



# Endogenous job destruction and job matching in cities <sup>☆</sup>

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## ABSTRACT

We propose a spatial search–matching model where both job creation and job destruction are endogenous. Workers are ex ante identical but not ex post since their jobs can be hit by a technological shock which decreases their productivity. They reside in a city, and commuting to the job center involves both pecuniary and time costs. As a result, workers with high wages are willing to live closer to jobs to save on time commuting costs. We show that, in equilibrium, there is a one-to-one correspondence between the productivity space and the urban location space. Workers with high productivities and wages reside close to jobs, have low per distance commuting costs and pay high land rents. We also show that higher per distance commuting costs and higher unemployment benefits lead to more job destruction.

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## 1. Introduction

It is commonly observed that unemployment varies both within and between regions and cities (Holzer, 1989; Blanchflower and Oswald, 1994; Overman and Puga, 2002; Patacchini and Zenou, 2007). There are also stark spatial differences in incomes. For example, in the United States, the median income of central city residents is 40 percent lower than that of suburban residents (Glaeser et al., 2008; Watson, 2006). In Europe (and in some American cities like New York), the reverse is observed since households residing in city centers tend to be richer than those living in suburbs (see e.g. Dujardin et al., 2008 and Table 1 in Brueckner et al., 1999). There is also strong evidence that the labor market is characterized by large flows and job turnover (Davis et al., 1996) and the dynamics of job flows vary across geographic space (Essletzbichler, 2004, 2007; Faberman, 2002, 2007).

The aim of this paper is to provide a search–matching model where endogenous job creation and job destruction are endogenous and the land market is explicitly modeled. We will focus on the European case, where rich households tend to live close to jobs while poorer families live in the suburbs far away from jobs.

In recent years a great deal of research has emerged examining gross job flows resulting in a much greater understanding of the dynamics of job reallocation (Davis et al., 1996). The main focus has been on how job flows vary across the firm size and age distributions and the business cycle. However, very little attention has been paid to the dynamics of job flows across geographic space. Some papers have examined whether agglomeration improves labor market coordination and facilitates greater learning and human capital formation (Duranton and Puga, 2004). Other papers have studied how geographic clustering by firms in particular industries interacts with local labor market dynamics. Using longitudinal employee–employer matched data in the U.S., Freedman (2008) explores the relationship between agglomeration among establishments and labor mobility, earnings levels, and earnings growth rates in one large and dynamic high-technology sector. The empirical analysis reveals that clustering among establishments in the software publishing industry is associated with shorter job durations and greater job-hopping among individuals within the sector. Duranton (2007) develops a theoretical framework trying to explain the observed fact that there is considerable “churning” of industries across cities. He shows that industries move across cities following endogenous cross-sector innovations. Cities experience gains and losses of industries, which partly offset each other, so that entire cities “churn” less than single industries, and urban growth goes along with industry turnover. Findeisen and Südekum (2008) test this (and other) results from Duranton (2007) for German cities. Using data on West German local industries over the

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time period 1977–2002, they find support for considerable industry churning in Germany, even after controlling for structural change.

In this paper, we take a different approach by analyzing job flows within cities using an urban search model. There is by now a small literature on the theoretical aspects of spatial search models that explicitly incorporates a land market into a dynamic labor market with job flows (see Zenou, forthcoming, 2009).<sup>1</sup> Wasmer and Zenou (2002, 2006) and Smith and Zenou (2003) introduce a land market (as in Fujita, 1989) in a standard search-matching model (as in Pissarides, 2000). They all assume that the city is monocentric, i.e. all jobs are located in one place. The interaction between land and labor markets is realized through search intensity. Indeed, in their models, distance to jobs affects search intensity because remote workers are less likely to obtain good information about job opportunities. As a result, the land market equilibrium depends on aggregate variables (such as wages and labor market tightness) since these variables affect location choices of workers. On the other hand, the labor market equilibrium crucially depends on the land market equilibrium configuration because the efficiency of aggregate matching depends on the average location of the unemployed. Sato (2004) extends this framework by introducing workers' heterogeneity in terms of specific training costs. It is assumed that ex post all workers have been trained and are thus identical in terms of productivities. The analysis is then relatively similar to what is obtained in the models above. Sato (2001) develops a similar framework but allows ex post heterogeneity in productivities. However, this heterogeneity in the productivity space, which implies different wages, does not translate into an heterogeneity in the urban space since, in equilibrium, whatever their location in the city, all workers experience the same costs of living and there is thus no direct correspondence between heterogeneity in the productivity space and in the urban space. Another interesting contribution is Coulson et al. (2001) who develop a search model in a duocentric city. They assume that the fixed entry cost of firms is greater in the Central Business District (CBD) than in the Suburban Business District (SBD), and that workers are heterogeneous in their disutility of transportation (or equivalently in their search costs). These two fundamental assumptions are sufficient to generate an equilibrium in which central city residents experience a higher rate of unemployment than suburban residents, while suburban firms create more jobs than central firms. The interaction between the land and labor market is due to the fact that workers and firms are mobile and look for a "partner" to form a match in both areas.

None of these approaches have, however, endogenized job destruction and workers' ex post heterogeneity. The aim of the present paper is to investigate these issues by proposing a spatial version of the search-matching model developed by Mortensen and Pissarides (1994). By endogenizing job destruction, we obtain new results as compared to the urban search models mentioned above.

More precisely, following Mortensen and Pissarides (1994), we assume that each job is characterized by a fixed irreversible technology, so that each time a technological shock occurs, the quality of the match and the resulting productivity deteriorate. New filled jobs start at the highest productivity value and a job is destroyed only if the idiosyncratic component of their productivity falls below some critical value. Workers reside and work in the city and commute to the employment center. Both pecuniary and time costs

are taken into account. Workers with the highest wages (productivity) have the steepest bid-rent curves and reside the closest to the employment center. The unemployed workers have the lowest value of time and live at the edge of the city. This urban pattern has thus the structure of European cities where richer workers live closer to job centers. These properties of the model are standard features of urban economic models in which the heterogeneity of workers leads to differences in the steepness of bid rent functions. What is less standard is that such differences are not related to structural worker characteristics (such as education, age or family composition) but to productivity levels that are only related to the job to which a worker is randomly matched. Each change in the productivity of the job immediately leads to a change in his/her wage, value of time and through these to his/her residential location. This mechanism creates a relationship between productivity and location that is central to the analysis of the model.

We first show that, in equilibrium, there is a one-to-one correspondence between the productivity space and the urban location space since high-productivity workers bid away low-productivity workers in order to occupy central locations. We then characterize the labor-market equilibrium. Wages are set according to a bilateral bargaining between workers and firms and we show that wages depend on labor market variables such as the unemployment benefit or the cost of maintaining a vacancy but also on spatial variables such as the commuting cost. In fact, as compared to the non-spatial model, there is a new element here. Firms need to compensate the employed workers for the difference in transport cost between employed and unemployed workers in order for them to accept a job offer. We also determine the job creation and destruction conditions, which depend on the commuting cost. We then characterize the urban land-use equilibrium. The difficulty here is that ex post all workers are heterogeneous in terms of productivities and thus in terms of wages. Since the latter are the opportunity cost of leisure, we show that higher productivity and higher wage workers locate closer to jobs than those with lower productivities and wages. We can finally calculate the equilibrium utilities of all workers as well as the land rent at each location in the city. The interaction between the land and labor market is realized through wages because, as stated above, firms need to compensate workers for their spatial costs.

By running numerical simulations, we are able to highlight the main differences between the spatial (i.e. big cities) and the non-spatial (i.e. small cities and rural areas) model. We show that the unemployment rate is higher while the vacancy rate and the reservation productivity are lower in bigger cities. There are indeed more spatial frictions in bigger cities since workers are differentiated both in the productivity and urban spaces. An interesting feature of the model is that firms keep some currently unprofitable jobs occupied because of the possibility that a job productivity might change, which would enable firms to start production at the new productivity immediately after arrival, without having to pay the recruitment cost and forgo production during search. This feature is present in both models but is much more important in the spatial model. Another interesting difference is that, in the spatial model, each labor variable has an impact on both markets. Consider, for example, the unemployment benefit. An increase in the latter increases the wage, which, in turn, affects the time cost of traveling. This increases the competition in the land market since the access to the job center becomes more valuable, which, in turn, increases the wage since firms need to compensate more for spatial costs to induce workers to take a job. These amplifying effects leads to a higher unemployment rate and a higher reservation productivity but also to a lower labor market tightness in the spatial model.

As compared to the urban search models mentioned above, we have the following new results/features:

<sup>1</sup> It has been recognized for a long time that space affects search activities. For example, Stigler (1961) and Diamond (1982) have put geographical dispersion as one of the main determinants of price ignorance because the spatial dispersion of agents creates more frictions and thus more unemployment. See also Simpson (1992) for an early approach of the analysis of urban labor markets.

- (i) Bigger cities experience higher job creation and destruction than smaller cities.
- (ii) Higher commuting costs and higher unemployment benefits lead to more job destruction.
- (iii) Firms can keep some currently unprofitable jobs occupied because of the possibility that a job productivity might change in the future.

The rest of the paper is organized as follows. Section 2 presents the model. In Sections 3 and 4, we characterize the urban land-use and the labor equilibrium, respectively. The steady-state equilibrium, which solves simultaneously for the urban land use and the labor market equilibrium, is analyzed in Section 5. In Section 6, we present different numerical results. Finally, Section 7 concludes. All proofs are given in Appendix A.

**2. The model**

There is a continuum of ex ante identical workers whose mass is  $N$  and a continuum of ex ante identical firms. Among the  $N$  workers, there are  $L$  employed and  $U$  unemployed so that  $N = L + U$ . The workers are uniformly distributed along a *linear, closed and monocentric* city. We assume housing consumption to be fixed and normalized to 1. The density of workers at each location is taken to be 1. There is no vacant land in the city and all land is owned by absentee landlords. All firms are exogenously located in the Central Business District (CBD hereafter) and consume no space. Workers are assumed to be risk neutral, to live infinitely, and decide on their optimal place of residence between the CBD and the city fringe.

Let us describe the labor market, which follows Mortensen and Pissarides (1994). We assume that each job is characterized by a *fixed irreversible technology*. To be more precise, the value of a match is given by  $y + \epsilon$ , where  $y$  denotes the general productivity and  $\epsilon$  is a parameter that is drawn from a distribution  $G(\epsilon)$ , which has a finite support  $[\underline{\epsilon}, \bar{\epsilon}]$  and no mass point. To be consistent with the way we model the *linear* city, we assume a uniform distribution, so that  $G(\epsilon) = (\epsilon - \underline{\epsilon})/(\bar{\epsilon} - \underline{\epsilon})$ .<sup>2</sup> It is assumed that a job is created with the highest productivity value,  $y + \bar{\epsilon}$ , capturing the idea that firms with new filled jobs use the best available technology. Then, at rate  $\delta$ , a new value of productivity  $\epsilon$  is drawn from  $G(\epsilon)$  independently of the current value of  $\epsilon$ . Given that a new match starts with the highest productivity, its productivity can always decline with time. A job is destroyed only if the idiosyncratic component of its productivity falls below some critical number  $\bar{\epsilon} < \bar{\epsilon}$ . As a result, the rate at which jobs are destroyed is given by  $\delta G(\bar{\epsilon}) = \delta(\bar{\epsilon} - \underline{\epsilon})/(\bar{\epsilon} - \underline{\epsilon})$ . Because there will be a distribution of ex post productivities  $y + \epsilon$ , there will also be a distribution of wages  $w_L(\epsilon)$ .

A firm is a unit of production that can either be filled by a type- $\epsilon$  worker whose production is  $y + \epsilon$  units of output or be unfilled and thus unproductive. In order to find a worker, a firm posts a vacancy. A vacancy can be filled according to a random Poisson process. Similarly, workers searching for a job will find one according to a random Poisson process. In aggregate, these processes imply there to be a number of contacts per unit of time between the two sides of the market that are determined by the following matching function<sup>3</sup>:

$$M(U, V)$$

where  $U$  and  $V$  are the total number of unemployed workers and vacancies, respectively. As in the standard search-matching model (see e.g. Mortensen and Pissarides, 1999 and Pissarides, 2000), we assume that  $M(\cdot)$  is increasing in both its arguments, concave and homogeneous of degree 1 (or, equivalently, has constant return to scale). Thus, the rate at which vacancies are filled is  $M(U, V)/V$ . By constant returns to scale, it can be rewritten as

$$M(1/\theta, 1) \equiv q(\theta)$$

where  $\theta = V/(U)$  is the *labor market tightness* and  $q(\theta)$  is a Poisson intensity. By using the properties of  $M(\cdot)$ , it is easily verified that  $q'(\theta) \leq 0$ : the higher the labor market tightness, the lower the rate at which firm fill their vacancy. Similarly, the rate at which an unemployed worker leaves unemployment is

$$\frac{M(U, V)}{U} \equiv \theta q(\theta).$$

Again, by using the properties of  $M(\cdot)$ , it is easily verified that  $[\theta q(\theta)]' \geq 0$ : the higher the labor market tightness, the higher the rate at which workers leave unemployment since there are relatively more jobs than unemployed workers.

A steady-state equilibrium requires solving *simultaneously* an urban land use equilibrium and a labor market equilibrium. It is convenient to present first the former and then the latter.

**3. Urban land-use equilibrium**

To generate a location pattern by productivity type, as in Brueckner et al. (2002), we introduce a key assumption that links commuting costs to the wage paid. In a more general model, this link is achieved through a labor-leisure choice, which implies that a unit of commuting time is valued at the wage rate (see, for example, Fujita, 1989, Chapter 2). However, such a model is cumbersome to analyze, and it is likely not to yield additional insights beyond those available from our simpler approach. This one is based on a particular formulation of the labor-leisure choice, which is consistent with the empirical literature that shows that the time cost of commuting increases with the wage (see, e.g. Small, 1992 and Glaeser et al., 2008).

We assume that each worker consumes one unit of land in the city in which he/she lives, while providing a fixed amount of labor time  $T$ . With land consumption fixed, the worker's utility solely depends on the quantity of a consumption good  $c$  and on the time  $\Delta$  available for leisure.

In a model with labor-leisure choice, work hours are adjusted until the marginal value of leisure time equals the wage rate. While the explicit incorporation of flexible work hours would seriously complicate the model, we incorporate the spirit of this standard result by assuming that utility can be approximately by a function that values leisure time at the wage rate. In particular, we assume that workers consume a composite good  $z$  defined as follows:

$$z = c + w_L \Delta. \tag{1}$$

The marginal utility of  $\Delta$  is thus equal to the worker's wage, as would be the case if work hours were adjusted optimally in a more general setting. The worker purchases the good  $c$  produced and sold at the CBD and incurs  $\tau x$  in monetary commuting costs when he/she lives at distance  $x$  from the CBD. Letting  $R(x)$  denote rent per unit of land at distance  $x$ , the budget constraint of a type- $\epsilon$  worker at distance  $x$  can be written as follows:

$$w_L(\epsilon)T = c + R(x) + \tau x \tag{2}$$

where  $T$ , the amount of working hours, is assumed to be the same and constant across workers. Furthermore, commuting time from

<sup>2</sup> None of our results in the labor-market equilibrium are affected by this assumption.

<sup>3</sup> This matching function is written under the assumption that the city is monocentric, i.e. all firms are located in one fixed location.

distance  $x$  is equal to  $tx$ , where  $t > 0$  is time spent per unit of distance. Hence, the time constraint of a type- $\epsilon$  worker at distance  $x$  is given by

$$1 = T + \Lambda + tx \quad (3)$$

in which the total amount of time is normalized to 1 without loss of generality.

Substituting (2) and (3) in (1), the *instantaneous* (indirect) utility of a type- $\epsilon$  worker at distance  $x$ , denoted by  $W_L(\epsilon)$ , is then given by

$$\begin{aligned} W_L(\epsilon) &\equiv z_L = w_L(\epsilon)T - R(x) - \tau x + w_L(\epsilon)(1 - T - tx) \\ &= w_L(\epsilon)(1 - tx) - \tau x - R(x). \end{aligned} \quad (4)$$

Therefore, the time cost of commuting for a type- $\epsilon$  worker residing at a distance  $x$  from the CBD is  $tw_L(\epsilon)x$ ; in accordance with empirical observation, it increases with the income  $w_L(\epsilon)T$ . As usual,  $w_L(\epsilon)$  in (4) does not stand for the worker's actual income (which is equal to  $w_L(\epsilon)T < w_L(\epsilon)$ ) but for the income that would accrue to an individual working all the time ( $T = 1$ ). Rearranging (4) yields:

$$\Psi_L(\epsilon, x, W_L(\epsilon)) = w_L(\epsilon)(1 - tx) - \tau x - W_L(\epsilon) \quad (5)$$

which is the bid rent of a type- $\epsilon$  employed worker residing at a distance  $x$  from the CBD.<sup>4</sup>

Inspection of (5) shows that, as usual, the bid-rent function is decreasing in  $x$ , with  $\partial\Psi_L/\partial x < 0$ . In the present model, this reflects the combined influence of the time cost of commuting and the monetary cost. Since  $w'(\epsilon) \geq 0$  (see below), further inspection shows that, at a given  $x$ , an increase in  $\epsilon$  makes the bid-rent slope more negative ( $\partial^2\Psi_L/\partial\epsilon\partial x < 0$ ). This means that high- $\epsilon$  workers have steeper bid-rent curves than low- $\epsilon$  workers. The intuitive reason is that *an extra mile of commuting reduces income more for a high- $\epsilon$  worker than for a low- $\epsilon$  worker*, a consequence of the higher net wage. Therefore, the high- $\epsilon$  worker requires a larger decline in land rent than a high- $\epsilon$  worker to maintain a given utility level. In comparing the residential locations of two groups, it is well known that the group with the steeper bid-rent curve locates closer to the CBD (see, for example, Fujita, 1989, Chapter 2). In the present model, this implies that *high- $\epsilon$  workers locate closer to the CBD than low- $\epsilon$  workers*.<sup>5</sup>

Observe, however, that the result that the most productive workers live closest to jobs is a direct consequence of the *fixed lot sizes assumption*. If we had instead assumed variable consumption of land, then the equilibrium locations of workers would also depend on the income elasticity of land demand relative to the income elasticity of commuting costs (Wheaton, 1974; Brueckner, 1987; Fujita, 1989). In that case, the American urban pattern where high-productive workers live far away from employment centers could emerge. This analysis is, however, very cumbersome and only a model with a Cobb-Douglas utility function is tractable (Smith and Zenou, 2003). Moreover, the gist of the analysis would certainly be preserved in this more general setting since most results would hold. For these different reasons and

because we want to focus on the standard European urban pattern (or cities like New York), we keep the assumption of fixed lot sizes.

Let us now focus on the unemployed workers. Their budget constraint is given by

$$w_U = z + R(x) + \tau x \quad (6)$$

where  $w_U$  is the unemployment benefit. It is assumed that the unemployed workers commute as often to the CBD as the employed since they go there to search for jobs and to be interviewed.<sup>6</sup> We also assume that the unemployed's opportunity cost of time is negligible since they do not work, and thus time costs do not enter in their utility function.<sup>7</sup> As a result, and to be consistent with the utility function of the employed workers, the *instantaneous* (indirect) utility of an unemployed worker residing at distance  $x$  is

$$W_U \equiv z_U = w_U - R(x) - \tau x$$

while his/her bid rent is equal to:

$$\Psi_U(x, W_U) = w_U - \tau x - W_U. \quad (7)$$

Since the wage is never equal to zero, any employed worker (whatever his/her  $\epsilon$ -type) will have a steeper bid rent than an unemployed worker. To formalize this notion, we introduce the definition of residential equilibrium:

**Definition 1.** A land-use urban equilibrium consists of the following mapping  $\epsilon(x)$  that assigns a worker of skill type  $\epsilon$  to a location  $x$ , i.e.

$$\epsilon(x) = \bar{\epsilon} - \left( \frac{\bar{\epsilon} - \tilde{\epsilon}}{L} \right) x \quad \text{for } x \leq L \quad (8)$$

a set of utility levels  $W_L^*(\epsilon)$  and  $W_U$ , and a land rent curve  $R^*(x)$  such that:

(i) at each  $x \in [0, L]$ ,

$$R^*(x) = \Psi_L(\epsilon(x), x, W_L^*(\epsilon)) = \max_{\epsilon} \Psi_L(\epsilon, x, W_L^*(\epsilon)), \quad (9)$$

(ii)

$$\Psi_L(L, W_L(\tilde{\epsilon})) = \Psi_U(L, W_U), \quad (10)$$

(iii)

$$\Psi_U(N) = 0. \quad (11)$$

To define the urban equilibrium, we need to calculate the assignment rule between the location space defined on  $[0, L]$  and the productivity space defined on  $[\tilde{\epsilon}, \bar{\epsilon}]$ , since, in equilibrium, only individuals with productivity higher or equals than  $\tilde{\epsilon}$  will work. Because we know that the relationship between these two spaces is negative and because of the linearity assumption of both the distribution  $x$  of workers and the distribution  $\epsilon$  of productivities, this relationship has to be linear. So we have to find a linear and negative relationship between  $\epsilon$  and  $x$  so that  $\epsilon(0) = \bar{\epsilon}$  and  $\epsilon(L) = \tilde{\epsilon}$ .

<sup>4</sup> The bid rent is a standard concept in urban economics. It indicates the maximum land rent that a worker located at a distance  $x$  from the CBD is ready to pay to achieve a utility level. See Fujita (1989) for a formal definition.

<sup>5</sup> Observe that, in the real world, people with different levels of income may use different ways of commuting. For example, it is likely that high-income families prefer using cars, whereas low-income families use public transportation. We could introduce two modes of transportation but this will complicate an already quite complicated model without yielding new results. In particular, the location of workers in the city would be the same since high-income workers would still have the highest opportunity cost of time.

<sup>6</sup> It is easily verified that assuming less trips for the unemployed would not change any of our results.

<sup>7</sup> This is admittedly a simplifying assumption because the unemployed workers are assumed to commute to the CBD as well. However, because their wage (i.e. unemployment benefit) is fixed, does not vary with work hours, and is lower than any wage earned by an employed worker, they have the lowest opportunity cost of time when commuting. Hence, assuming that time costs do not enter in their utility function is just a normalization assumption.

The only equation that satisfies these requirements is given by (8). Moreover, Eq. (8) formalizes the claim above that high- $\epsilon$  workers locate closer to the CBD than low- $\epsilon$  workers. Indeed, the mapping between the physical and productivity “distances” of workers involves a correlation between these two distances. It should be noted that this result depends on the assumption of a fixed lot size and a uniform distribution of  $\epsilon$ . As it is well known, variable land consumption can overturn the present inverse association between residential distance and time cost of commuting (see, for example, Fujita, 1989, Chapter 2). With variable consumption, however, additional conditions could be imposed to guarantee that the two distances remain perfectly correlated. The same applies to a more general c.d.f.  $G(\epsilon)$ .

Let us interpret the other equations. Eq. (9) says that the land rent  $\Psi(x)$  at location  $x$  equals the maximum of the bid rents across productivity types, and that the productivity type offering the highest bid at  $x$  is occupying location  $x$ . Furthermore, at distance  $x = L$ , the bid rents between the employed with the lowest productivity, i.e.  $\bar{\epsilon}$ , and the unemployed worker has to be equal to ensure the continuity of the land rent in the city. This condition is given by (10). Finally, because of our assumptions of a fixed lot size and no vacant land, the land rent at the edge of the city (distance  $N$ ) is undetermined. Since the value of this constant does not affect our results, we say in Eq. (11) that it equals the opportunity cost of land, which is assumed to be zero without loss of generality.

It is important to observe that richer and more productive workers live closer to the job center but this does not necessarily imply that they live close to the city-center. What matters here is the distance to jobs. For example, in a very decentralized city, rich workers will live far away from the city center but relatively close to jobs, if they are located in the suburbs.

We cannot solve for the urban land-use equilibrium before knowing the exact value of the wage. So let us determine the labor-market equilibrium.

#### 4. Labor-market equilibrium

We first need to write the different lifetime expected utilities of workers and firms. The steady-state Bellman equations for unemployed and employed workers are given by

$$rI_U = w_U - \tau x - R(x) + \theta q(\theta)[I_L(\bar{\epsilon}) - I_U], \tag{12}$$

$$rI_L(\epsilon) = w_L(\epsilon)(1 - tx) - \tau x - R(x) + \delta \int_{\bar{\epsilon}}^{\bar{\epsilon}} I_L(s) dG(s) + \delta G(\bar{\epsilon})I_U - \delta I_L(\epsilon). \tag{13}$$

Eq. (12) states that an unemployed worker, who enjoys an instantaneous utility  $w_U - \tau x - R(x)$  today, can find a job at rate  $\theta q(\theta)$  and, in that case, will start a job at the highest productivity level  $y + \bar{\epsilon}$  and thus obtains an expected utility  $I_L(\bar{\epsilon})$  with the highest wage level  $w_L(\bar{\epsilon})$ . Eq. (13) has a similar interpretation. An employed worker, who is employed at a certain productivity level  $\epsilon$ , obtains an instantaneous utility given by (4), i.e.  $w_L(\epsilon)(1 - tx) - \tau x - R(x)$ , but can then be “hit” by a productivity shock at rate  $\delta$  and will continue to work in that job only if the match is still productive, i.e. the productivity is at least equal to  $y + \bar{\epsilon}$ . If not, that is if the productivity of the match is less than  $y + \bar{\epsilon}$ , which happens with probability  $G(\bar{\epsilon}) = \Pr[\epsilon \leq \bar{\epsilon}]$ , the worker becomes unemployed and loses  $I_L(\epsilon) - I_U$ . With our uniform distribution assumption, (13) can be written as:

$$rI_L(\epsilon) = w_L(\epsilon)(1 - tx) - \tau x - R(x) + \frac{\delta}{(\bar{\epsilon} - \bar{\epsilon})} \int_{\bar{\epsilon}}^{\bar{\epsilon}} I_L(s) ds + \delta \left( \frac{\bar{\epsilon} - \epsilon}{\bar{\epsilon} - \bar{\epsilon}} \right) I_U - \delta I_L(\epsilon). \tag{14}$$

For firms, with a filled job and a vacancy, we have respectively:

$$rI_F(\epsilon) = y + \epsilon - w_L(\epsilon) + \frac{\delta}{(\bar{\epsilon} - \bar{\epsilon})} \int_{\bar{\epsilon}}^{\bar{\epsilon}} I_F(s) ds + \delta \left( \frac{\bar{\epsilon} - \epsilon}{\bar{\epsilon} - \bar{\epsilon}} \right) I_V - \delta I_F(\epsilon), \tag{15}$$

$$rI_V = -c + q(\theta)[I_F(\bar{\epsilon}) - I_V] \tag{16}$$

where  $c$  is the cost of a vacancy. The interpretation of these equations are similar to that of (12) and (13).

##### 4.1. Free-entry condition and labor demand

We assume that firms post vacancies up to a point where:

$$I_V = 0 \tag{17}$$

which is a free-entry condition. From (16), the value of a new job is equal to:

$$I_F(\bar{\epsilon}) = \frac{c}{q(\theta)}. \tag{18}$$

Now using (17), (15) can be written as:

$$I_F(\epsilon) = \frac{y + \epsilon - w_L(\epsilon)}{(r + \delta)} + \frac{\delta}{(\bar{\epsilon} - \bar{\epsilon})(r + \delta)} \int_{\bar{\epsilon}}^{\bar{\epsilon}} I_F(s) ds. \tag{19}$$

Evaluating this last equation at  $\epsilon = \bar{\epsilon}$  and using (18) leads to the following decreasing relationship between labor market tightness and wages:

$$\frac{c(r + \delta)}{q(\theta)} = y + \bar{\epsilon} - w_L(\bar{\epsilon}) + \frac{\delta}{(\bar{\epsilon} - \bar{\epsilon})} \int_{\bar{\epsilon}}^{\bar{\epsilon}} I_F(s) ds. \tag{20}$$

##### 4.2. Wage determination

In each period, the total intertemporal surplus is shared through a generalized Nash-bargaining process between the firm and the worker. As a result, in each period, the wage is determined by

$$w_L = \arg \max_{w_L} [I_L(\epsilon) - I_U]^\beta [I_F(\epsilon) - I_V]^{1-\beta} \tag{21}$$

where  $0 \leq \beta \leq 1$  is the bargaining power of workers. Observe that  $I_U$ , the threat point for the worker does not depend on the current location of the worker, who will relocate if there is a transition in his/her employment status. The result of the bargaining is given by

$$(1 - \beta)[I_L(\epsilon) - I_U] = \beta I_F(\epsilon). \tag{22}$$

We have the following result.

**Lemma 1.** Assume

$$(1 - \beta)tL < \frac{\beta(\bar{\epsilon} - \bar{\epsilon})}{(1 - \beta)w_U + \beta(y + c\theta + \bar{\epsilon})}. \tag{23}$$

Then the wage  $w_L(\epsilon)$  is always strictly positive, given by

$$w_L(\epsilon) = \frac{(1-\beta)w_U + \beta(y + \epsilon + c\theta)}{1 - (1-\beta)tL(\bar{\epsilon} - \epsilon)/(\bar{\epsilon} - \bar{\epsilon})} \quad (24)$$

and increasing in  $\epsilon$ .

First, condition (23) guarantees that  $1 - (1-\beta)t\alpha > 0, \forall \alpha \in [0, L]$  so that the wage is always strictly positive in the area of the city where the employed workers live, i.e. in  $[0, L]$ . This condition also guarantees that  $w'(\epsilon) > 0$ . Second, the wage (24) is a generalization of the non-spatial wage obtained by Mortensen and Pissarides (1994) (see, for example, Eq. (2.10) page 42 in Pissarides (2000), which is (24) when  $t = 0$ ). As in the standard search-matching model, we have the same positive effects of  $w_U, y, c$  and  $\theta$  on the wage since they all increase the outside option of workers. The new element here is the spatial aspect of the negotiation where the employer has to take into account the location  $\alpha$  of workers. As in Wasmer and Zenou (2002), firms must compensate workers for the difference in transportation costs between employed and unemployed workers for them to accept the job offer. However, the main difference with Wasmer and Zenou is that here workers are ex post heterogeneous in terms of productivity  $\epsilon$  and, because of the assignment rule (8), this translates into heterogeneity in terms of distance to jobs  $\alpha$ . So the compensation is more complex and this leads to a non-linear relationship between  $w$  and  $\epsilon$ .

Third, as compared to the non-spatial model, the range of the distribution  $\bar{\epsilon} - \bar{\epsilon}$  does here affect the wage. Indeed, in the non-spatial model, wages depend on the general productivity  $y$  but not on the specific productivities  $\epsilon$  of other workers. Here, because of the spatial compensation, the highest and lowest specific productivity has an impact on the wage of every worker, whatever his/her own specific productivity. Fourth, because employed and unemployed workers have the same monetary commuting costs, only time cost (i.e.  $t$ ) affects the wage. Not surprisingly, the higher is  $t$ , the higher is the wage since firms need to compensate more workers to induce them to accept the job offer. Naturally, this interpretation (like the ones above) holds for a given  $\theta$  and a given  $L$  and are not necessarily true in general equilibrium since  $\theta$  and  $L$  will, themselves, be affected by  $t$ . Finally, the wage obtained in (24) is independent of the uniform distribution assumption of  $G(\epsilon)$ .

#### 4.3. Endogenous job creation and job destruction

To determine the job-acceptance rule, we have to solve the following equations

$$I_F(\bar{\epsilon}) = 0,$$

$$I_L(\bar{\epsilon}) - I_U = 0.$$

These two equations are equivalent by the sharing rule (22) so we only solve the first one. We show in Appendix A that, by solving this equation and using (18), the job-creation condition can be written as:

$$\frac{c}{q(\theta)} = \left( \frac{1-\beta}{r+\delta} \right) \times \frac{(\bar{\epsilon} - \bar{\epsilon})(1-tL) + [(1-\beta)w_U + \beta(y + \bar{\epsilon} + c\theta)]tL}{[1 - (1-\beta)tL]} \quad (25)$$

while the job-destruction condition is given by

$$(y + \bar{\epsilon})(1-tL) - w_U - \frac{\beta c}{1-\beta} \theta + \frac{\delta(1-tL)}{2(r+\delta)} (\bar{\epsilon} - \bar{\epsilon}) = 0. \quad (26)$$

The job-creation equation comes from the zero-profit condition while the job-destruction condition stems from the workers' arbitrage between working and being unemployed. Eq. (25) states that

the firm's expected gain from a new job is equal to the expected hiring cost. In the non-spatial case, when  $t = 0$ , we obtain

$$\frac{c}{q(\theta)} = \left( \frac{1-\beta}{r+\delta} \right) (\bar{\epsilon} - \bar{\epsilon}) \quad (27)$$

which is Eq. (2.14) page 43 in Pissarides (2000). First, in the non-spatial case (see (27)), the relationship between the reservation productivity  $\bar{\epsilon}$  and the labor market tightness  $\theta$  is negative because, for higher  $\bar{\epsilon}$ , the expected lifetime of a job is shorter since, during any short interval of time  $dt$ , the job is destroyed at rate  $\delta G(\bar{\epsilon}) dt$ . As a result, firms create fewer jobs, which leads to a fall in  $\theta$ . This is not anymore true in the spatial model since, by differentiating (25), we obtain:

$$\frac{\partial \theta}{\partial \bar{\epsilon}} \geq 0 \Leftrightarrow (1-\beta)tL \geq \frac{\eta(\theta)(r+\delta)}{\theta q(\theta)\beta + \eta(\theta)(r+\delta)} \quad (28)$$

where

$$\eta(\theta) = -\frac{q'(\theta)}{q(\theta)} \theta > 0$$

is the elasticity of  $q(\theta)$ . Indeed, when  $\bar{\epsilon}$  increases, jobs last shorter but workers need to be compensated less. The net effect is thus ambiguous. If the spatial compensation effect is not too strong, i.e. if  $t$  is not too high, one can see from (28) that the relationship between  $\theta$  and  $\bar{\epsilon}$  is negative, as in the non-spatial model. This is what we assume now. Second, the impact of the discount rate  $r$  or the mismatch rate  $\delta$  is similar to that of the non-spatial model since higher  $r$  or  $\delta$  decreases job creation because the future returns from new jobs are discounted at a higher rate. Third, contrary to the non-spatial model, the general productivity  $y$ , the cost of maintaining a vacancy  $c$  as well as the unemployment benefit  $w_U$  enter in the equation and thus affect the relationship between  $\bar{\epsilon}$  and  $\theta$ . This is due to the spatial aspect of wages, which implies that the firm's expected revenues and costs are not anymore proportional to  $y$ . Finally, the spatial variable  $t$  does affect the relationship between  $\bar{\epsilon}$  and  $\theta$ . If  $t$  increases, then firms will create less jobs because wages need to be raised to spatially compensate workers. Observe that all our comments were made at a given  $L = (1-u)N$ . In general equilibrium, there will be additional effects since  $u$  will also be affected by  $\theta$  and  $\bar{\epsilon}$  (see below).

Let us now comment on the job-destruction equation (26) but comparing it to the non-spatial case, which equation is given by

$$y + \bar{\epsilon} - w_U - \frac{\beta c}{1-\beta} \theta + \frac{\delta}{2(r+\delta)} \frac{(\bar{\epsilon} - \bar{\epsilon})^2}{(\bar{\epsilon} - \bar{\epsilon})} = 0 \quad (29)$$

and corresponds to Eq. (2.15) page 44 (for the uniform distribution) in Pissarides (2000). Contrary to the two job-creation equations, (26) and (29) are quite similar. This is mainly because job creation is directly affected by wages while job destruction is not. First, as in the non-spatial model, there is a positive relationship between  $\theta$  and  $\bar{\epsilon}$  because, at higher  $\theta$ , the worker's outside opportunities are better, wages are thus higher, and more marginal jobs are destroyed, which increases  $\bar{\epsilon}$ . Second, all the variables except  $t$  enter in a similar way and have the same effects on the relationship between  $\theta$  and  $\bar{\epsilon}$  as in the non-spatial model. Finally, the new element is  $t$ , the commuting cost per unit of distance. At given  $\theta$  and  $L$ , it is easy to verify that an increase in  $t$  increases  $\bar{\epsilon}$ , and thus more marginal jobs are destroyed. Indeed, when  $t$  is higher, net wages increase and thus the value of employment at each  $\epsilon$ , i.e.  $I_L(\epsilon)$ , increases. As a result, since the value of unemployment  $I_U$  is not affected,  $\bar{\epsilon}$  has to increase to satisfy the condition  $I_L(\bar{\epsilon}) = I_U$ .

4.4. Unemployment rate

The number of workers entering unemployment is  $\delta G(\bar{\epsilon})(1 - u)N$  and the number leaving unemployment is  $\theta q(\theta)uN$ . The evolution of unemployment is thus given by the difference between these two flows,

$$\dot{u} = \delta G(\bar{\epsilon})(1 - u) - \theta q(\theta)u \tag{30}$$

where  $\dot{u}$  is the variation in unemployment with respect to time. In steady state, the rate of unemployment is constant and thus, these two flows are equal. We have:

$$u^* = \frac{\delta G(\bar{\epsilon})}{\delta G(\bar{\epsilon}) + \theta q(\theta)} = \frac{\delta(\bar{\epsilon} - \underline{\epsilon})}{\delta(\bar{\epsilon} - \underline{\epsilon}) + (\bar{\epsilon} - \underline{\epsilon})\theta q(\theta)}. \tag{31}$$

This equation is exactly the same as in the non-spatial model. All spatial variables will affect the unemployment rate only indirectly through both job creation  $\theta$  (see (25)) and job destruction  $\bar{\epsilon}$  (see (26)).

5. Steady-state equilibrium

The steady-state equilibrium is a 7-tuple  $(\theta^*, \bar{\epsilon}^*, u^*, R_L^*(x), W_L^*(\epsilon), W_U^*, L^*)$  such that Eqs. (25), (26), (31), (9), (10), (11), and

$$L^* = (1 - u^*)N \tag{32}$$

are satisfied. We can in fact solve separately the urban and the labor equilibrium. We have a first result:

**Proposition 1.** Assume (23) and

$$tN < 1. \tag{33}$$

Then, there exists a unique residential equilibrium characterized by the assignment rule (8). The equilibrium instantaneous utilities of employed workers of type  $\epsilon$  are given by

$$W_L^*(\epsilon) = \frac{\beta^2(\bar{\epsilon} - \bar{\epsilon})}{(1 - \beta)^2 tL} \log \left[ \frac{\bar{\epsilon} - \bar{\epsilon} - (1 - \beta)tL(\bar{\epsilon} - \epsilon)}{\bar{\epsilon} - \bar{\epsilon} - (1 - \beta)tL(\bar{\epsilon} - \bar{\epsilon})} \right] - \tau N + \frac{\beta(\epsilon - \bar{\epsilon})}{(1 - \beta)} + \frac{[(1 - \beta)w_U + \beta(y + \epsilon + c\theta)](1 - tL)}{1 - (1 - \beta)tL} \tag{34}$$

which are increasing and convex in  $\epsilon$ , while the instantaneous unemployed's utility is

$$W_U^* = w_U - s\tau N. \tag{35}$$

The equilibrium land rent in the employment zone  $R_L^*(x)$ , i.e. for all  $x \in [0, L]$ , is equal to

$$R_L^*(x) = \frac{[(1 - \beta)w_U + \beta(y + \bar{\epsilon}(1 - 1/L) + x\bar{\epsilon}/L + c\theta)](1 - tx)}{1 - (1 - \beta)tx} - \frac{[(1 - \beta)w_U + \beta(y + \bar{\epsilon} + c\theta)](1 - tL)}{1 - (1 - \beta)tL} + \tau(N - x) + \frac{\beta(\bar{\epsilon} - \bar{\epsilon})}{(1 - \beta)L} \left[ \frac{\beta}{(1 - \beta)t} \log \left[ \frac{1 - (1 - \beta)tx}{1 - (1 - \beta)tL} \right] - 1 + x \right] \tag{36}$$

and is decreasing and convex in  $x$ . In the unemployment zone, i.e. for all  $x \in [L, N]$ , the equilibrium land rent  $R_U^*(x)$  is given by

$$R_U^*(x) = \tau(N - x). \tag{37}$$

This result establishes that there is a unique urban land-use equilibrium and gives the exact values of all the endogenous variables as a function of the exogenous parameters and the endogenous variables in the labor market, i.e.  $\theta^*, \bar{\epsilon}^*, u^*$ . Condition (33) is a natural requirement because it says that commute time from the edge of the city, which equals  $tN$ , is less than the total time available (unity). As show in Appendix A, condition (33) ensures that, at each location  $x$ , the bid rent function of a type  $\epsilon(x)$  worker is maximal. Condition (33) also guarantees that the utility function (34) is an increasing function of  $\epsilon$ . Observe that the way the labor-market variables  $\theta^*, \bar{\epsilon}^*, u^*$  affect the spatial variables  $W_L^*(\epsilon), W_U^*$  and  $R^*(x)$  are through the wage setting. Indeed, as we have seen above, the wage (24) is affected by  $\theta^*, \bar{\epsilon}^*, u^*$ , but it also affects the land market through the bid rent. Indeed, workers with higher wages want to reduce their commuting time, and thus out-bid workers with lower wages at the outskirts of the city. Concerning the labor market, we have three unknowns  $\theta^*, \bar{\epsilon}^*, u^*$ , and three Eqs. (25), (26), and (31). By imposing (28), we obtain a decreasing relationship between  $\theta$  and  $\bar{\epsilon}$  in (25). Then, using similar arguments as in Mortensen and Pissarides (1994), one can show that the steady-state equilibrium exists and is unique. It is now interesting to analyze the interaction between the land and the labor market. Because the model is quite cumbersome, we turn to numerical simulations.

6. Numerical simulations

We run some numerical simulations to obtain reasonable values of the unemployment rate and the job creation rate. We will, in particular, focus our discussion on the differences between the spatial and the non-spatial model. As it is usual, we use the following Cobb-Douglas function for the matching function:

$$M(uN, V) = (uN)^{0.5}(vN)^{0.5}$$

where  $v$  is the vacancy rate. This implies that  $q(\theta) = \theta^{-0.5}$ ,  $\theta q(\theta) = \theta^{0.5}$  and, the elasticity of the matching rate (defined as  $\eta(\theta) = -q'(\theta)\theta/q(\theta)$ ) is equal to 0.5. The values of the parameters (in yearly terms) are the following: the output  $y$  is normalized to unity, as is the scale parameter of the matching function. The relative bargaining power of workers is equal to  $\eta(\theta)$ , i.e.  $\beta = \eta(\theta) = 0.5$ . Unemployment benefits have a value of 2 and the costs of maintaining a vacancy  $c$  are equal to 2 per unit of time while the general productivity is 4. The uniform distribution of  $\epsilon$  is defined on the support  $[0, 2]$ , which implies that new jobs have a productivity equal to  $y + \bar{\epsilon} = 6$  while for jobs with the lowest productivity, it is  $y + \underline{\epsilon} = 4$ . Pecuniary commuting costs  $\tau$  are equal to 0.1 whereas time cost are set to 0.07. The discount rate is  $r = 0.05$ , whereas the mismatch rate is  $\delta = 0.15$ , which means that, on average, there is a technological shock every six and half years. Finally, the total population is normalized to 1. Table 1 summarizes these different values.

6.1. Steady-state equilibrium

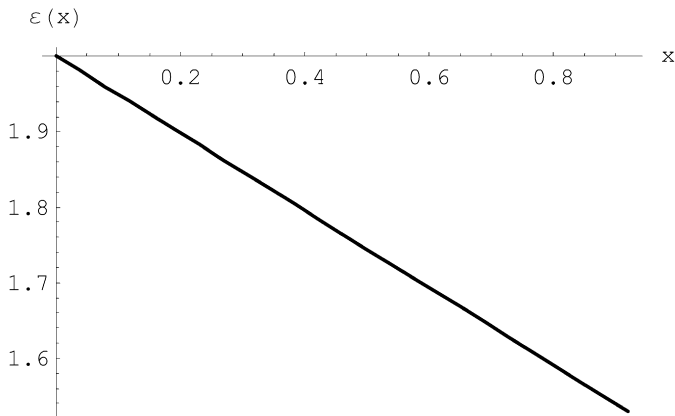
Let us calculate the steady-state equilibrium for the non-spatial ( $t = 0$ ) and the spatial models ( $t > 0, \tau > 0$ ) using the parameter values given in Table 1. These comparisons are about how big cities,

**Table 1**  
Parameter values.

$y = 4$ General productivity	$r = 0.01$ Pure discount rate
$\delta = 0.15$ Job-specific shock arrival rate	$\beta = 0.5$ Workers' share of surplus
$\epsilon = 0, \bar{\epsilon} = 2$ Parameters of the distribution	$\eta(\theta) = 0.5$ Search elasticity of matching
$N = 1$ Total population	$t = 0.07$ Time commuting cost
$w_U = 2$ Unemployment benefit	$\tau = 0.1$ Pecuniary commuting cost
$c = 2$ Cost of a vacant job	

**Table 2**  
Steady-state equilibrium.

	Non-spatial model ( $t = 0$ )	Spatial model ( $t > 0, \tau > 0$ )
$u^*$ (%)	6.07	8.11
$\theta^*$	1.77	1.69
$v^*$ (%)	10.77	13.71
$\tilde{\epsilon}^*$	1.15	1.53
$\epsilon^*(x)$	–	$2 - 0.51x$
$w_L^*(\epsilon)$	$4.69 + 0.5\epsilon$	$(4.69 + 0.5\epsilon)/(0.86 - 0.068\epsilon)$
$I_F(\epsilon)$	$-3.59 + 3.13\epsilon$	$-19.96 + 6.25\epsilon + 146.83/(12.60 + \epsilon)$
$W_L^*(\epsilon)$	$4.69 + 0.5\epsilon$	$2.95 + 1.48\epsilon + 7.30 \log[0.89 + 0.07\epsilon]$
$W_U^*$	2	1.95



**Fig. 1.** Equilibrium assignment rule.

for which  $t$  and  $\tau$  are large, differ from *smaller cities* (or even rural areas), for which  $t$  and  $\tau$  are very small (obviously they do not literally have  $t = 0$ ). The numerical results of these two equilibria are displayed in Table 2.

We find that in big cities, the unemployment rate  $u^*$ , the job-creation rate (i.e. vacancy rate  $v^*$ ) and the job destruction rate (i.e. the reservation productivity  $\tilde{\epsilon}^*$ ) are higher than in small cities where spatial frictions are negligible. As stated in the introduction, there is little evidence on the spatial aspects of job flows. For the United States, Essletzbichler (2004) only looks at *manufacturing employment* and finds that, both between 1967 and 1982 and between 1982 and 1997, rates of job creation were higher in rural areas, while rural job destruction rates were lower than for urban areas. Faberman (2007) also looks at this issue using data from the Business Employment Dynamics (BED) program of the U.S. Bureau of Labor Statistics (BLS). This data are longitudinally linked administrative records for all establishments covered by state unemployment insurance programs. Faberman (2007) uses a sample of all private-sector establishments in 53 Metropolitan Statistical Areas (MSAs) and Primary Metropolitan Statistical Areas (MSAs) across five U.S. states. As in our model, Faberman (2007) find that big cities are characterized by higher unemployment rates, higher wages and more commuting. Concerning the job destruction and job creation rates, the picture is less clear. Flows in the labor market are important in both types of cities but no clear pattern emerges. It would be very interesting to properly test econometrically the relationship between city size or commuting costs and job flows.

In our model, it is because the urban space through commuting costs and land rent creates additional frictions in the labor market that job flows are more important in bigger cities. Let us understand more closely these differences from a theoretical viewpoint.

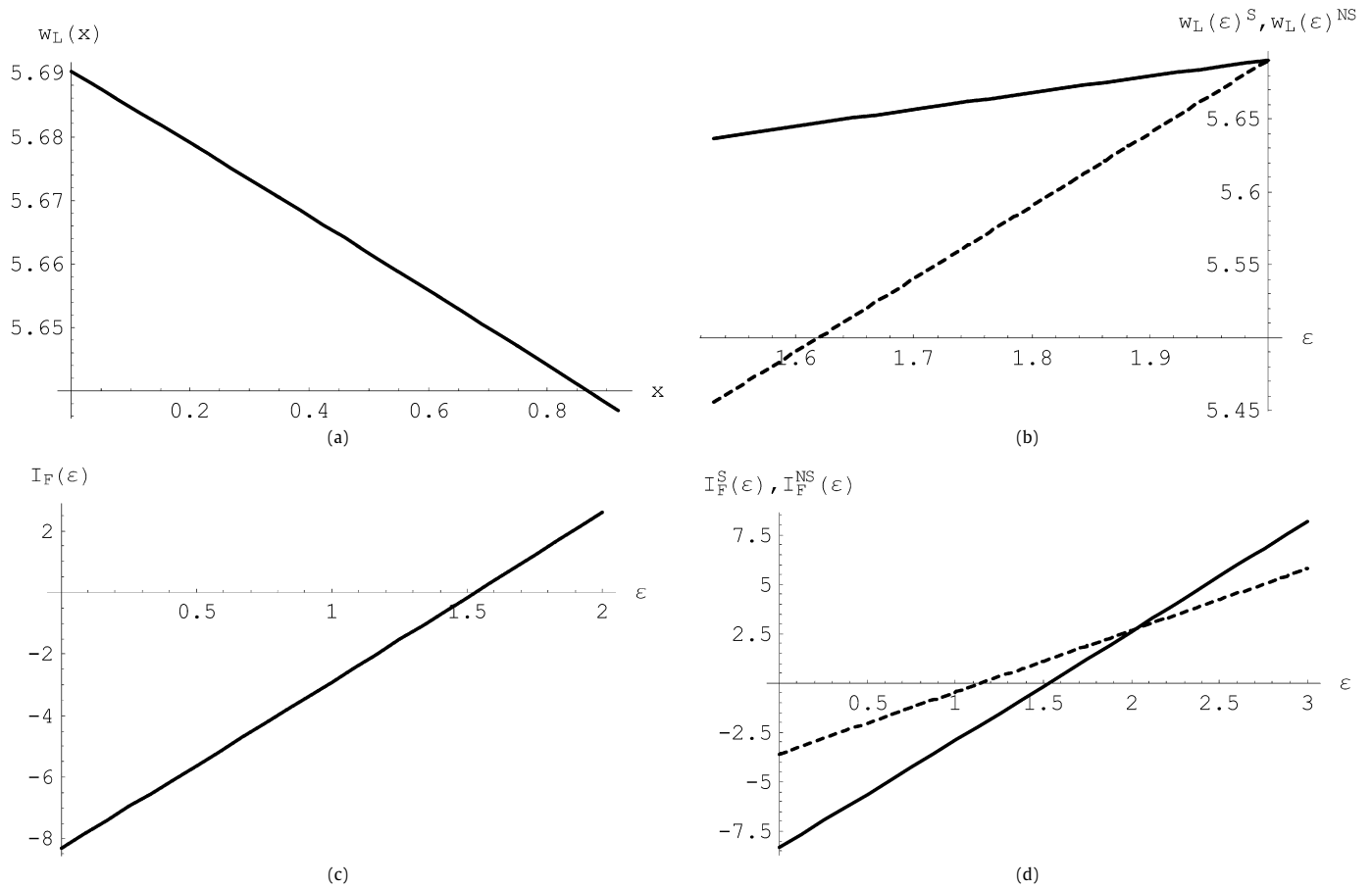
Fig. 1 displays the assignment rule  $\epsilon^*(x) = 2 - 0.51x$ , which establishes a perfect negative correlation between the urban and the productivity space. This provides evidence of the double differentiation of workers in both the productivity and urban spaces.

To better understand the differences between the spatial and the non-spatial model, let us examine the wage setting. In the non-spatial model, the wage, given by  $4.69 + 0.5\epsilon$ , varies between  $w_L^*(\tilde{\epsilon}^*) = 5.27$  and  $w_L^*(\bar{\epsilon}) = 5.69$  while the total productivity varies between  $y + \tilde{\epsilon}^* = 5.15$  and  $y + \bar{\epsilon} = 6$ . This means that firms keep some currently unprofitable jobs occupied. In fact, it is easily verified that all jobs with a productivity  $\epsilon \leq 1.38$  are not currently profitable because  $y + \epsilon \leq w_L^*(\epsilon)$  while job with a  $\epsilon \in ]1.38, 2]$  are profitable. As observed by Pissarides (2000), this means that some firms keep some currently unprofitable jobs occupied because of the possibility that a job productivity might change, which would enable firms to start production at the new productivity immediately after arrival, without having to pay the recruitment cost (which is here quite high since  $c = 2$ ) and forgo production during search. This is confirmed by looking at  $I_F(\epsilon)$ , the lifetime expected utility of a firm with a filled job, which is calculated using (43), and given by  $-3.59 + 3.13\epsilon$ . It is easily checked that  $I_F(\epsilon)$  is always strictly positive for all  $\epsilon \in [\tilde{\epsilon}^*, \bar{\epsilon}]$ . So even if jobs with  $\epsilon \in [1.15, 1.38[$  are not currently profitable, firms keep them because their lifetime expected utility  $I_F(\epsilon)$  are strictly positive in this interval. This is even more true in the spatial model. Indeed, because firms need to compensate for spatial costs, wages are always higher but productivity is the same, and it is easily verified that, in our example, *all jobs* are currently unprofitable since  $w_L^*(\epsilon) > y + \epsilon, \forall \epsilon \in [\tilde{\epsilon}^*, \bar{\epsilon}]$ . However, in the same interval, it is also easily checked that the lifetime expected utility  $I_F(\epsilon)$  is strictly positive. Fig. 2a displays the wage distribution in the urban space, showing that workers living further away have a lower wage because they also have a lower productivity. Fig. 2b confirms what we said above. The spatial wage  $w_L^{*S}(\epsilon)$  (solid curve) is always higher than the non-spatial one  $w_L^{*NS}(\epsilon)$  (dotted curve) because of the need for firms to spatially compensate workers for commuting and land costs. Fig. 2c shows that, in the spatial model,  $I_F(\epsilon)$  is always strictly positive for any  $\epsilon > \tilde{\epsilon}^*$  and  $I_F(\tilde{\epsilon}^*) = 0$ . Finally, Fig. 2d compares the two expected lifetime utilities  $I_F(\epsilon)$  (solid and dotted curves for the spatial and non-spatial, respectively) and shows that it is always higher in the spatial model for  $\epsilon \in [\tilde{\epsilon}^*, \bar{\epsilon}]$  because wages are much lower. To summarize, when comparing the spatial and the non-spatial model, we find that, because spatial wages are higher and there are more frictions since workers are differentiated both in the productivity and urban spaces, the unemployment rate as well as the reservation productivity are higher and job creation lower.

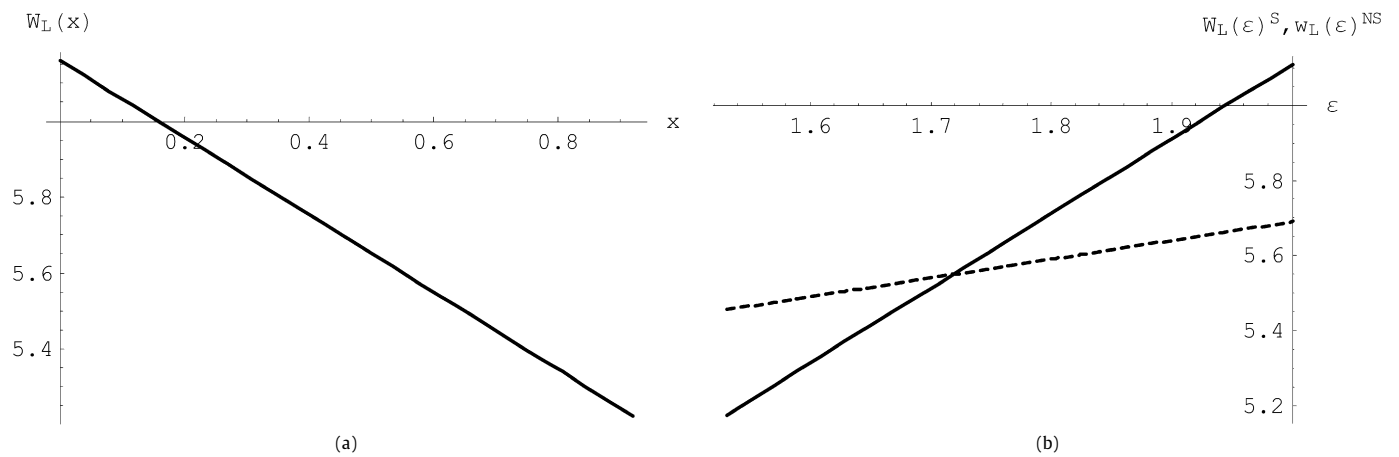
Let us now focus on the land market. Inspection of Fig. 3a indicates that the instantaneous utility of the employed workers is decreasing with distance to jobs because workers who live further away have a lower productivity and earn a lower wage. Fig. 3b compares the equilibrium instantaneous utilities in the spatial model (solid curve), given by  $W_L^*(\epsilon)$ , and in the non-spatial model (dotted curve), given by  $w_L^*(\epsilon)$ . It is not clear which utility is higher because, on the one hand, spatial wages are higher but workers incur commuting and land costs. In Fig. 3b, one can see that for low values of  $\epsilon$ , the non-spatial utility is higher while we have the reverse for high productivity values. This is quite intuitive because when  $\epsilon$  is low, wage differences are not too high but spatial costs exist, and thus the non-spatial utility has a higher value. For high  $\epsilon$ , we have the reverse because the wage difference is sufficiently high to compensate for the spatial cost difference. This is because the spatial wage (24) is non-linear in  $\epsilon$ . Finally, in Fig. 4, we have plotted the land rent for type- $\tilde{\epsilon}$  workers. It shows that the land rent decreases with distance to jobs.

## 6.2. Comparative statics

We would like to pursue our analysis of the interaction between land and labor markets by analyzing the impact of the key spatial



**Fig. 2.** (a) Equilibrium wage distribution in the urban space. (b) Equilibrium wage distribution for the spatial and non-spatial models in the productivity space. (c) Equilibrium distribution of lifetime expected utility of firms with filled jobs in the productivity space. (d) Comparison of lifetime expected utilities of firms with filled jobs between the spatial and non-spatial models.



**Fig. 3.** (a) Equilibrium instantaneous utility distribution of the employed workers in the urban space. (b) Comparison of equilibrium instantaneous utilities between the spatial and non-spatial models.

variable, the commuting cost  $t$ , on the equilibrium labor market variables,  $u^*$ ,  $\tilde{\epsilon}^*$  and  $\theta^*$ . Since  $t$  captures both the quality of the transport system and the size of the city, we will be able to interpret our results in both directions. From a theoretical viewpoint, the effects are complex since  $t$  directly affects the land market through the land rent and the instantaneous utilities  $W_L(\epsilon)$  and  $W_U$ , but also indirectly affects the labor market through the wage. Figs. 5a, 5b and 5c display the comparative statics results of the

impact of an increase in  $t$  on  $u^*$ ,  $\tilde{\epsilon}^*$  and  $\theta^*$ .<sup>8</sup> The relationships between  $u^*$  and  $t$  and between  $\tilde{\epsilon}^*$  and  $t$  are clearly increasing while the one between  $\theta^*$  and  $t$  is decreasing. This should not be a sur-

<sup>8</sup> For robustness checks, we have performed these comparative statics exercises for lower and higher values of unemployment benefits and productivity as well as for lower and higher range of the support of the distribution of the  $\epsilon$ . The simulations results are quantitatively unchanged. These results are available upon request.

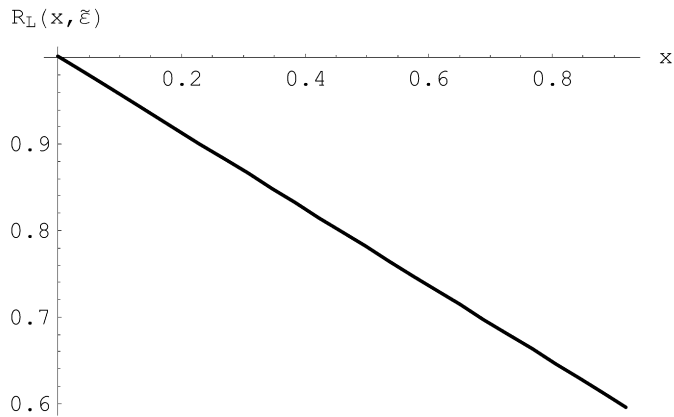


Fig. 4. Equilibrium land rent of the employed workers of type  $\tilde{\epsilon}$ .

prise since higher commuting costs lead to an increase in wages (firms need to compensate more workers for their spatial costs), which makes traveling to the job center even more costly. As a result, land rents increase and firms must compensate even more workers. Therefore, firms enter less the labor market, thus creating less jobs, which decreases  $\theta^*$ . Also, since wages have increased,  $I_L(\epsilon)$ , the lifetime expected value of employment, increases for each job  $\epsilon$ , which implies that  $\tilde{\epsilon}^*$  must increase for the condition  $I_L(\tilde{\epsilon}^*) = I_U$  to be satisfied. Because more marginal jobs are destroyed and less jobs are created, the equilibrium unemployment  $u^*$  is reduced following an increase in  $t$ .

One can also notice that the equilibrium values are quite sensitive to a variation in the commuting cost  $t$ . Indeed, inspection of Figs. 5a, 5b, and 5c indicates that an increase in the commuting cost from  $t = 0$  to  $t = 0.15$  leads to both an increase in the unemployment rate from 6.07% to 10.59%, an increase in the reservation productivity from 1.15 to 1.995, and a decrease in the job-creation rate from 1.77 to 1.6 (which corresponds to an increase in the vacancy rate from 10.77% to 16.91%). These are important effects due to the fact that the interaction between land and labor markets tends to amplify the effect of one variable on the other. To illustrate this point, let us consider an increase in the unemployment benefit  $w_U$  on  $u^*$ ,  $\tilde{\epsilon}^*$  and  $\theta^*$  both in the spatial and non-spatial models. In both models, we obtain without surprise that an increase in  $w_U$  leads to an increase in  $u^*$  and  $\tilde{\epsilon}^*$  and a decrease in  $\theta^*$ . What is interesting is the difference in the values of the endogenous variables. Table 3 reports these values for  $w_U = 0, 1, 2, 3, 4$ .

It is easily seen that the values of the endogenous variables are much higher in the spatial model (big cities) than in the non-spatial one (small cities) because of amplifying effects due to the interaction between the land and labor market. Indeed, when  $w_U = 0$ , the unemployment rate is 4.12% and the vacancy rate is 11.20% in the non-spatial model while, in the spatial model, these figures are 5.58% and 15.17%, respectively. When  $w_U = 4$ , the differences are even larger, with for example 3 percent more unemployment in the spatial model. This is because, in the spatial model, each labor variable, like for example  $w_U$ , has an impact on both markets. Indeed, the unemployment benefit directly affects the wage since, at given  $\theta$  and  $L$ , higher  $w_U$  implies higher wages, which, in turn, affects the time cost of traveling. This increases the competition in the land market since the access to the job center becomes more valuable, which, in turn, increases the wage since firms need to compensate more for spatial costs in order to induce workers to take a job. These amplifying effects lead to higher unemployment rate and reservation productivity and lower labor market tightness in the spatial model.

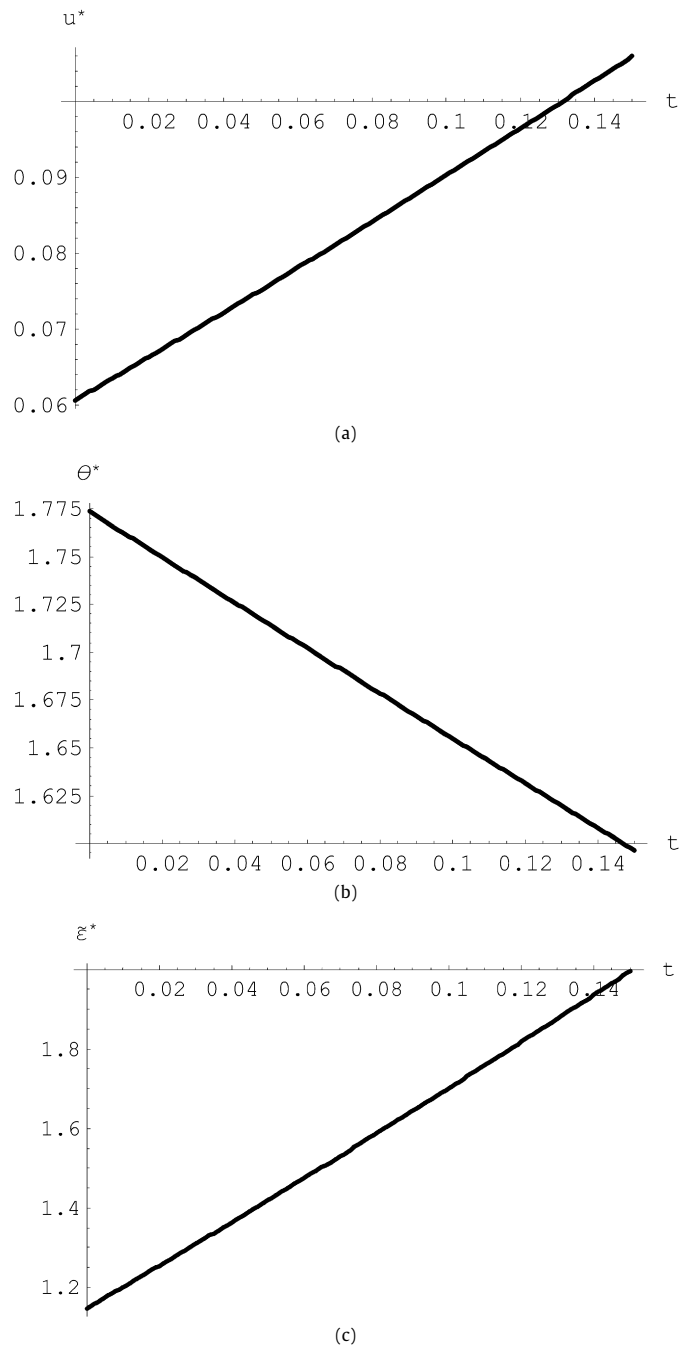


Fig. 5. (a) Impact of the per distance commuting cost on the equilibrium unemployment. (b) Impact of the per distance commuting cost on the equilibrium labor-market tightness. (c) Impact of the per distance commuting cost on the equilibrium reservation productivity.

### 7. Conclusion

Most people live and work in cities, particularly in developed countries. Because of the complexity of the urban world, understanding its problems requires the application of many economic subdisciplines. In the present paper, we focus on one of them, namely the labor market. We embed a search-matching model a la Mortensen–Pissarides into a land market a la Alonso–Fujita. To the best of our knowledge, this is the first paper that brings together a search matching model with endogenous job destruction and a land market in a unified framework. We characterize the general equilibrium of this economy. In the urban space, workers

**Table 3**  
Variation of the unemployment benefit  $w_U$ .

	Non-spatial model ( $t = 0$ )				Spatial model ( $t > 0, \tau > 0$ )			
	$u^*$ (%)	$\theta^*$	$v^*$ (%)	$\bar{\epsilon}^*$	$u^*$ (%)	$\theta^*$	$v^*$ (%)	$\bar{\epsilon}^*$
$w_U = 0$	4.12	2.72	11.20	0.94	5.58	2.63	15.17	1.32
$w_U = 1$	4.95	2.25	11.12	1.04	6.76	2.16	14.61	1.42
$w_U = 2$	6.07	1.77	10.77	1.15	8.11	1.69	13.71	1.53
$w_U = 3$	7.69	1.31	10.04	1.27	10.07	1.22	12.33	1.65
$w_U = 4$	10.34	0.84	8.72	1.41	13.32	0.77	10.21	1.79

with high productivities and wages reside close to jobs, have low commuting costs but pay high land rents. In the productivity space, ex ante identical workers are ex post heterogeneous and a wage distribution is obtained. In this respect, workers are heterogeneous in both the urban and the productivity space. We show that, in equilibrium, there is a perfect negative correlation between these two spaces since high-productivity workers are also those who occupy locations close to jobs. We also show that in the bargaining process, there is a spatial element in the wage setting since firms need to compensate workers for their spatial costs. As compared to the non-spatial model, the unemployment rate and the reservation productivity are lower and the job-creation rate is higher in the non-spatial model because the urban space through commuting costs and land rent create additional frictions in the labor market.

Our model provides some mechanisms of why there may be more flows and mobility in the labor market in bigger than smaller cities. Indeed, we have shown that higher commuting costs lead to more job destruction. Concerning job creation, if smaller cities (or rural areas) are characterized by lower levels of productivity or at least a lower productivity gap between workers, then job creation can be higher in bigger cities even if commuting costs are also higher. This is an interesting issue and a proper econometric test of these predictions will indicate how empirically relevant are these results.

**Appendix A**

**Proof of Lemma 1.** We start the demonstration of this Lemma by using a general c.d.f.  $G(\epsilon)$ .

The sharing rule (22) implies in particular that:

$$I_L(\bar{\epsilon}) - I_U = \frac{\beta}{1 - \beta} I_F(\bar{\epsilon}). \tag{38}$$

Note that the sharing rule (22) can also be written as

$$(1 - \beta)I_L(\epsilon) - \beta I_F(\epsilon) = (1 - \beta)I_U. \tag{39}$$

Now, using (38) and (18), we have:

$$\begin{aligned} rI_U &= w_U - \tau x - R(x) + \theta q(\theta) [I_L(\bar{\epsilon}) - I_U] \\ &= w_U - \tau x - R(x) + \theta q(\theta) \frac{\beta}{1 - \beta} I_F(\bar{\epsilon}) \\ &= w_U - \tau x - R(x) + \frac{\beta}{1 - \beta} c\theta. \end{aligned} \tag{40}$$

Now multiplying the asset equation  $I_F(\epsilon)$  defined in (15) by  $\beta$ , we obtain:

$$r\beta I_F(\epsilon) = \beta [y + \epsilon - w_L(\epsilon)] + \beta \delta \int_{\bar{\epsilon}}^{\bar{\epsilon}} I_F(s) dG(s) - \delta \beta I_F(\epsilon).$$

Also, multiplying the asset equation  $I_F(\epsilon)$  defined in (14) by  $(1 - \beta)$ , we get:

$$\begin{aligned} r(1 - \beta)I_L(\epsilon) &= (1 - \beta) [w_L(\epsilon)(1 - tx) - \tau x - R(x)] \\ &\quad + (1 - \beta) \delta \int_{\bar{\epsilon}}^{\bar{\epsilon}} I_L(s) dG(s) + (1 - \beta) \delta G(\bar{\epsilon}) I_U \\ &\quad - \delta (1 - \beta) I_L(\epsilon). \end{aligned}$$

Subtracting the second equation from the first, we obtain:

$$\begin{aligned} r[(1 - \beta)I_L(\epsilon) - \beta I_F(\epsilon)] &= (1 - \beta) [w_L(\epsilon)(1 - tx) - \tau x - R(x)] - \beta [y + \epsilon - w_L(\epsilon)] \\ &\quad + (1 - \beta) \delta \int_{\bar{\epsilon}}^{\bar{\epsilon}} I_L(s) dG(s) - \beta \delta \int_{\bar{\epsilon}}^{\bar{\epsilon}} I_F(s) dG(s) \\ &\quad - \delta [(1 - \beta)I_L(\epsilon) - \beta I_F(\epsilon)] + (1 - \beta) \delta G(\bar{\epsilon}) I_U. \end{aligned}$$

Observing that

$$\begin{aligned} (1 - \beta) \delta \int_{\bar{\epsilon}}^{\bar{\epsilon}} I_L(s) dG(s) - \beta \delta \int_{\bar{\epsilon}}^{\bar{\epsilon}} I_F(s) dG(s) &= \delta \int_{\bar{\epsilon}}^{\bar{\epsilon}} [(1 - \beta)I_L(s) - \beta I_F(s)] dG(s) \end{aligned}$$

then using the sharing rule (39), we obtain:

$$\begin{aligned} r(1 - \beta)I_U &= (1 - \beta) [w_L(\epsilon)(1 - tx) - \tau x - R(x)] - \beta [y + \epsilon - w_L(\epsilon)] \\ &\quad + \delta (1 - \beta) I_U \int_{\bar{\epsilon}}^{\bar{\epsilon}} G(s) - \delta (1 - \beta) I_U + (1 - \beta) \delta G(\bar{\epsilon}) I_U. \end{aligned}$$

Observing that  $\int_{\bar{\epsilon}}^{\bar{\epsilon}} G(s) = 1 - G(\bar{\epsilon})$ , we get:

$$\begin{aligned} r(1 - \beta)I_U &= (1 - \beta) [w_L(\epsilon)(1 - tx) - \tau x - R(x)] - \beta [y + \epsilon - w_L(\epsilon)] \\ &\quad + \delta (1 - \beta) I_U [1 - G(\bar{\epsilon}) - 1 + G(\bar{\epsilon})] \end{aligned}$$

which is equivalent to:

$$\begin{aligned} r(1 - \beta)I_U &= (1 - \beta) [w_L(\epsilon)(1 - tx) - \tau x - R(x)] \\ &\quad - \beta [y + \epsilon - w_L(\epsilon)]. \end{aligned}$$

Finally, using the value of  $rI_U$  in (40), we obtain

$$\begin{aligned} (1 - \beta) [w_U - \tau x - R(x)] + \beta c\theta &= (1 - \beta) [w_L(\epsilon)(1 - tx) - \tau x - R(x)] - \beta [y + \epsilon - w_L(\epsilon)] \end{aligned}$$

which after some manipulations leads to:

$$w_L(\epsilon) = \frac{(1 - \beta)w_U + \beta(y + \epsilon + c\theta)}{1 - (1 - \beta)tx}. \tag{41}$$

Now we need to use the assignment rule given by (8), which assumes a uniform distribution in  $G(\epsilon)$ . The assignment rule (8) implies that:

$$x(\epsilon) = \left( \frac{\bar{\epsilon} - \epsilon}{\bar{\epsilon} - \bar{\epsilon}} \right) L \quad \text{for } 0 \leq x \leq L.$$

The wage is then given by

$$w_L(\epsilon) = \frac{(1 - \beta)w_U + \beta(y + \epsilon + c\theta)}{1 - (1 - \beta)tL(\bar{\epsilon} - \epsilon)/(\bar{\epsilon} - \bar{\epsilon})}$$

which is (24). For this wage to be strictly positive, it has to be that:

$$(1 - \beta)tL \frac{(\bar{\epsilon} - \epsilon)}{(\bar{\epsilon} - \bar{\epsilon})} < 1.$$

This has to be true for all  $\epsilon \in [\bar{\epsilon}, \bar{\epsilon}]$ , which means that this condition must be

$$(1 - \beta)tL < 1$$

since  $\bar{\epsilon} - \epsilon < \bar{\epsilon} - \bar{\epsilon}$ . This condition is always true if (23) holds since

$$\frac{\beta(\bar{\epsilon} - \bar{\epsilon})}{(1 - \beta)w_U + \beta(y + c\theta + \bar{\epsilon})} < 1.$$

Indeed, the latter inequality is equivalent to:

$$(1 - \beta)w_U + \beta(y + c\theta + \bar{\epsilon}) > 0$$

which is always true. Furthermore, we have:

$$w'(\epsilon) = \frac{\beta[1 - (1 - \beta)tL \frac{(\bar{\epsilon} - \epsilon)}{(\bar{\epsilon} - \bar{\epsilon})}] - [(1 - \beta)w_U + \beta(y + \epsilon + c\theta)]tL \frac{(1 - \beta)}{(\bar{\epsilon} - \bar{\epsilon})}}{[1 - (1 - \beta)t \frac{(\bar{\epsilon} - \epsilon)}{(\bar{\epsilon} - \bar{\epsilon})}]^2}$$

For  $w'(\epsilon) > 0$ , it must be that

$$\beta \left[ 1 - (1 - \beta)tL \left( \frac{\bar{\epsilon} - \epsilon}{\bar{\epsilon} - \bar{\epsilon}} \right) \right] > [(1 - \beta)w_U + \beta(y + \epsilon + c\theta)](1 - \beta) \left( \frac{tL}{\bar{\epsilon} - \bar{\epsilon}} \right)$$

which is equivalent to

$$(1 - \beta)tL < \frac{\beta(\bar{\epsilon} - \bar{\epsilon})}{(1 - \beta)w_U + \beta(y + c\theta + \bar{\epsilon})}$$

which is condition (23). □

**A.1. Determination of the job-creation and job-destruction conditions**

We derive the results for a general c.d.f.  $G(\epsilon)$  and then determine them for the uniform distribution. By plugging the wage  $w_L(\epsilon)$  given by (24) into (15) and using the fact that  $I_V = 0$ , we obtain:

$$(r + \delta)I_F(\epsilon) = \frac{(1 - \beta)(y + \epsilon)[1 - tL \frac{(\bar{\epsilon} - \epsilon)}{(\bar{\epsilon} - \bar{\epsilon})}] - (1 - \beta)w_U - \beta c\theta}{[1 - (1 - \beta)tL \frac{(\bar{\epsilon} - \epsilon)}{(\bar{\epsilon} - \bar{\epsilon})}]} + \delta \int_{\bar{\epsilon}}^{\bar{\epsilon}} I_F(s) dG(s). \tag{42}$$

First, evaluating (42) at  $\epsilon = \bar{\epsilon}$ , we obtain:

$$(r + \delta)I_F(\bar{\epsilon}) = \frac{(1 - \beta)(y + \bar{\epsilon})(1 - tL) - (1 - \beta)w_U - \beta c\theta}{[1 - (1 - \beta)tL]} + \delta \int_{\bar{\epsilon}}^{\bar{\epsilon}} I_F(s) dG(s).$$

Now, subtracting this equation from (42) and noting that  $I_F(\bar{\epsilon}) = 0$ , we get:

$$(r + \delta)I_F(\epsilon) = \frac{(1 - \beta)(y + \epsilon)[1 - tL \frac{(\bar{\epsilon} - \epsilon)}{(\bar{\epsilon} - \bar{\epsilon})}] - (1 - \beta)w_U - \beta c\theta}{[1 - (1 - \beta)tL \frac{(\bar{\epsilon} - \epsilon)}{(\bar{\epsilon} - \bar{\epsilon})}]} - \frac{(1 - \beta)(y + \bar{\epsilon})(1 - tL) - (1 - \beta)w_U - \beta c\theta}{[1 - (1 - \beta)tL]}.$$

Replacing this value into the  $I_F(s)$  in the integral in (42) yields:

$$(r + \delta)I_F(\epsilon) = \frac{(1 - \beta)(y + \epsilon)[1 - tL \frac{(\bar{\epsilon} - \epsilon)}{(\bar{\epsilon} - \bar{\epsilon})}] - (1 - \beta)w_U - \beta c\theta}{[1 - (1 - \beta)tL \frac{(\bar{\epsilon} - \epsilon)}{(\bar{\epsilon} - \bar{\epsilon})}]} + \frac{\delta}{(r + \delta)} \times \int_{\bar{\epsilon}}^{\bar{\epsilon}} \frac{(1 - \beta)(y + \epsilon)[1 - tL \frac{(\bar{\epsilon} - \epsilon)}{(\bar{\epsilon} - \bar{\epsilon})}] - (1 - \beta)w_U - \beta c\theta}{[1 - (1 - \beta)tL \frac{(\bar{\epsilon} - \epsilon)}{(\bar{\epsilon} - \bar{\epsilon})}]} dG(\epsilon) - \frac{\delta}{(r + \delta)} \int_{\bar{\epsilon}}^{\bar{\epsilon}} \frac{(1 - \beta)(y + \bar{\epsilon})(1 - tL) - (1 - \beta)w_U - \beta c\theta}{[1 - (1 - \beta)tL]} dG(\epsilon). \tag{43}$$

Let us now determine the conditions for job creation and job destruction. For job creation, we evaluate (43) at  $\epsilon = \bar{\epsilon}$  and obtain:

$$(r + \delta)I_F(v) = (1 - \beta) \frac{(y + \bar{\epsilon})\beta tL + (\bar{\epsilon} - \bar{\epsilon})(1 - tL) + [(1 - \beta)w_U + \beta c\theta]tL}{[1 - (1 - \beta)tL]}.$$

By combining this equation with (18), we obtain

$$\frac{c}{q(\theta)} = \left( \frac{1 - \beta}{r + \delta} \right) \times \frac{(y + \bar{\epsilon})\beta tL + (\bar{\epsilon} - \bar{\epsilon})(1 - tL) + [(1 - \beta)w_U + \beta c\theta]tL}{[1 - (1 - \beta)tL]}$$

which is (25). This is the job-creation condition.

Let us now determine the job-destruction condition. For that, we evaluate (43) at  $\epsilon = \bar{\epsilon}$  and using the reservation rule  $I_F(\bar{\epsilon}) = 0$ , we get:

$$(y + \bar{\epsilon})(1 - tL) - w_U - \frac{\beta c}{1 - \beta} \theta + \frac{\delta(1 - tL)}{(r + \delta)} \int_{\bar{\epsilon}}^{\bar{\epsilon}} (\epsilon - \bar{\epsilon}) dG(\epsilon) = 0. \tag{44}$$

Let us calculate this result for the case of a uniform distribution. We obtain:

$$(y + \bar{\epsilon})(1 - tL) - w_U - \frac{\beta c}{1 - \beta} \theta + \frac{\delta(1 - tL)}{2(r + \delta)} (\bar{\epsilon} - \bar{\epsilon}) = 0$$

which is (26).

**Proof of Proposition 1.** This proof follows from that of Brueckner et al. (2002).

*Existence:* We establish existence by construction. In order for

$$\Psi_L(\epsilon(x), x, W_L^*(\epsilon(x))) = \max_{\epsilon} \Psi_L(\epsilon, x, W_L^*(\epsilon)),$$

it must be true that:

$$\frac{d\Psi_L(\epsilon, x, W_L^*(\epsilon))}{d\epsilon} \Big|_{\epsilon=\epsilon(x)} = 0. \tag{45}$$

We use this equation to solve for the unknown function  $W_L^*(\epsilon)$ , which ensures that the bid rent of a type  $\epsilon(x)$  worker is maximal at location  $x$ . Using (41), the bid rent can be written as:

$$\Psi_L(\epsilon, x, W_L(\epsilon)) = w_L(\epsilon)(1 - tx) - \tau x - W_L(\epsilon) = \left[ \frac{(1 - \beta)w_U + \beta(y + \epsilon + c\theta)}{1 - (1 - \beta)tx} \right] (1 - tx) - \tau x - W_L(\epsilon).$$

Thus (45) is equivalent to:

$$w'(\epsilon)(1 - tx) - W'_L(\epsilon) = 0 \tag{46}$$

that is

$$\frac{\beta(1-tx)}{1-(1-\beta)tx} - W'_L(\epsilon) = 0. \tag{47}$$

Because we want to solve for  $W_L^*(\epsilon)$ , this equation must be rewritten in terms of  $\epsilon$ . Given our mapping (8), we have:

$$x(\epsilon) = \left(\frac{\bar{\epsilon} - \epsilon}{\bar{\epsilon} - \bar{\epsilon}}\right)L \quad \text{for } 0 \leq x \leq L$$

and thus:

$$W'_L(\epsilon) = \frac{\beta[\bar{\epsilon} - \bar{\epsilon} - tL(\bar{\epsilon} - \epsilon)]}{\bar{\epsilon} - \bar{\epsilon} - (1-\beta)tL(\bar{\epsilon} - \epsilon)}. \tag{48}$$

Eq. (48) constitutes a differential equation involving the unknown function  $W_L^*(\cdot)$ . Integrating, the solution is

$$\begin{aligned} W_L^*(\epsilon) &= \int \frac{\beta[1-tL(\frac{\bar{\epsilon}-\epsilon}{\bar{\epsilon}-\bar{\epsilon}})]}{1-(1-\beta)tL(\frac{\bar{\epsilon}-\epsilon}{\bar{\epsilon}-\bar{\epsilon}})} d\epsilon \\ &= \int \frac{\beta(\bar{\epsilon} - \bar{\epsilon}) - \beta tL(\bar{\epsilon} - \epsilon)}{\bar{\epsilon} - \bar{\epsilon} - (1-\beta)tL(\bar{\epsilon} - \epsilon)} d\epsilon \\ &= \beta(\bar{\epsilon} - \bar{\epsilon}) \int \frac{1}{\bar{\epsilon} - \bar{\epsilon} - (1-\beta)tL(\bar{\epsilon} - \epsilon)} d\epsilon \\ &\quad + \beta tL \int \frac{-(\bar{\epsilon} - \epsilon)}{\bar{\epsilon} - \bar{\epsilon} - (1-\beta)tL(\bar{\epsilon} - \epsilon)} d\epsilon \\ &= \frac{\beta(\bar{\epsilon} - \bar{\epsilon})}{(1-\beta)tL} \log[\bar{\epsilon} - \bar{\epsilon} - (1-\beta)tL(\bar{\epsilon} - \epsilon)] \\ &\quad + \beta tL \left[ -\left(\frac{\bar{\epsilon} - \epsilon}{(1-\beta)tL}\right) \right. \\ &\quad \left. - \frac{(\bar{\epsilon} - \bar{\epsilon}) \log[\bar{\epsilon} - \bar{\epsilon} - (1-\beta)tL(\bar{\epsilon} - \epsilon)]}{[(1-\beta)tL]^2} \right] + K \\ &= -\frac{\beta^2(\bar{\epsilon} - \bar{\epsilon})}{(1-\beta)^2 tL} \log[\bar{\epsilon} - \bar{\epsilon} - (1-\beta)tL(\bar{\epsilon} - \epsilon)] \\ &\quad - \frac{\beta tL(\bar{\epsilon} - \epsilon)}{(1-\beta)tL} + K \end{aligned}$$

where  $K$  is a constant of integration. We thus have:

$$\begin{aligned} W_L^*(\epsilon) &= -\frac{\beta^2(\bar{\epsilon} - \bar{\epsilon})}{(1-\beta)^2 tL} \log[\bar{\epsilon} - \bar{\epsilon} - (1-\beta)tL(\bar{\epsilon} - \epsilon)] \\ &\quad - \frac{\beta(\bar{\epsilon} - \epsilon)}{(1-\beta)} + K. \end{aligned} \tag{49}$$

To verify that the above solution indeed maximizes  $\Psi_L(\cdot)$ , we need to check that the second-order condition holds at any solution to the first-order condition (47). This condition requires that  $d^2\Psi_L/d\epsilon^2 < 0$ , which implies from (47) that  $W_L''(\epsilon) > 0$ . By differentiating (48), we obtain:

$$W_L''(\epsilon) = \frac{\beta^2(\bar{\epsilon} - \bar{\epsilon})tL}{[\bar{\epsilon} - \bar{\epsilon} - (1-\beta)tL(\bar{\epsilon} - \epsilon)]^2} > 0.$$

Let us check that  $W_L^*(\epsilon) > 0$ . Using (48), this is equivalent to:

$$\frac{\bar{\epsilon} - \bar{\epsilon} - tL(\bar{\epsilon} - \epsilon)}{\bar{\epsilon} - \bar{\epsilon} - (1-\beta)tL(\bar{\epsilon} - \epsilon)} > 0.$$

First, let us show that the numerator is positive, that is:

$$\bar{\epsilon} - \bar{\epsilon} > tL(\bar{\epsilon} - \epsilon).$$

Since the highest value that  $\epsilon$  can take is  $\bar{\epsilon}$ , a sufficient condition for this inequality to be true is  $tL < 1$ . This is always guaranteed by condition (33). Let us now show that the denominator is also positive, that is:

$$\bar{\epsilon} - \bar{\epsilon} - (1-\beta)tL(\bar{\epsilon} - \epsilon) > 0.$$

Since the highest value that  $\epsilon$  can take is  $\bar{\epsilon}$ , a sufficient condition is again  $tL < 1$ , which is always guaranteed by condition (33).

Now, substituting the value of  $W_L^*(\epsilon)$ , defined in (49), in (46), and using the assignment rule (8), we obtain the equilibrium land rent  $R_L^*(x)$  at the given  $x \in [0, L]$ , which equals:

$$\begin{aligned} R_L^*(x) &= \left[ \frac{(1-\beta)w_U + \beta(y + \bar{\epsilon} - \frac{\bar{\epsilon}}{L} + \frac{\bar{\epsilon}}{L}x + c\theta)}{1-(1-\beta)tx} \right] (1-tx) - \tau x - K \\ &\quad + \frac{\beta^2(\bar{\epsilon} - \bar{\epsilon})}{(1-\beta)^2 tL} \log[\bar{\epsilon} - \bar{\epsilon} - (1-\beta)tx(\bar{\epsilon} - \bar{\epsilon})] \\ &\quad + \frac{\beta}{(1-\beta)L} (\bar{\epsilon} - \bar{\epsilon})x. \end{aligned}$$

We can calculate the equilibrium utility of the unemployed. Indeed, by solving (11) using (7), we obtain

$$W_U^* = w_U - \tau N$$

which is Eq. (35). By plugging this value in (7), we get the equilibrium land rent  $R_U^*(x)$  at the given  $x \in [L, N]$ , that is:

$$R_U^*(x) = \tau(N - x)$$

which is (37). Then, using  $R_U^*(x)$  evaluated at  $x = L$  and  $R_L^*(x)$ , defined by (50), evaluated at  $x = L$ , Eq. (10) can be written as:

$$\begin{aligned} K &= \left[ \frac{(1-\beta)w_U + \beta(y + \bar{\epsilon} + c\theta)}{1-(1-\beta)tL} \right] (1-tL) - \tau N \\ &\quad + \frac{\beta^2(\bar{\epsilon} - \bar{\epsilon})}{(1-\beta)^2 tL} \log\{(\bar{\epsilon} - \bar{\epsilon})[1-(1-\beta)tL]\} + \frac{\beta(\bar{\epsilon} - \bar{\epsilon})}{(1-\beta)}. \end{aligned} \tag{50}$$

Substituting this value of  $K$  in (49), we obtain  $W_L^*(\epsilon)$ . Finally, by plugging this value of  $W_L^*(\epsilon)$  in (46), we obtain the city's equilibrium land-rent function  $R^*(x)$ .

By plugging the value of the constant  $K$  defined by (50) into (49) and in (50), we finally obtain:

$$\begin{aligned} W_L^*(\epsilon) &= \beta \frac{(\bar{\epsilon} - \bar{\epsilon})}{(1-\beta)tL} \log \left[ \frac{\bar{\epsilon} - \bar{\epsilon} - (1-\beta)tL(\bar{\epsilon} - \epsilon)}{\bar{\epsilon} - \bar{\epsilon} - (1-\beta)tL(\bar{\epsilon} - \bar{\epsilon})} \right] - \tau N \\ &\quad + \beta tL \frac{(\bar{\epsilon} - \epsilon/2)(1-\epsilon)}{(\bar{\epsilon} - \bar{\epsilon})} \\ &\quad + \frac{[(1-\beta)w_U + \beta(y + \epsilon + c\theta)](1-tL)}{1-(1-\beta)tL} \end{aligned}$$

and

$$\begin{aligned} R_L^*(x) &= \frac{[(1-\beta)w_U + \beta(y + \bar{\epsilon}(1-1/L) + x\bar{\epsilon}/L + c\theta)](1-tx)}{1-(1-\beta)tx} \\ &\quad - \frac{[(1-\beta)w_U + \beta(y + \bar{\epsilon} + c\theta)](1-tL)}{1-(1-\beta)tL} + \tau(N-x) \\ &\quad + \frac{\beta(\bar{\epsilon} - \bar{\epsilon})}{(1-\beta)L} \left[ \frac{\beta}{(1-\beta)t} \log \left[ \frac{1-(1-\beta)tx}{1-(1-\beta)tL} \right] - 1 + x \right] \end{aligned}$$

which are (34) and (36), respectively.

Let us now calculate  $R_L^{*'}(x)$ . For that, we use (5), that is:

$$R_L^*(x) = w_L(\epsilon)(1-tx) - \tau x - W_L(\epsilon)$$

with the assignment rule (8), i.e.  $\epsilon(x) = \bar{\epsilon} - (\frac{\bar{\epsilon}-\bar{\epsilon}}{L})x$ , that links negatively  $\epsilon$  and  $x$ ,  $\epsilon'(x) < 0$ , we have:

$$\begin{aligned} \frac{\partial R_L^*(x)}{\partial x} &= w'(\epsilon)\epsilon'(x)(1-tx) - w_L(\epsilon)t - \tau - W'_L(\epsilon)\epsilon'(x) \\ &= \epsilon'(x)[w'(\epsilon)(1-tx) - W'_L(\epsilon)] - w_L(\epsilon)t - \tau \\ &= -w_L(\epsilon)t - \tau \end{aligned}$$

which is always negative. To obtain the last equality, we use (46), which gives the value of  $W'_L(\epsilon)$ . Differentiating this expression leads to:

$$\frac{\partial R_L^*(x)}{\partial x} = -w'(\epsilon)\epsilon'(x)t \geq 0. \quad (51)$$

Since the land rent function is the upper envelope of downward-sloping bid-rent curves, we know that it must be downward sloping and convex. Convexity of  $R_L^*(x)$  is clear from inspection of (51), and we have shown that the slope is negative, i.e.  $R_L^{*'}(x) < 0$ .

**Uniqueness:** To show uniqueness of the equilibrium, suppose that productivity types  $\epsilon_0$  and  $\epsilon_1 > \epsilon_0$  reside at distances  $x_0$  and  $x_1$  where  $x_1 > x_0$ . This pattern differs from the mapping in (8). For workers of skill type  $\epsilon_0$  to reside at the close-in location  $x_0$ , they must outbid workers of type  $\epsilon_1$  for land at this location. But since the bid rent curve of type  $\epsilon_0$  is flatter than that of type  $\epsilon_1$ , it follows that type  $\epsilon_0$  will also outbid type  $\epsilon_1$  for land at the more-distant location  $x_1$ , where that type is assumed to live. This is a contradiction, and it rules out any location pattern in which productivity type and location distance are not perfectly correlated. □

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