

Proposition 4. *In a Harris–Todaro model with search externalities and bargained wages in the formal sector, an increase in the hiring subsidy H^F in the formal sector, leads to:*

- *(i) an increase in job creation in the formal sector θ^{F*};*
- *(ii) a decrease in employment in the informal sector L^{I*} and an increase in the informal wage w_L^{I*}.*
- *(iii) an increase in employment in the formal sector L^{F*}.*
- *(iv) an increase in wages in the formal sector w_L^{F*}.*

These results are comparable to that of Proposition 3 where an employment/wage subsidy policy was implemented. Indeed, increasing the hiring subsidy induces firms to hire more workers, which reduces the employment in the informal sector because the latter is less attractive. Now the new aspect here is that this policy has no direct effect on wages because the hiring subsidy is paid only once and thus cannot be used in the negotiation while the employment/wage subsidy policy is paid to firms throughout the duration of the job. It has however an indirect positive effect on wage through job creation θ^{F*}.

7.2. Steady-state equilibrium with a government budget constraint and policy implications

Let us write the (local) government's budget constraint. It is given by:

$$t^F L^F = w_U^F(N - L^F - L^I) + H^F\delta L^F.$$

Indeed, the hiring subsidy H^F is only paid to new employed workers, whose total number in steady-state is:

$$\theta^F q(\theta^F)U^F = \delta L^F.$$

Solving in w_U^F, we obtain:

$$w_U^F = \frac{(t^F - \delta H^F)L^F}{N - L^F - L^I}. \tag{7.6}$$

The equilibrium values (w_L^{F*}, θ^{F*}, L^{I*}) are now defined by the following equations:

$$w_L^{F*} = (1-\beta)\frac{(t^F - \delta H^F)L^F}{N - L^F - L^I} + \beta(y^F - t^F + \gamma\theta^F) \tag{7.7}$$

$$y^F - \left[\frac{t^F(N - L^I) - H^F\delta L^F}{N - L^F - L^I}\right] = \frac{\gamma}{q(\theta^F)}\left[\frac{r+\delta+\beta\theta^F q(\theta^F)}{1-\beta}\right] - \frac{H^F(r+\delta)}{1-\beta} \tag{7.8}$$

$$\frac{\left[\frac{(t^F-\delta H^F)L^F}{N-L^F-L^I}\right][r+\delta+(1-\beta)\theta^F q(\theta^F)] + \beta\theta^F q(\theta^F)[y^F - t^F + \gamma\theta^F]}{r+\delta+\theta^F q(\theta^F)} = F'(L^I) \tag{7.9}$$

while L^F is still given by Eq. (3.1). Using the same parameters as in Table 1, let us now see how an increase in H^F affects the equilibrium outcomes. Fig. 5 displays the results.[12]

Compared to Proposition 4, we have nearly the same results, but the effects are much stronger. Take for example the positive impact of the subsidy H^F on the job creation rate θ^F. In the exogenous unemployment benefit case, there were only one effect: increasing H^F increased the lifetime expected value of having a vacancy I_V (see Eq. (7.1)) and thus more firms were entering the labor market. In the endogenous unemployment benefit case, there is an additional indirect positive effect that goes through w_U^{F*}.

Indeed, when H^F increases, not only I_V increases as before, but the unemployment benefit has to decrease to balance the budget (see the lowest panel of Fig. 5). Since w_U^{F*} decreases, the formal wage also decreases and the lifetime expected value of a filled job I_J increases. The effects are thus much stronger with a financed policy since both I_V and I_J increase. For example, in the latter case, if H^F increases from 0 to 1, then θ^{F*} increases by more than 100% (from 0.98 to 2.09). If we do the same

[12] Again, to save some space, we do not report the results on wages and unemployment, but we comment them in the text.

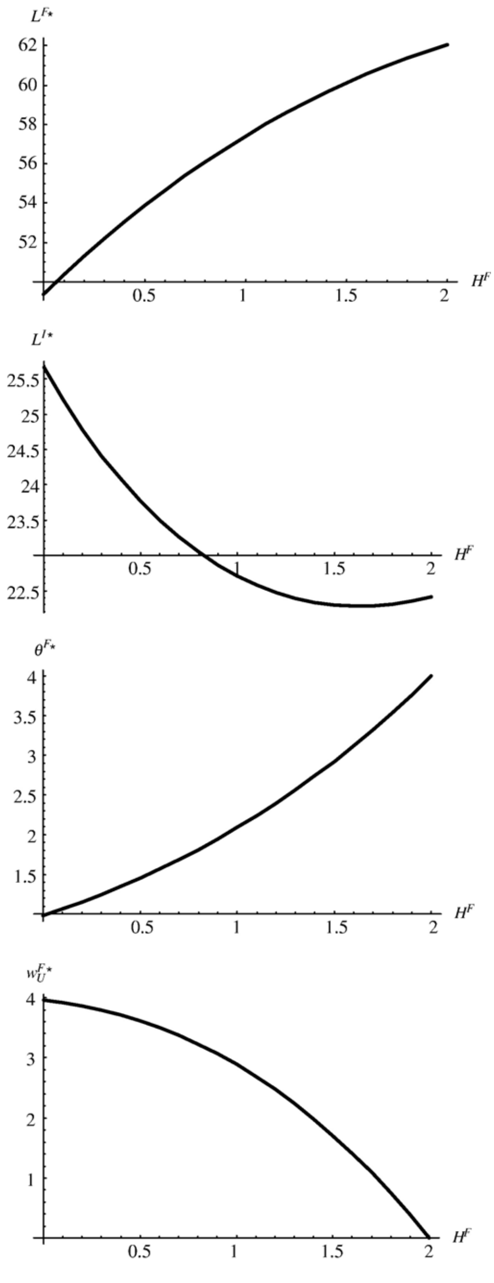

Fig. 5. Subsidizing hiring: General equilibrium effects.

exercise for the no-financing case where w_U^F is exogenously fixed at 3.96 (as in Table 2), then θ^{F*} "only" increases by 21.56% (from 1.67 to 2.03). Furthermore, as in Proposition 4, we still have the positive impact of H^F on L^{F*} and on w_L^{F*} and the resulting decreasing impact on L^{I*}. The impact on unemployment can be signed here and it is negative since higher subsidy creates formal jobs and reduces formal employment, which in turn decreases the unemployment rate both in the economy and in the formal sector. Finally, observe that in the employment subsidy policy, an increase in the subsidy S^F leads to an increase in the size of the informal sector (Fig. 4) while in the hiring subsidy policy, we have exactly the opposite result (Fig. 5). If one looks at the lowest panel of Figs. 4 and 5, one sees that, for the same increase of the subsidy S^F or H^F, say from 0 to 0.8, the reduction in the equilibrium unemployment benefit w_U^{F*} is much starker in the employment subsidy policy (from 3.96 to 2) than in the hiring subsidy policy (from 3.96 to 3.23). As a result, since the mobility decision from the informal to the formal sector is based on the expected lifetime utility I_U, defined by Eq. (2.10), then when the subsidy increases, w_U^{F*} decreases but w_L^{F*} increases. As seen above, the negative effect on the unemployment benefit is so strong in the employment subsidy policy that this negative effect dominates the positive effect on wages and thus increasing S^F reduces I_U, which deters mobility and thus increases the size of the informal sector L^{I*}. On the contrary, for the hiring subsidy policy, the negative effect on w_U^{F*} is not that strong and thus we end up with a negative effect on L^{I*} when increasing H^F.

8. Concluding remarks

In this paper, we consider different policies in a dual labor market where the formal sector is characterized by search frictions and wage bargaining while the informal sector is competitive. We show that there exists a unique steady-state equilibrium in this dual economy. We then consider different policies financed by a tax on firms' profits. We find that reducing the unemployment benefit or the firms' entry cost in the formal sector induces higher job creation and formal employment, reduces the size of the informal sector but has an ambiguous effect on wages. We also find that an employment/wage subsidy policy and a hiring subsidy policy have different effects. In particular, the former policy increases the size of the informal sector while the latter decreases it. Interestingly, in all policies, a Todaro paradox can exist under some condition of the parameters. This means that an increase or decrease of a policy variable can lead to an increase in the equilibrium values of both formal employment and unemployment.

We believe that this paper gives some answers to important questions about mobility between formal and informal sectors in developing countries. In particular, even if the informal sector is unregulated and cannot be directly targeted by a government's policy, the latter can indirectly affect the wage, the employment and thus the size of the informal sector. Thus, any policy implemented, especially in cities where the informal sector is large, should take into account not only the direct effect on the formal sector, but also the induced effect on the informal sector.

Appendix A. Proof of Propositions

Proof of Proposition 1. Effect of w_U^F on θ^F

By differentiating Eq. (3.4), it is easy to verify that

$$\frac{\partial\theta^F}{\partial w_U^F}=\frac{(1-\beta)q(\theta^F)}{(1-\beta)(y^F-w_U^F)q'(\theta^F)-\beta\gamma\frac{\partial[\theta^F q(\theta^F)]}{\partial\theta^F}}<0. \tag{9.1}$$

Effect of w_U^F on L^I

By differentiating Eq. (3.5), we have:

$$\frac{\partial L^I}{\partial\theta^F}=\frac{\frac{\partial[\theta^F q(\theta^F)]}{\partial\theta^F}\left[(1-\beta)w_U^F+\beta(y^F+\gamma\theta^F)-\frac{F'(L^I)}{r}\right]+\beta\gamma\theta^F q(\theta^F)}{F''(L^I)[r+\delta+\theta^F q(\theta^F)]}r$$

Since $w_L^F=(1-\beta)\,w_U^F+\beta\,(y^F+\gamma\theta^F)$, and by assumption $w_L^F>w^I$, then

$$\frac{\partial L^I}{\partial\theta^F}=\frac{\frac{\partial[\theta^F q(\theta^F)]}{\partial\theta^F}\left[w_L^F-\frac{F'(L^I)}{r}\right]+\beta\gamma\theta^F q(\theta^F)}{F''(L^I)[r+\delta+\theta^F q(\theta^F)]}r<0. \tag{9.2}$$

Now, by differentiating Eq. (3.5) and using the fact that $w_L^F=(1-\beta)\,w_U^F+\beta\,(y^F+\gamma\theta^F)$, we obtain:

$$\frac{\partial L^I}{\partial w_U^F}=\frac{r+\delta+\frac{\partial[\theta^F q(\theta^F)]}{\partial\theta^F}\frac{\partial\theta^F}{\partial w_U^F}\left[w_L^F-\frac{F'(L^I)}{r}\right]+\theta^F q(\theta^F)\left[1-\beta+\beta\gamma\frac{\partial\theta^F}{\partial w_U^F}\right]}{F''(L^I)[r+\delta+\theta^F q(\theta^F)]}r$$

Therefore,

$$\operatorname{sgn}\frac{\partial L^I}{\partial w_U^F}=-\operatorname{sgn}\left\{r+\delta+\frac{\partial[\theta^F q(\theta^F)]}{\partial\theta^F}\frac{\partial\theta^F}{\partial w_U^F}\left[w_L^F-\frac{F'(L^I)}{r}\right]+\theta^F q(\theta^F)\left[1-\beta+\beta\gamma\frac{\partial\theta^F}{\partial w_U^F}\right]\right\}$$

As a result, if Eq. (4.1) holds, then

$$\frac{\partial L^I}{\partial w_U^F}>0$$

Otherwise, the sign of $\frac{\partial L^I}{\partial w_U^F}$ is indeterminate.

Effect of w_U^F on w_L^I

Using Eq. (2.5), we have

$$\frac{\partial w_L^I}{\partial w_U^F}=\frac{\partial w_L^I}{\partial L^I}\frac{\partial L^I}{\partial w_U^F}$$

which implies that

$$\operatorname{sgn}\left[\frac{\partial w_L^I}{\partial w_U^F}\right]=-\operatorname{sgn}\left[\frac{\partial L^I}{\partial w_U^F}\right]$$

Effect of w_U^F on L^F

$$L^F(\theta^F,L^I)=\frac{\theta^F q(\theta^F)}{\delta+\theta^F q(\theta^F)}(N-L^I)$$

By differentiating Eq. (3.1), we have:

$$\frac{\partial L^F}{\partial w_U^F}=\frac{\partial[\theta^F q(\theta^F)]}{\partial\theta^F}\frac{\partial\theta^F}{\partial w_U^F}\frac{\delta}{[\delta+\theta^F q(\theta^F)]^2}(N-L^I)-\frac{\theta^F q(\theta^F)}{\delta+\theta^F q(\theta^F)}\frac{\partial L^I}{\partial w_U^F}.$$

Using Eq. (9.1), we have the following. If Eq. (4.1) holds, then

$$\frac{\partial L^F}{\partial w_U^F}<0.$$

Otherwise the sign of $\frac{\partial L^F}{\partial w_U^F}$ is indeterminate.

Effect of w_U^F on U^F

Differentiating Eq. (2.1) leads to:

$$\frac{\partial U^F}{\partial w_U^F}=\frac{\partial L^F}{\partial w_U^F}-\frac{\partial L^I}{\partial w_U^F} \tag{9.3}$$

Thus the impact on U^F is indeterminate.

Effect of w_U^F on w_L^F

Differentiating Eq. (2.18) yields:

$$\frac{\partial w_L^F}{\partial w_U^F}=1-\beta+\beta\gamma\frac{\partial\theta^F}{\partial w_U^F}$$

which is ambiguous because of Eq. (9.1). □

Proof of Proposition 2. Effect of i on θ^F

Differentiating Eq. (3.4) yields:

$$\frac{\partial\theta^F}{\partial\gamma}=\frac{\beta\theta^F q(\theta^F)+r+\delta}{(1-\beta)(y^F-w_U^F)q'(\theta^F)-\beta\gamma\frac{\partial[\theta^F q(\theta^F)]}{\partial\theta^F}}<0.$$

Effect of γ on L^I

Differentiating Eq. (3.5) yields:

$$\frac{\partial L^I}{\partial\theta^F}=\frac{\frac{\partial[\theta^F q(\theta^F)]}{\partial\theta^F}\left[w_L^F-\frac{F'(L^I)}{r}\right]+\beta\gamma\theta^F q(\theta^F)}{F''(L^I)[r+\delta+\theta^F q(\theta^F)]}r<0.$$

Again, differentiating Eq. (3.5) leads to

$$\frac{\partial L^I}{\partial\gamma}=\frac{\frac{\partial[\theta^F q(\theta^F)]}{\partial\theta^F}\frac{\partial\theta^F}{\partial\gamma}\left[w_L^F-\frac{F'(L^I)}{r}\right]+\beta\theta^F q(\theta^F)\left[\theta^F+\gamma\frac{\partial\theta^F}{\partial\gamma}\right]}{F''(L^I)[r+\delta+\theta^F q(\theta^F)]}r.$$

Thus

$$\text{sgn}\frac{\partial L^{\text{I}}}{\partial \gamma} = -\text{sgn}\left\{\frac{\partial[\theta^{\text{F}}q(\theta^{\text{F}})]}{\partial\theta^{\text{F}}}\frac{\partial\theta^{\text{F}}}{\partial\gamma}\left[w_{\text{L}}^{\text{F}} - \frac{F'(L^{\text{I}})}{r}\right] + \beta\theta^{\text{F}}q(\theta^{\text{F}})\left[\theta^{\text{F}} + \gamma\frac{\partial\theta^{\text{F}}}{\partial\gamma}\right]\right\}$$

Under which condition

$$\frac{\partial[\theta^{\text{F}}q(\theta^{F})]}{\partial\theta^{\text{F}}}\frac{\partial\theta^{\text{F}}}{\partial\gamma}\frac{1}{\theta^{\text{F}}}\left[w_{\text{L}}^{\text{F}} - \frac{F'(L^{\text{I}})}{r}\right] + \beta\theta^{\text{F}}q(\theta^{\text{F}})\left[1 + \frac{\partial\theta^{\text{F}}}{\partial\gamma}\frac{\gamma}{\theta^{\text{F}}}\right] < 0.$$

Define $\eta \equiv -\frac{\partial\theta^{\text{F}}}{\partial\gamma}\frac{\gamma}{\theta^{\text{F}}} > 0$. Then this inequality is equivalent to:

$$\frac{\partial[\theta^{\text{F}}q(\theta^{\text{F}})]}{\partial\theta^{\text{F}}}\frac{\partial\theta^{\text{F}}}{\partial\gamma}\frac{1}{\theta^{\text{F}}}\left[w_{\text{L}}^{\text{F}} - \frac{F'(L^{\text{I}})}{r}\right] + \beta\theta^{\text{F}}q(\theta^{\text{F}})(1-\eta) < 0.$$

Therefore, if $\eta > 1$, then

$$\frac{\partial L^{\text{I}}}{\partial\gamma} > 0.$$

If $\eta < 1$, the sign of $\frac{\partial L^{\text{I}}}{\partial\gamma}$ becomes indeterminate.

Effect of γ on w_{L}^{I}

Using Eq. (2.5), we have

$$\frac{\partial w_{\text{L}}^{\text{I}}}{\partial\gamma} = \frac{\partial w_{\text{L}}^{\text{I}}}{\partial L^{\text{I}}}\frac{\partial L^{\text{I}}}{\partial\gamma}$$

which implies that

$$\text{sgn}\left[\frac{\partial w_{\text{L}}^{\text{I}}}{\partial\gamma}\right] = -\text{sgn}\left[\frac{\partial L^{\text{I}}}{\partial\gamma}\right]$$

Effect of γ on L^{F}

Differentiating Eq. (3.1) yields:

$$\frac{\partial L^{\text{F}}}{\partial\gamma} = \frac{\partial[\theta^{\text{F}}q(\theta^{\text{F}})]}{\partial\theta^{\text{F}}}\frac{\partial\theta^{\text{F}}}{\partial\gamma}\delta(N - L^{\text{I}}) - \frac{\theta^{\text{F}}q(\theta^{\text{F}})}{\delta + \theta^{\text{F}}q(\theta^{\text{F}})}\frac{\partial L^{\text{I}}}{\partial\gamma}.$$

Again, if $\eta > 1$, then $\frac{\partial L^{\text{I}}}{\partial\gamma} > 0$, and thus since $\frac{\partial\theta^{\text{F}}}{\partial\gamma} < 0$, we have

$$\frac{\partial L^{\text{F}}}{\partial\gamma} < 0$$

If $\eta < 1$, the sign of $\frac{\partial L^{\text{F}}}{\partial\gamma}$ is indeterminate.

Effect of γ on U^{F}

Differentiating Eq. (2.1) leads to:

$$\frac{\partial U^{\text{F}}}{\partial\gamma} = -\frac{\partial L^{\text{F}}}{\partial\gamma} - \frac{\partial L^{\text{I}}}{\partial\gamma}.$$

If $\eta > 1$, then $\frac{\partial L^{\text{I}}}{\partial\gamma} > 0$ and $\frac{\partial L^{\text{F}}}{\partial\gamma} < 0$, thus the impact on U^{F} is indeterminate.

Effect of γ on w_{L}^{F}

$$\frac{\partial w_{\text{L}}^{\text{F}}}{\partial\gamma} = \beta\left[\theta^{\text{F}} + \gamma\frac{\partial\theta^{\text{F}}}{\partial\gamma}\right]$$

which has an ambiguous sign. □

Proof of Proposition 3. Effect of S^{F} on θ^{F}

Differentiating Eq. (6.4) gives:

$$\frac{\partial\theta^{\text{F}}}{\partial S^{\text{F}}} = -\frac{(1-\beta)q(\theta^{\text{F}})}{(1-\beta)(y^{\text{F}} - w_{\text{u}}^{\text{F}} - S^{\text{F}})q'(\theta^{\text{F}}) - \beta\gamma\frac{\partial[\theta^{\text{F}}q(\theta^{\text{F}})]}{\partial\theta^{\text{F}}}} > 0.$$

Effect of S^{F} on L^{I}

Differentiating Eq. (6.5) yields:

$$\frac{\partial L^{\text{I}}}{\partial S^{\text{F}}} = \frac{\frac{\partial[\theta^{\text{F}}q(\theta^{\text{F}})]}{\partial\theta^{\text{F}}}\frac{\partial\theta^{\text{F}}}{\partial S^{\text{F}}}\left[(1-\beta)w_{\text{U}}^{\text{F}} + \beta(y^{\text{F}} + S^{\text{F}} + \gamma\theta^{\text{F}}) - \frac{F'(L^{\text{I}})}{r}\right] + \beta\theta^{\text{F}}q(\theta^{\text{F}})\left[1 + \gamma\frac{\partial\theta^{\text{F}}}{\partial S^{\text{F}}}\right]}{F''(L^{\text{I}})[r + \delta + \theta^{\text{F}}q(\theta^{\text{F}})]}r$$

Since $w_{\text{L}}^{\text{F}} = (1-\beta)\ w_{\text{U}}^{\text{F}} + \beta\ (y^{\text{F}} + S^{\text{F}} + \gamma\theta^{\text{F}})$, and by assumption $w_{\text{L}}^{\text{F}} > w^{\text{I}}$, then

$$\frac{\partial L^{\text{I}}}{\partial S^{\text{F}}} = \frac{\frac{\partial[\theta^{\text{F}}q(\theta^{\text{F}})]}{\partial\theta^{\text{F}}}\frac{\partial\theta^{\text{F}}}{\partial S^{\text{F}}}\left[w_{\text{L}}^{\text{F}} - \frac{F'(L^{\text{I}})}{r}\right] + \beta\theta^{\text{F}}q(\theta^{\text{F}})\left[1 + \gamma\frac{\partial\theta^{\text{F}}}{\partial S^{\text{F}}}\right]}{F''(L^{\text{I}})[r + \delta + \theta^{\text{F}}q(\theta^{\text{F}})]}r < 0$$

Effect of S^{F} on w_{L}^{I}

Using Eq. (2.5), we have

$$\frac{\partial w_{\text{L}}^{\text{I}}}{\partial S^{\text{F}}} = \frac{\partial w_{\text{L}}^{\text{I}}}{\partial L^{\text{I}}}\frac{\partial L^{\text{I}}}{\partial S^{\text{F}}}$$

which implies that

$$\frac{\partial w_{\text{L}}^{\text{I}}}{\partial S^{\text{F}}} > 0.$$

Effect of S^{F} on L^{F}

Differentiating Eq. (3.1) leads to:

$$\frac{\partial L^{\text{F}}}{\partial S^{\text{F}}} = \frac{\partial[\theta^{\text{F}}q(\theta^{\text{F}})]}{\partial\theta^{\text{F}}}\frac{\partial\theta^{\text{F}}}{\partial S^{\text{F}}}\delta(N - L^{\text{I}}) - \frac{\theta^{\text{F}}q(\theta^{\text{F}})}{\delta + \theta^{\text{F}}q(\theta^{\text{F}})}\frac{\partial L^{\text{I}}}{\partial S^{\text{F}}} > 0.$$

Effect of S^{F} on U^{F}

Differentiating Eq. (2.1) gives:

$$\frac{\partial U^{\text{F}}}{\partial S^{\text{F}}} = -\frac{\partial L^{\text{F}}}{\partial S^{\text{F}}} - \frac{\partial L^{\text{I}}}{\partial S^{\text{F}}}$$

which is indeterminate.

Effect of S^{F} on w_{L}^{F}

Differentiating Eq. (6.3), we have:

$$\frac{\partial w_L^F}{\partial S^F} = \beta\left(1+\gamma\frac{\partial\theta^F}{\partial S^F}\right) > 0 \quad \square$$

Proof of Proposition 4. Effect of H^F on θ^F

Differentiating Eq. (7.5) gives:

$$\frac{\partial\theta^F}{\partial H^F} = -\frac{(r+\delta)q(\theta^F)}{(1-\beta)[y^F - w_U^F + H^F(r+\delta)]q'(\theta^F) - \beta\gamma\frac{\partial[\theta^F q(\theta^F)]}{\partial\theta^F}} > 0.$$

Effect of H^F on L^I

Differentiating Eq. (3.5) yields:

$$\frac{\partial L^I}{\partial H^F} = \frac{\frac{\partial[\theta^F q(\theta^F)]}{\partial\theta^F}\frac{\partial\theta^F}{\partial H^F}\left[(1-\beta)w_U^F + \beta(y^F+\gamma\theta^F) - \frac{F'(L^I)}{r}\right] + \beta\gamma\theta^F q(\theta^F)\frac{\partial\theta^F}{\partial H^F}}{F''(L^I)[r+\delta+\theta^F q(\theta^F)]}r.$$

Since $w_L^F=(1-\beta)\, w_U^F+\beta\,(y^F+\gamma\theta^F)$, and by assumption $w_L^F > w^I$, then

$$\frac{\partial L^I}{\partial H^F} = \frac{\frac{\partial[\theta^F q(\theta^F)]}{\partial\theta^F}\frac{\partial\theta^F}{\partial H^F}\left[w_L^F - \frac{F'(L^I)}{r}\right] + \beta\theta^F q(\theta^F)\frac{\partial\theta^F}{\partial H^F}}{F''(L^I)[r+\delta+\theta^F q(\theta^F)]}r < 0$$

Effect of H^F on w_L^I

Using (2.5), we have

$$\frac{\partial w_L^I}{\partial H^F} = \frac{\partial w_L^I}{\partial L^I}\frac{\partial L^I}{\partial H^F}$$

which implies that

$$\frac{\partial w_L^I}{\partial H^F} > 0.$$

Effect of H^F on L^F

Differentiating Eq. (3.1) leads to:

$$\frac{\partial L^F}{\partial H^F} = \frac{\partial[\theta^F q(\theta^F)]}{\partial\theta^F}\frac{\partial\theta^F}{\partial H^F}\delta(N-L^I) - \frac{\theta^F q(\theta^F)}{\delta+\theta^F q(\theta^F)}\frac{\partial L^I}{\partial H^F} > 0.$$

Effect of H^F on U^F

Differentiating Eq. (2.1) gives:

$$\frac{\partial U^F}{\partial H^F} = -\frac{\partial L^F}{\partial H^F} - \frac{\partial L^I}{\partial H^F}$$

which is indeterminate.

Effect of H^F on w_L^F

Differentiating Eq. (7.4), we have:

$$\frac{\partial w_L^F}{\partial H^F} = \frac{\partial w_L^F}{\partial\theta^F}\frac{\partial\theta^F}{\partial H^F} > 0. \quad \square$$

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